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CONSTRUCTION OF THE EFFECTIVE VERTICES AND THE GAUGE INVARIANT CURRENTS
IN THE GAUGED WESS-ZUMINO ACTION

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ABSTRACT

The vector flavour current is derived directly from the recently proposed gauged Wess-Zumino action. This gauge covariant expression is used to compute the baryonic and electromagnetic currents. We perform a small field expansion of the pure vector Wess-Zumino action and present the first few terms in a gauge invariant form. Special attention is paid to the two flavour action; in the electromagnetic and the vector-axial vector cases, the results are presented in a manifestly gauge invariant form in terms of the physical pion field.

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1. INTRODUCTION

After the proposal by Witten¹⁾ that the chiral model, including the Wess Zumino anomalous term²⁾ provides a good framework for studying the low energy limit of QCD, increasing attention has been devoted to these models.

The key ingredient is that the solitonic sectors of the model (stabilized by a Skyrme term) can be identified with the baryons³⁾ (the mesons being in the trivial soliton sector). However, in order to describe the interactions of the hadrons with the vector bosons (say $\gamma, W^\pm, Z_0$) one can imagine to construct a gauged version of the Wess-Zumino-Witten (WZW) effective action invariant under global $U(N)_L \otimes U(N)_R$ transformations.

While the gauging of the non-anomalous part is straightforward the gauging of the anomalous part needs more sophisticated techniques because this term is written as a topologically non-trivial five-dimensional integral. Moreover, the gauging has to be such that it reproduces the Bardeen anomaly; that is, the final action has to be invariant under local vector transformations only, and to reproduce the anomaly under more general transformations.

This task was accomplished by several groups^{1), 4-6)} which obtained the final effective action gauged for arbitrary subgroups of $U(N)_L \otimes U(N)_R$. Despite its elegant formulation, the anomalous part of the action is not suitable for direct physical applications; indeed, the field of interest, say π , appears only through the function $U = \exp i\pi f_\pi^{-1}$; as a consequence, the identification of the vertices is not directly obvious. Furthermore, the action does not exhibit explicitly the local invariance under vector transformations.

So, it is clearly desirable to work again on the WZW action and try to turn it in a more conventional form, explicitly written in terms of the pion field, the covariant derivative D_μ and the field strength $F_{\mu\nu}$.

This programme was realized by Di Vecchia et al.⁷⁾ for the two-dimensional WZW action. In four-dimensions their technique involves too large a number of integrations; moreover, working with the action term by term is confusing because of many possibilities of partial integrations. On the other hand, it can be useful to study the anomalous currents that are not affected by the problem of partial integration.

The purpose of this paper is to report the progress we made in this direction on some particular cases of physical interest. In the first part, we study the case of pure vector gauging, compute the currents (baryonic, isospin, etc,) and we develop an expansion of the effective action in powers of $1/f_\pi$, the first terms of which being written in a conventional way. Afterwards, special attention is paid to the purely electromagnetic action in the two flavour case; it is presented in the form discussed above. In the third part, we treat in detail the vector - axial vector gauging for the two flavour case.

The WZW effective action has the form¹⁾ [see (1.6) for notations].

$$\begin{aligned}\Gamma_{\text{eff}}(U) &= \Gamma_{\text{chiral}} - i W_{\text{WZW}} \\ &= \frac{f_\pi^2}{4} \int d^4x \text{Tr}(\partial_\mu U \partial_\mu U^\dagger) + \frac{i N_c}{240 \pi^2} \int d^5x \epsilon_{ijklm} \text{Tr}(L_i L_j L_k L_l L_m)\end{aligned}\quad (1.1)$$

and is globally invariant under $U(N)_L \otimes U(N)_R$. The second integral is on a five-dimensional disc Q whