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VECTOR MESONS WITHIN THE EFFECTIVE LAGRANGIAN APPROACH

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A B S T R A C T

We construct an effective Lagrangian which describes the non-anomalous and the anomalous interactions between the low-lying pseudoscalar mesons and their vector and axial vector partners.

By first observing that the approximate non-Abelian gauging of the Wess-Zumino-Witten action does not reproduce the purely pionic and electromagnetic low energy theorems through the vector meson dominance, we develop a systematic functional subtraction formalism to implement the vector meson dominance unambiguously. This enables us to construct the unique effective π -V and π -V-A actions which reproduce all the low energy theorems exactly without any constraints on the fundamental parameters of the model.

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1. - INTRODUCTION

In the absence of a derivation of the correct effective low energy Lagrangian from QCD, we must content ourselves with guessing an effective Lagrangian with the guidance from phenomenology, and whatever hints we can gather from QCD itself. In this pursuit, interest has recently refocussed¹⁾ on the idea that the non-linear sigma model of Goldstone pions, by identifying its soliton sectors with the nucleons, may be a good candidate to be the effective Lagrangian for QCD for the pion-nucleon sector²⁾. The aim of the present work is to complete the picture by including the other known low-lying hadrons, vector and axial vector mesons, into this scheme. This, in principle, leads to a complete low energy description of the strong interactions in accord with the general principles deduced from QCD³⁾. It is worth noting that the identification of baryons with the solitons of the pseudoscalar sector is more natural in the presence of vector mesons; because stability of the solitons is guaranteed by the vector mesons⁴⁾, without further need for higher order terms which do not seem to have a natural explanation in the current algebra.

Our work is organized as follows. In Section 2 we introduce the vector and axial-vector mesons by identifying them with the gauge bosons of a minimally broken gauged $SU(2) \times SU(2) \times U(1)$ pion theory. The anomalous interactions of pions with the vector and axial vector mesons are worked in detail. In Section 3 the vector meson dominance principle is introduced. First, by observing that the naive effective action obtained in Section 2 does not reproduce the electromagnetic low energy theorems, we developed a systematic functional subtraction scheme to remove the spurious local vertices introduced by the propagation of the vector mesons. This, in turn, enables us to construct the unique low energy π -V and π -V-A actions which reproduce all the low energy theorems exactly. Finally, in the light of our results we comment on the question of extracting experimentally testable constraints on the fundamental parameters of the scheme from the anomalous electromagnetic low energy theorems.

2. - NAIVE CONSTRUCTION OF EFFECTIVE LAGRANGIAN

Our starting point is the effective Lagrangian

$$\begin{aligned} \mathcal{L}_0 &= \frac{f_\pi^2}{4} \text{Tr} (\partial_\mu U^\dagger \partial_\mu U) + \dots \\ U &= \exp \frac{i}{f_\pi} \vec{\tau} \cdot \vec{\pi} \end{aligned} \quad f_\pi \simeq 93 \text{ MeV} \quad (1)$$

which describes satisfactorily the pseudoscalar interactions at low energies.

The photon being a true gauge particle, the coupling to electromagnetism can be carried out unambiguously using the gauge principle. Thus we can safely state that low energy photon-pseudoscalar interactions can be described by the Wess-Zumino-Witten⁵⁾ effective Lagrangian. Restricting our attention to the two-flavour case, this simply reads^{6),7)}:

$$\begin{aligned} \mathcal{L}_{\text{eff}} &= \frac{f_\pi^2}{4} \text{Tr} (D_\mu U^\dagger D_\mu U) \\ &\quad - \frac{N_c}{3} \left(\frac{e}{48\pi^2} \right) \epsilon_{\mu\nu\rho\sigma} a_\mu \text{Tr} (L_\nu L_\rho L_\sigma) \\ &\quad - i \frac{N_c}{3} \left(\frac{e^2}{64\pi^2} \right) \epsilon_{\mu\nu\rho\sigma} f_{\mu\nu} a_\rho \text{Tr} [\tau_3 (L_\sigma + R_\sigma)] \end{aligned} \quad (2)$$

where N_c is the number of QCD colours, and

$$\begin{aligned} D_\mu U &= \partial_\mu U - ie a_\mu [Q, U] \\ Q &= \frac{1}{6} + \frac{1}{2} \tau_3 \\ L_\mu &= \partial_\mu U U^\dagger \quad R_\mu = U^\dagger \partial_\mu U \end{aligned} \quad (3)$$

This Lagrangian is shown to incorporate all the information on the low energy (ordinary and anomalous) behaviour of the $\gamma\pi$ -interactions derived from the current algebra⁸⁾.

An equivalent form (for the two-flavour case), which is very handy in making small field expansions is

$$\begin{aligned} \mathcal{L}_{\text{eff}} &= \frac{f_\pi^2}{2} (D_\mu \vec{\varphi})^2 + \frac{f_\pi^2}{2} \left[\frac{\sin^2 \varphi - \varphi^2}{\varphi^4} \right] (\vec{\varphi} \wedge D_\mu \vec{\varphi})^2 \\ &\quad + \frac{N_c e^2}{96\pi^2} \epsilon_{\mu\nu\rho\sigma} f_{\mu\nu} f_{\rho\sigma} \varphi_0 \\ &\quad - \frac{N_c e}{96\pi^2} \epsilon_{\mu\nu\rho\sigma} f_{\mu\nu} \left[\frac{2\varphi - \sin^2 \varphi}{2\varphi^3} \right] \vec{\varphi} \cdot (D_\rho \vec{\varphi} \wedge D_\sigma \vec{\varphi}) \end{aligned} \quad (4)$$

where we have defined

$$\vec{\varphi} = \frac{\vec{\pi}}{f_\pi} \quad (D_\mu \varphi)^a = \partial_\mu \varphi^a + e a_\mu \epsilon^{3ab} \varphi^b$$

The introduction of the vector and axial vector mesons is not as clean and unambiguous as in the electromagnetic case, because of the question of vector meson masses. A reasonable way seems to be⁹⁾ to introduce them first in the most symmetrical way possible as gauge particles of $SU(2)_L \times SU(2)_R \times U(1)_V$ [we ignore the $U_A(1)$ sector for simplicity and economy of space; extension is straightforward and unambiguous), and then break this gauge symmetry minimally through degenerate mass terms. As we shall see later, the crucial signature which distinguishes this way of introducing vector and axial vector mesons from a true gauge theory is a partial Higgs mechanism which eventually yields the mass difference for the physical ρ and A particles⁹⁾.

Again we start with (1) and first replace $\partial_\mu U$ by

$$D_\mu U = \partial_\mu U + g(A_\mu^L U - U A_\mu^R) \quad (5)$$

Making the identification in the anti-Hermitian matrix notation,

$$\begin{aligned} A_\mu^L &= V_\mu + A_\mu & A_\mu^R &= V_\mu - A_\mu \\ V_\mu &= -\frac{i}{2}(\bar{\psi} \cdot \vec{V}_\mu + \omega_\mu) & A_\mu &= -\frac{i}{2}\bar{\psi} \cdot \vec{A}_\mu \end{aligned} \quad (6)$$

the non-anomalous part of the π , V , A systems can be written as

$$\Gamma_0 = \frac{f_\pi^2}{4} \text{Tr} (D_\mu U^\dagger D_\mu U) - m_\pi^2 \text{Tr} (V_\mu^2 + A_\mu^2) + \frac{1}{2} \text{Tr} (V_{\mu\nu}^2 + A_{\mu\nu}^2) \quad (7)$$

where $V_{\mu\nu}$ and $A_{\mu\nu}$ are the chiral covariant curls^{*)}. The gauge coupling constant g can be fixed by relating it to the $\rho \rightarrow 2\pi$ width:

$$\frac{g^2}{4\pi} \cong 3.0 \quad (8)$$

For the $SU(2)$ case, Eq. (7) can be rewritten explicitly as

$$\begin{aligned} \Gamma_{\pi\pi} &= \frac{f_\pi^2}{2(1-\alpha)} \left\{ (D_\mu \vec{\psi} - g \vec{A}_\mu)^2 + \left[\frac{\sin^2 \psi - \psi^2}{\psi^4} \right] [\vec{\psi} \wedge (D_\mu \vec{\psi} - g \vec{A}_\mu)]^2 \right\} \\ &+ \frac{f_\pi^2 g^2}{2\alpha} (\vec{V}_\mu^2 + \omega_\mu^2) + \frac{f_\pi^2 g^2}{2\alpha} \vec{A}_\mu^2 - \frac{1}{4} \vec{A}_{\mu\nu}^2 - \frac{1}{4} \vec{V}_{\mu\nu}^2 \end{aligned} \quad (9)$$

*) Throughout we use the notation of Ref. 5).

where

$$D_\mu \bar{\psi} = \partial_\mu \bar{\psi} + g \bar{V}_\mu \wedge \bar{\psi} \quad (10)$$

where we have introduced a new parameter $\alpha = (fg/m_\rho)^2$ which will come out as the natural dimensionless parameter as a result of diagonalization which we will discuss soon.

We have also introduced for convenience the factor $1/1-\alpha$ in front of f_π^2 in order to avoid the introduction of unphysical parameters that would need a renormalization after the diagonalization is carried out.

Let us now concentrate on the anomalous part of the effective action^{6),9)}. Although the non-Abelian anomalous effective action is extremely complicated, one can easily observe that for the two-flavour case [as a result of SU(2) being an anomaly-free group is respected by that expression] only a piece involving the coupling of the singlet current

$$\frac{1}{2} \text{Tr} \left(\frac{\delta \Gamma^{\text{an}}}{\delta A_\mu^R} + \frac{\delta \Gamma^{\text{an}}}{\delta A_\mu^L} \right) = -\frac{iN_c}{3} \frac{1}{16\pi^2} \epsilon_{\mu\nu\rho\sigma} \text{Tr} \left\{ L_\nu L_\rho L_\sigma - 3\partial_\nu [g A_\rho^R R_\sigma + g A_\rho^L L_\sigma + g^2 U A_\rho^R U^\dagger A_\sigma^L - g^2 A_\rho^R A_\sigma^L] \right\} \quad (11)$$

to the singlet vector meson (ω_μ) survives, leading to a contribution

$$-\frac{N_c}{48\pi^2} \epsilon_{\mu\nu\rho\sigma} g \omega_\mu \text{Tr} \left\{ L_\nu L_\rho L_\sigma - 3\partial_\nu [g A_\rho^R R_\sigma + g A_\rho^L L_\sigma + g^2 U A_\rho^R U^\dagger A_\sigma^L - g^2 A_\rho^R A_\sigma^L] \right\} \quad (12)$$

Next, using the SU(2) property

$$\text{Tr} (L_\nu L_\rho L_\sigma) = 3\partial_\nu \left[\frac{2\psi - \sin 2\psi}{2\psi^3} \bar{\psi} \cdot (\partial_\rho \bar{\psi} \wedge \partial_\sigma \bar{\psi}) \right] \quad (13)$$

we get the rather manageable looking effective action

$$\begin{aligned} \Gamma^{\text{an}} = & -\frac{N_c}{16\pi^2} g \epsilon_{\mu\nu\rho\sigma} \partial_\mu \omega_\nu \left\{ \frac{2\psi - \sin 2\psi}{2\psi^3} \bar{\psi} \cdot (D_\rho \bar{\psi} \wedge D_\sigma \bar{\psi}) \right. \\ & + \frac{\sin 2\psi}{2\psi} \bar{\psi} \cdot (g \bar{A}_\rho \wedge g \bar{A}_\sigma) - 2 \frac{\sin^2 \psi}{\psi^2} \bar{\psi} \cdot (g \bar{A}_\rho \wedge D_\sigma \bar{\psi}) \\ & \left. - g \bar{\psi} \cdot (\partial_\rho \bar{V}_\sigma - \partial_\sigma \bar{V}_\rho + g \bar{V}_\rho \wedge \bar{V}_\sigma) \right\} \quad (14) \end{aligned}$$

The series expansion, including vertices involving up to 4π can now immediately be carried out for both the non-anomalous and the anomalous parts:

$$\begin{aligned}\Gamma^{\text{na}} &\simeq \frac{1}{1-\alpha} \left\{ \frac{1}{2} (D_\mu \vec{\pi} - g f_\pi \vec{A}_\mu)^2 - \frac{1}{6 f_\pi^2} [\vec{\pi} \wedge (D_\mu \vec{\pi} - g f_\pi \vec{A}_\mu)]^2 \right\} + \frac{f_\pi^2 g^2}{2\alpha} (\vec{V}_\mu^2 + \vec{\omega}_\mu^2 + \vec{A}_\mu^2) \\ \Gamma^{\text{an}} &\simeq -\frac{N_c g}{16\pi^2} \varepsilon_{\mu\nu\rho\sigma} \partial_\mu \omega_\nu \left\{ \frac{2}{3 f_\pi^2} \vec{\pi} (D_\rho \vec{\pi} \wedge D_\sigma \vec{\pi}) - \frac{2g}{f_\pi^2} \vec{\pi} (\vec{A}_\rho \wedge D_\sigma \vec{\pi}) \right. \\ &\quad \left. - \frac{g}{f_\pi} \vec{\pi} (2\partial_\rho \vec{V}_\sigma + g \vec{V}_\rho \wedge \vec{V}_\sigma - g \vec{A}_\rho \wedge \vec{A}_\sigma) \right\} \quad (15)\end{aligned}$$

Because of the bilinear term, $\text{tr}(\partial_\mu \pi A^\mu)$ introduced to lowest order in Γ^{na} through gauging, a diagonalization is necessary to define the physical π and A_μ fields. The standard diagonalization procedure consists in redefining the $\vec{A}_\mu$ field as

$$\vec{A}_\mu = \vec{A}_\mu^{(\text{phys})} + \frac{\alpha}{f_\pi g} \partial_\mu \vec{\pi} \quad (16)$$

This is discussed in detail in Ref. 9). Thus we will not discuss it in detail here, except on one example: the $\rho\pi$ scattering, where we compare it to an alternative scheme that we have introduced as a computational device in order to simplify the evaluation of Feynman diagrams not involving A_μ particles in the asymptotic states (and therefore insensitive to the diagonalization scheme assumed, that is to the definition of the "physical" A_μ).

Our alternative diagonalization procedure is suggested by the appearance of the combination $D_\mu \vec{\pi} - g f_\pi \vec{A}_\mu$ in the non-anomalous part of the Lagrangian. We choose

$$\vec{A}_\mu = \vec{A}_\mu' + \frac{\alpha}{f_\pi g} D_\mu \vec{\pi} \quad (17)$$

The diagonalized effective Lagrangian (dropping the tilde for simplicity) takes the form

$$\begin{aligned}\Gamma &= \frac{1}{2} (D_\mu \vec{\pi})^2 + \frac{(f_\pi g)^2}{2\alpha} (\vec{V}_\mu^2 + \vec{\omega}_\mu^2) + \frac{(f_\pi g)^2}{2\alpha(1-\alpha)} \vec{A}_\mu'^2 - \frac{(1-\alpha)}{6 f_\pi^2} [\vec{\pi} \wedge D_\mu \vec{\pi} - \frac{f_\pi g}{1-\alpha} \vec{\pi} \wedge \vec{A}_\mu']^2 \\ &\quad - \frac{N_c g}{24\pi^2} \varepsilon_{\mu\nu\rho\sigma} \partial_\mu \omega_\nu \left[\frac{1}{f_\pi^2} (1-3\alpha + \frac{3}{2}\alpha^2) \vec{\pi} \cdot (D_\rho \vec{\pi} \wedge D_\sigma \vec{\pi}) \right. \\ &\quad \left. - \frac{3g}{f_\pi^2} (1-2\alpha) \vec{\pi} \cdot (\vec{A}_\rho \wedge D_\sigma \vec{\pi}) - \frac{3}{2} \frac{g}{f_\pi} \vec{\pi} \cdot (\partial_\rho \vec{V}_\sigma - \partial_\sigma \vec{V}_\rho + g \vec{V}_\rho \wedge \vec{V}_\sigma - g \vec{A}_\rho \wedge \vec{A}_\sigma) \right] \quad (18)\end{aligned}$$

The superiority of our "covariant" diagonalization procedure over the standard one in actual computations will now be illustrated on the example of π - ρ scattering. The result is the following gauge invariant amplitude which is analogous to the one produced by the electromagnetic Lagrangian, Eq. (4):

$$2g^2 \varepsilon_1^\mu \varepsilon_2^\mu + g^2 \frac{\varepsilon_1^\mu (2p_1^\mu - k_1^\mu) \varepsilon_2^\nu (2p_2^\nu - k_2^\nu)}{(p_1 - k_1)^2} + g^2 \frac{\varepsilon_1^\mu (2p_2^\mu - k_1^\mu) \varepsilon_2^\nu (2p_1^\nu - k_2^\nu)}{(p_1 - k_2)^2} \quad (19)$$

each term corresponding to the individual diagrams in Fig. 1.

In the standard diagonalization scheme, however, this result is recovered after a substantial amount of work. Indeed, although the three-point vertex, $V\pi\pi$, is unaffected (so are the last two diagrams in Fig. 1), the four-point vertex ($VV\pi\pi$) becomes

$$\frac{2g^2}{1-\alpha} \varepsilon_1^\mu \varepsilon_2^\mu \quad (20)$$

However, a new π VA vertex is generated through the diagonalization:

$$\frac{g^2 f_\pi}{1-\alpha} \pi \cdot (\vec{V}_\mu \wedge \vec{A}_\mu) \quad (21)$$

The corresponding contribution from the diagrams in Fig. 2 is

$$-\frac{2}{(1-\alpha)^2} g^4 f_\pi^2 \varepsilon_1^\mu \varepsilon_2^\mu \frac{\alpha(1-\alpha)}{(gf_\pi)^2} = -2g^2 \left(\frac{\alpha}{1-\alpha} \right) \varepsilon_1^\mu \varepsilon_2^\mu \quad (22)$$

The sum of (20) and (22) is again exactly $2g^2 \varepsilon_1^\mu \varepsilon_2^\mu$, reproducing (19).

A general remark is due in conclusion of this section. The absence of a coupling of the vector mesons to external sources makes it in principle impossible to provide their physical identification, and therefore it might seem that the choice of a diagonalization procedure reflects some intrinsic ambiguity of the model. However, as we shall see, vector meson dominance will provide the coupling of V_μ to the external electromagnetic source and therefore an unambiguous identification of the physical V_μ . For what concerns the axial meson, in turn, we can always imagine to couple it to an external axial current $\vec{J}_\mu^5$ introducing the Lagrangian term $g \bar{A}_\mu \vec{J}_\mu^5$ in the Lagrangian.

We can now use Eq. (16) to obtain

$$g \bar{A}_\mu \bar{J}_\mu^5 \rightarrow g \bar{A}'_\mu \bar{J}_\mu^5 + \frac{\alpha}{f_\pi} \partial_\mu \bar{\pi} \bar{J}_\mu^5 = g \bar{A}'_\mu \bar{J}_\mu^5 - \frac{\alpha}{f_\pi} \bar{\pi} \partial_\mu \bar{J}_\mu^5 \quad (23)$$

We can now recall the PCAC relationship

$$\partial_\mu \bar{J}_\mu^5 = m_\pi^2 f_\pi \bar{\pi}$$

and obtain, from Eq. (23),

$$g \bar{A}_\mu \bar{J}_\mu^5 = g \bar{A}'_\mu \bar{J}_\mu^5 - \alpha m_\pi^2 \bar{\pi} \cdot \bar{\pi}$$

showing that the standard diagonalization generates the mass of the pion field as an effective mass. This argument could not be repeated with Eq. (17) because there is no "covariant" PCAC relation and therefore a spurious $V\pi J$ coupling would be generated. As a consequence, only Eq. (16) is physically acceptable when computing amplitudes involving explicitly the axial mesons.

3. - VECTOR MESON DOMINANCE AND THE SUBTRACTED LAGRANGIAN

The sort of (approximate) gauging that has been employed in order to construct the effective Lagrangian suffers from some lack of physical motivation and it would be hard, if not impossible, to trace its origin from the underlying QCD structure of the theory. It is therefore proper to look for a consistency criterion for this choice, starting from some established physical property of the system.

The criterion we shall adopt in this paper is the request that in the low energy region the vector meson dominance (VMD)¹⁰⁾ holds exactly, that is, every process described by the effective photon-meson Lagrangian⁸⁾, (2), should be reproduced with the same amplitude by the corresponding processes involving vector mesons. VMD is the hypothesis that all the electromagnetic couplings can be obtained from the vector meson couplings through the insertion of vertices derived from an extra term in the Lagrangian⁹⁾ (making use of the new parameter α that we have introduced)

$$\mathcal{L}_{VMD} = -f_\pi^2 \left(\frac{e g}{\alpha} \right) \left(V_\mu^0 + \frac{1}{3} \omega_\mu \right) a_\mu + O(a_\mu^2) \quad (24)$$

$\mathcal{L}_{\text{VMD}}$ couples the vector meson to the external e.m. sources therefore allowing for an unambiguous physical identification of V_μ .

3.1. - Vector Case

As emphasized already by Zumino¹¹⁾, implementation of VMD at the effective Lagrangian level calls for the introduction of counter-terms removing the spurious "local" vertices introduced by the propagator of the vector meson. To set up the stage for our discussion, let us show how these counter-terms are generated for a Lagrangian which is at most quadratic in the external (vector) sources (for simplicity of notation, we remove all the internal indices, and absorb the couplings in the definition of the fields).

Let us start from an effective action, $\Gamma_0(\phi, a_\mu)$, which reproduces the low energy theorems:

$$\Gamma_0(\phi, a_\mu) = \Gamma_0(\phi, 0) + a_\mu \frac{\delta \Gamma_0(\phi, 0)}{\delta a_\mu} + \frac{1}{2} a_\mu a_\nu \frac{\delta^2 \Gamma_0(\phi, 0)}{\delta a_\mu \delta a_\nu} \quad (25)$$

we are looking for an action, $\Gamma(\phi, V_\mu)$ such that, in the functional integral formalism, VMD reads

$$e^{i\Gamma_0(\phi, a_\mu)} = \int \mathcal{D}V_\mu e^{i\Gamma(\phi, V_\mu) + i\frac{m^2}{2}(V_\mu - a_\mu)^2} \quad (26)$$

By expanding around $V_\mu = a_\mu$:

$$\Gamma(\phi, V_\mu) = \Gamma(\phi, a_\mu) + (V_\mu - a_\mu) \frac{\delta \Gamma(\phi, a_\mu)}{\delta V_\mu} + \frac{1}{2} (V_\mu - a_\mu)(V_\nu - a_\nu) \frac{\delta^2 \Gamma(\phi, a_\mu)}{\delta V_\mu \delta V_\nu} \quad (27)$$

the functional integral over V_μ can be performed in (26), yielding the following functional equation defining the effective action $\Gamma(\phi, V_\mu)$:

$$\left(\Gamma(\phi, V_\mu) - \frac{1}{2} \frac{\delta \Gamma}{\delta V_\mu} \left[m^2 g_{\mu\nu} + \frac{\delta^2 \Gamma}{\delta V_\mu \delta V_\nu} \right]^{-1} \frac{\delta \Gamma}{\delta V_\nu} \right)_{V_\mu = a_\mu} = \Gamma_0(\phi, a_\mu) \quad (28)$$

The physical interpretation of this equation is very simple (actually it is easier to derive it heuristically). All it is telling to us is that the effective vertices in the vector meson dominated theory are obtained by adding to the action of irreducible vertices all the diagrams generated by the exchange of a "dressed" V-propagator $i[g_{\mu\nu} m^2 + (\delta^2 \Gamma / \delta V_\mu \delta V_\nu)]^{-1}$ between two external currents

$\delta\Gamma/\delta V_\mu(\phi, a)$. Under the assumption that Γ is at most quadratic in V_μ , it is easy to find the solution to this equation:

$$\Gamma(\varphi, V_\mu) + \frac{1}{2} m^2 V_\mu V_\mu = \Gamma_0(\varphi, 0) + \frac{1}{2} \left[\frac{\delta\Gamma_0(\varphi, 0) + m^2 V_\mu}{\delta a_\mu} \right] \left[m^2 g_{\mu\nu} - \frac{\delta^2 \Gamma_0}{\delta a_\mu \delta a_\nu} \right]^{-1} \left[\frac{\delta\Gamma_0(\varphi, 0) + m^2 V_\nu}{\delta a_\nu} \right] \quad (29)$$

An expansion in powers of m^{-2} yields:

$$\Gamma(\varphi, V_\mu) = \Gamma_0(\varphi, V_\mu) + \frac{1}{2m^2} \left[\frac{\delta\Gamma_0(\varphi, V_\mu)}{\delta a_\mu} \right]^2 + O(m^{-4}) \quad (30)$$

where it is easy to recognize the subtraction of the contact current-current term generated by the $(i/m^2)g_{\mu\nu}$ propagator of V_μ . Equation (30) is exact when only terms linear in a_μ are present¹¹⁾.

The above discussion suits to the non-anomalous part exactly (since this part is quadratic in external sources). The anomalous part, however, requires further special care. We will first illustrate the method with some examples before turning our attention to the discussion of the full action (non-anomalous and anomalous).

The simplest non-trivial example which illustrates the above general discussion is the $\pi\pi$ scattering amplitude. The low energy theorems of current algebra dictate that 4π -interaction must be generated by an effective Lagrangian

$$\Gamma_{4\pi} = - \frac{1}{6f_\pi^2} (\bar{\pi} \wedge \partial_\mu \bar{\pi})^2 \quad (31)$$

However, after a little algebra, one can deduce from Eq. (18) that the sum of the diagrams in Fig. 3 [where, as usual, we replace $T\langle V_\mu(x) V_\nu(y) \rangle$ by $i(\alpha/f^2 g^2) \delta^4(x-y) g_{\mu\nu}$ in order to extract the low energy content] leads to the effective vertex

$$\left[- \frac{(1-\alpha)}{6f_\pi^2} - \frac{3\alpha}{6f_\pi^2} \right] (\bar{\pi} \wedge \partial_\mu \bar{\pi})^2 = - \frac{1+2\alpha}{6f_\pi^2} (\bar{\pi} \wedge \partial_\mu \bar{\pi})^2 \quad (32)$$

The only way out of this contradiction is, as suggested by Eq. (30), the introduction of a Lagrangian counter term

$$\frac{\alpha}{3f_\pi^2} (\bar{\pi} \wedge \partial_\mu \bar{\pi})^2 \quad (33)$$

leading, together with (32), to the correct $\pi\pi$ amplitude.

If we now use the effective electromagnetic Lagrangian as a testing ground for the computations in the $\pi\text{-}\rho$ Lagrangian, making use of the VMD hypothesis, we can systematically determine, by simple comparison, all the subtracted effective vertices for the $\pi\text{-}\rho$ theory required in order to reproduce the low energy theorems. We apply this method to the first few interactions and extract physical predictions concerning the $\gamma \rightarrow 3\pi$, $\gamma\gamma \rightarrow 3\pi$ processes⁸⁾.

As already witnessed in the case of $\pi\rho$ -scattering, terms not involving a V-meson exchange are unaffected by the counter term problem. This is reflected in the fact that the anomalous vertices

$$\begin{aligned} & \frac{iN_c g}{24\pi^2} \varepsilon_{\mu\nu\rho\sigma} \partial_\mu \omega_\nu \left\{ g \bar{\psi} (2\partial_\rho \bar{V}_\sigma + g \bar{V}_\rho \wedge \bar{V}_\sigma) - \frac{g}{\varphi^2} (D_\rho \bar{\psi} \wedge D_\sigma \psi) \right\} = \\ & = \frac{iN_c g}{24\pi^2} \varepsilon_{\mu\nu\rho\sigma} \partial_\mu \omega_\nu \left\{ 2g (\bar{V}_\rho \hat{\psi}) \partial_\sigma \psi - \bar{\psi} (\partial_\rho \hat{\psi} \wedge \partial_\sigma \hat{\psi}) \right\} \end{aligned} \quad (34)$$

are left invariant and reproduce the corresponding low energy theorems, in particular the $\pi^0 \rightarrow \gamma\gamma$ amplitude, exactly.

The first interesting non-trivial example is the process $\omega \rightarrow 3\pi$. The direct anomalous vertex, Fig. 4a,

$$-\frac{iN_c g}{24\pi^2 f_\pi^3} \varepsilon_{\mu\nu\rho\sigma} \partial_\mu \omega_\nu \bar{\pi} \cdot (\partial_\rho \pi \wedge \partial_\sigma \pi) (1 - 3\alpha + \frac{3}{2}\alpha^2) \quad (35)$$

is accompanied by the one in Fig. 4b, which involves a vector exchange contribution¹²⁾:

$$\begin{aligned} & \left(\frac{i3g^2 N_c}{24\pi^2 f_\pi} \varepsilon_{\mu\nu\rho\sigma} \partial_\mu \omega_\nu \bar{\pi} \partial_\rho \bar{V}_\sigma \right) \times \left(ig \bar{V}_\sigma \cdot (\pi \wedge \partial_\sigma \pi) \right) \rightarrow \\ & \rightarrow -\frac{iN_c g}{24\pi^2 f_\pi^3} (3\alpha) \varepsilon_{\mu\nu\rho\sigma} \partial_\mu \omega_\nu \bar{\pi} \cdot (\partial_\rho \pi \wedge \partial_\sigma \pi) \end{aligned} \quad (36)$$

The total effective vertex is therefore

$$-\frac{iN_c g}{24\pi^2 f_\pi^3} \left(1 + \frac{3}{2}\alpha^2 \right) \varepsilon_{\mu\nu\rho\sigma} \partial_\mu \omega_\nu \bar{\pi} \cdot (\partial_\rho \pi \wedge \partial_\sigma \pi) \quad (37)$$

Now, comparison with the electromagnetic Lagrangian, through VMD, leads to the introduction of a Lagrangian counter term

$$\frac{N_c g}{24\pi^2 f_\pi^3} \left(\frac{3}{2} \alpha^2 \right) \varepsilon_{\mu\nu\rho\sigma} \partial_\mu \omega_\nu \bar{\pi} \cdot (\partial_\rho \bar{\pi} \wedge \partial_\sigma \bar{\pi}) \quad (38)$$

and thus eventually leads to an expression where the low energy theorem⁸⁾

$$\frac{e F^{3\pi}}{F^\pi} = f_\pi^2 \quad (39)$$

is satisfied without any constraint on the parameter α , contrary to a recent claim¹³⁾.

Our last example is the process $\gamma\gamma \rightarrow 3\pi$. The relevant piece in the anomalous effective Lagrangian is

$$\frac{N_c g^2}{12\pi^2 f_\pi^3} \varepsilon_{\mu\nu\rho\sigma} \partial_\mu \omega_\nu [(\bar{V}_8 \wedge \bar{\pi}) \wedge \bar{\pi}] D_\sigma \bar{\pi} \quad (40)$$

We classify the large number of contributing diagrams according to the corresponding ones in $\gamma\gamma \rightarrow 3\pi$ process⁸⁾, Fig. 5. The corresponding α -weights are also explicitly depicted. The diagrams in the vertical column are related to each other by insertion of the vector meson exchange, whereas the first two rows are associated by gauge invariance. After a bit of work, one can show that the gauge invariant version of the previous counter term is:

$$\frac{N_c g}{24\pi^2 f_\pi^3} \left(\frac{3}{2} \alpha^2 \right) \varepsilon_{\mu\nu\rho\sigma} \partial_\mu \omega_\nu \bar{\pi} (D_\rho \bar{\pi} \wedge D_\sigma \bar{\pi}) \quad (41)$$

Equation (41), together with the insertion of the 4π -counter term in the last column is all that is needed to reproduce the low energy theorems on $\gamma\gamma \rightarrow \pi\pi\pi$ from Fig. 5^{7),14)}.

In the discussion of these explicit examples, we applied the formalism sketched at the beginning of the section in order to construct the general low-energy effective action for pions and vector mesons which satisfies the low energy theorems through vector meson dominance. In other words, we are seeking for a $\Gamma(\vec{\phi}, \vec{V}, \omega)$ such that the electromagnetic vertices, generated by $\Gamma(\vec{\phi}, \vec{V}, \omega) + \Gamma_{\text{VMD}}$ reproduce those already contained in the original Γ_{em} , Eq. (2). It is straightforward to deduce the non-anomalous part of Γ , Γ_{na} , using (29). We only state the resulting expression:

$$\Gamma_{\text{an}}(\vec{\psi}, \vec{V}_\mu, \omega_\mu) = \frac{f_\pi^2}{2} (\partial_\mu \psi)^2 + \frac{f_\pi^2}{2} \frac{1}{1 - \alpha \sin^2 \varphi} \frac{\sin^2 \varphi}{\varphi^2} \frac{[\vec{\psi} \wedge D_\mu \vec{\psi}]^2}{\varphi^2} + \frac{f_\pi^2 g^2}{2\alpha} (\vec{V}_\mu^2 + \omega_\mu^2) \quad (42)$$

The only essential ingredient in the derivation was the decomposition of $D_\mu \vec{\phi}$ along the orthogonal directions

$$D_\mu \vec{\psi} = \frac{\vec{\psi}}{\varphi} (D_\mu \varphi) - \frac{\vec{\psi} \wedge (\vec{\psi} \wedge D_\mu \vec{\psi})}{\varphi^2} = \partial_\mu \psi \hat{\psi} - \hat{\psi} \wedge (\hat{\psi} \wedge D_\mu \vec{\psi}) \quad (43)$$

The analysis of the anomalous part requires special care. In principle, we are not obeying our original assumption of being second order in the external sources because of the existence of cubic terms in the vector fields in the anomalous effective action. However, keeping in mind that we are dealing with a low energy effective action, it is easy to recognize that, to lowest order in momentum, only diagrams containing only one anomalous vertex will contribute. In particular no ω -propagation is allowed. With this new piece of information, we observe that the non-anomalous part of the Lagrangian is not affected by the presence of the anomalous vertices. Thus it is possible to write down the functional equation which is an extension of Eq. (28):

$$\left(\Gamma^{\text{an}}(\vec{\psi}, \vec{V}_\mu, \omega_\mu) - \frac{\delta \Gamma^{\text{an}}}{\delta V_\mu}(\vec{\psi}, \vec{V}_\mu) \left[m^2 g_{\mu\nu} + \frac{\delta^2 \Gamma^{\text{an}}}{\delta V_\mu \delta V_\nu} \right]^{-1} \frac{\delta \Gamma^{\text{an}}}{\delta V_\nu}(\vec{\psi}, \vec{V}_\mu, \omega_\mu) + \frac{1}{2} \frac{\delta \Gamma^{\text{an}}}{\delta V_\mu}(\vec{\psi}, \vec{V}_\mu) \left[m^2 g_{\mu\nu} + \frac{\delta^2 \Gamma^{\text{an}}}{\delta V_\mu \delta V_\nu} \right]^{-1} \frac{\delta^2 \Gamma^{\text{an}}}{\delta V_\nu \delta V_\rho}(\vec{\psi}, \omega_\rho) \left[m^2 g_{\rho\sigma} + \frac{\delta^2 \Gamma^{\text{an}}}{\delta V_\rho \delta V_\sigma} \right]^{-1} \frac{\delta \Gamma^{\text{an}}}{\delta V_\sigma}(\vec{\psi}, \vec{V}_\mu) \right)_{\substack{V_\mu = a_\mu \\ \omega_\mu = \frac{1}{2} a_\mu}} = \Gamma_0^{\text{an}}(\vec{\psi}, a_\mu) \quad (44)$$

Once again, this equation has a rather simple interpretation: the second term corresponds to the "dressed" propagation of the $\vec{V}_\mu$ between an ordinary and anomalous current and the third term is just the insertion of an anomalous vertex in the previously derived propagation of a $\vec{V}$ between two non-anomalous currents. Again the solution is straightforward. The result is:

$$\begin{aligned} \Gamma^{\text{em}}(\vec{\psi}, \vec{V}_\mu, \omega_\mu) = & \frac{N_c}{16\pi^2} \epsilon_{\mu\nu\rho\sigma} \partial_\mu \omega_\nu \left\{ g \vec{\psi} (\partial_\rho \vec{V}_\sigma - \partial_\sigma \vec{V}_\rho + g \vec{V}_\rho \wedge \vec{V}_\sigma) - \frac{\vec{\psi}}{\varphi^2} (D_\rho \vec{\psi} \wedge D_\sigma \vec{\psi}) \right. \\ & \left. + \frac{\sin 2\varphi}{2\varphi} \frac{1}{(1-\alpha \sin^2 \varphi)^2} \frac{\vec{\psi}}{\varphi^2} (D_\rho \vec{\psi} \wedge D_\sigma \vec{\psi}) \right\} \end{aligned} \quad (45)$$

where the longitudinal and transverse components of the vector field are naturally separated. The combination $\Gamma^{\text{na}} + \Gamma^{\text{a}} + \Gamma^{\text{VMD}}$ is one of the main results of this work. It is the unique and complete low energy Lagrangian of the π and ρ , ω mesons (all the other degrees of freedom are integrated over) which reproduces exactly all the low energy theorems incorporated into Γ^{em} . The interaction with the nucleons can be incorporated into a straightforward way by turning to the soliton degrees of freedom for the pseudoscalar sector.

3.2. - Vector-Axial Vector Case

As we already noticed at the end of Section 2, the procedure that led to the construction of the effective π - ρ - ω action is made intrinsically unambiguous by the presence of the photon-vector meson coupling which identifies the "physical ρ -particle" through its interaction with the external electromagnetic sources.

However, when we introduce axial vector mesons we do not have such an explicit external source and we must therefore appeal to some other physical principle in order to construct the effective Lagrangian. We indeed recognize that the simple request that VMD is respected allows for the very general expression:

$$\begin{aligned}
 \Gamma(\bar{\psi}, \bar{V}_\mu, \omega_\mu, \bar{A}_\mu) = & \Gamma^{n2}(\bar{\psi}, \bar{V}_\mu, \omega_\mu) + \Gamma^{n3}(\bar{\psi}, \bar{V}_\mu, \omega_\mu) \\
 & + \frac{1}{2} G_2(\psi) [\hat{\psi} \hat{A}_\mu - H_1(\psi) \partial_\mu \psi]^2 \\
 & + \frac{1}{2} G_2(\psi) [(\hat{\psi} \wedge \bar{A}_\mu) - H_2(\psi) \hat{\psi} \wedge \partial_\mu \bar{\psi} - K_2(\psi) \hat{\psi} \wedge (\bar{V}_\mu \wedge \bar{\psi})]^2 \\
 & + \varepsilon_{\mu\nu\rho\sigma} \partial_\mu \omega_\nu \{ M(\psi) \bar{\psi} [(\bar{A}_3 - H_2 \partial_3 \bar{\psi} - K_2 \bar{V}_3 \wedge \bar{\psi}) \wedge D_\sigma \bar{\psi}] \\
 & + \frac{1}{2} N(\psi) \bar{\psi} [\bar{A}_3 \wedge \bar{A}_\sigma - (H_2 \partial_3 \bar{\psi} + K_2 \bar{V}_3 \wedge \bar{\psi}) \wedge (H_2 \partial_\sigma \bar{\psi} + K_2 \bar{V}_\sigma \wedge \bar{\psi})] \}
 \end{aligned} \tag{46}$$

to be assumed as the complete π - ω - ρ - A effective Lagrangian. The functions G , H , K , M , N appearing in Eq. (46) are arbitrary (even) functions of $\phi = |\vec{\phi}|$.

We can, however, introduce some external currents coupled to the $\bar{A}_\mu$ fields and require that the resulting non-anomalous interaction between the external currents obeys the request of chiral invariance: this choice reflects the original "naive" prescription that the $SU(2)_L \times SU(2)_R \times U(1)_V$ symmetry be minimally broken through globally chiral symmetric degenerate mass terms. The computation is a simple generalization of our previous analysis and leads to the following effective Lagrangian

$$\begin{aligned}
 \Gamma^{n2} = & \frac{f_\pi^2}{2(1-\alpha)} [\hat{\psi} (D_\mu \bar{\psi} - g \bar{A}_\mu)]^2 + \frac{f_\pi^2}{2} \frac{\sin^2 \psi}{\psi^2} \frac{[\hat{\psi} \wedge (D_\mu \bar{\psi} - g \bar{A}_\mu)]^2}{1 - \alpha \sin^2 \psi - \alpha \frac{\sin^2 \psi}{\psi^2}} \\
 & + \frac{f_\pi^2 g^2}{2\alpha} (\bar{V}_\mu^2 + \omega_\mu^2 + \bar{A}_\mu^2) - \frac{f_\pi^2}{\alpha} e g (V_\mu^0 + \frac{1}{3} \omega_\mu) a_\mu + g \bar{A}_\mu \bar{J}_\mu^5
 \end{aligned} \tag{47}$$

and the diagonalization, Eq. (16), must still be performed leading to an effective

$$\frac{f^2}{2\alpha(1-\alpha)} \bar{A}_\mu'^2 - \alpha m_\pi^2 \bar{\pi} \cdot \bar{\pi}$$

mass term for the axials and pions respectively^{*}). Comparing with Eq. (9), we show that in some sense the naive Lagrangian is really only minimally modified; the dependence on the combination $D_\mu \bar{\phi} - \bar{A}_\mu$ is retained even after the introduction of the mass terms.

^{*}) Notice that the request that the pion mass be generated only through this mechanism fixes the value of the parameter α , $\alpha = \frac{1}{2}$. By the way, this is exactly the KSFR relationship¹⁵⁾.

We cannot, however, make use of chiral invariance in the construction of the anomalous part of the effective action. One way of resolving this ambiguity (that is, determining the arbitrary functions) is to require that the interaction between the axial sources J_μ^5 reproduce the anomalous Lagrangian, Eq. (14), after the integration over V and A degrees of freedom is taken into account.

The computation is by now standard within the scheme we have designed and the resulting expression is:

$$\begin{aligned} \Gamma^{an}(\bar{\psi}, \bar{V}_\mu, \omega_\mu, \bar{A}_\mu) = & \frac{N_c}{16\pi^2} \epsilon_{\mu\nu\rho\sigma} \partial_\mu \omega_\nu \left[g \bar{\psi} (2 \partial_\rho \bar{V}_\sigma + g \bar{V}_\rho \wedge \bar{V}_\sigma) - \frac{\bar{\psi}}{\varphi^2} (D_\rho \bar{\psi} \wedge D_\sigma \bar{\psi}) \right. \\ & + \frac{\sin 2\varphi}{2\varphi^3} \bar{\psi} (\bar{C}_3^1 \wedge \bar{C}_\sigma^1) - \frac{\sin 2\varphi}{2\varphi} \bar{\psi} (\bar{C}_3^2 \wedge \bar{C}_\sigma^2) \\ & \left. + 2 \frac{\sin^2 \varphi}{\varphi^2} \bar{\psi} (\bar{C}_3^1 \wedge \bar{C}_\sigma^2) \right] \end{aligned} \quad (48)$$

where

$$\begin{aligned} \bar{C}_\mu^1 = & \frac{(1 - \alpha \frac{\sin^2 \varphi}{\varphi^2}) D_\mu \bar{\psi} - \alpha \sin^2 \varphi g \bar{A}_\mu}{1 - \alpha \sin^2 \varphi - \alpha \frac{\sin^2 \varphi}{\varphi^2}} \\ \bar{C}_\mu^2 = & \frac{(1 - \alpha \sin^2 \varphi) g \bar{A}_\mu - \alpha \frac{\sin^2 \varphi}{\varphi^2} D_\mu \bar{\psi}}{1 - \alpha \sin^2 \varphi - \alpha \frac{\sin^2 \varphi}{\varphi^2}} \end{aligned} \quad (49)$$

with the important property

$$\bar{C}_\mu^1 - \bar{C}_\mu^2 = \frac{D_\mu \bar{\psi} - g \bar{A}_\mu}{1 - \alpha \sin^2 \varphi - \alpha \frac{\sin^2 \varphi}{\varphi^2}} \quad (50)$$

We propose the combination of Eqs. (47) and (48) evaluated at $J_\mu^5 = 0$ as our complete effective Lagrangian for the $\pi\omega\rho A_1$ interactions in the low energy regime (essentially below A_1) after the diagonalization is performed. This Lagrangian is unique in reproducing the purely pionic and electromagnetic low energy theorems and in generating a chiral invariant interaction between external vector and axial sources, supplemented with an interaction term reproducing the standard (Bardeen) non-Abelian anomaly. Indeed, in the limit in which the vector mesons are identified with the external sources, $\alpha \rightarrow 0$, it goes smoothly to Lagrangian, Eq. (14). Furthermore, all the counter terms needed in the naive construction to eliminate the spurious local vertices generated by the vector meson propagation (proportional to α) is built in by construction. This can easily be illustrated on the familiar example, $\omega \rightarrow 3\pi$. The contact term is obtained as (first diagram in Fig. 4):

$$- i \frac{2}{8\pi^2 f_\pi^3} \varepsilon_{\mu\nu\rho\sigma} \partial_\mu \omega_\nu \bar{\pi} (\partial_\rho \pi \wedge \partial_\sigma \pi) (1-3\alpha)$$

whereas the Gell Mann-Sharp-Wagner contribution (second diagram in Fig. 4) is the same as before (this diagram is not affected by the introduction of axials)

$$- i \frac{2}{8\pi^2 f_\pi^3} \varepsilon_{\mu\nu\rho\sigma} \partial_\mu \omega_\nu \bar{\pi} (\partial_\rho \pi \wedge \partial_\sigma \pi) (3\alpha)$$

These together yield the correct amplitude without any need for any counter term.

4. - CONCLUDING REMARKS

In the light of our results, we would like to comment on a recent attempt¹³⁾ of "deriving" the KSFR relation¹⁵⁾ in the context of the effective Lagrangian formalism. Let us first recall that the KSFR relation, in our notation, would correspond to the statement that $\alpha = \frac{1}{2}$. What we claim here is that this relation cannot follow from the effective action formalism.

If one assumes, along with Ref. 12), that only the propagation of the vector meson in $\gamma \rightarrow 3\pi$ process is relevant¹²⁾, that is all the contact terms disappear, we get^{*)}, using VMD

$$3\alpha = 1 \tag{51}$$

The left-hand side represents the result of the computation of the non-contact terms we have presented in Section 3, and the right-hand side represents the rescaled low energy theorem, (39). That the prediction $\alpha = 1/3$ is experimentally unsatisfactory (apart from being in disagreement with KSFR) is not surprising, given the theoretically unfounded assumption involved in the derivation. One naive attempt of remedy could have been to include the contact contribution of the naive Lagrangian (non-subtracted), along with the non-contact one. We get, as already mentioned, from Eqs. (35) and (36),

*) Compare with Ref. 13).

$$3\alpha + (1 - 3\alpha + \frac{3}{2}\alpha^2) = 1$$

$$\frac{3}{2}\alpha^2 = 0 \tag{52}$$

an even more unsatisfactory result. The correct Lagrangian to use, however, is the subtracted one which yields the identity, $1 = 1$. As illustrated on several examples in Section 3, within the subtraction scheme developed here, the low energy theorems are identically satisfied for any value of α , and thus no derivation of KSFR can follow from them. Perhaps a reasonable line of thought could be the discussion following Eq. (47) which reproduces the KSFR relation together with the Weinberg's mass formula ($M_{A_1} = \sqrt{2}m_\rho$).

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FIGURE CAPTIONS

- Fig. 1 Contribution to the $\pi\rho$ -scattering amplitude in the covariant diagonalization scheme.
- Fig. 2 Additional contributions to the $\pi\rho$ -scattering amplitude in the standard diagonalization scheme.
- Fig. 3 Contributions to the $\pi\pi$ -scattering amplitude.
- Fig. 4 Contact and V-exchange (Gell Mann-Sharp-Wagner) contributions to the $\omega \rightarrow 3\pi$ amplitude.
- Fig. 5 The dominant $VV \rightarrow \pi\pi\pi$ diagrams.

- Figure 1 -

- Figure 2 -

- Figure 3 -

- Figure 4 -

$1-3\alpha+3/2 \alpha^2$	$3 (1-3\alpha+3/2 \alpha^2)$	$1-\alpha$
$3/2 \alpha$	3α	3α
$3/2 \alpha$	6α	

- Figure 5 -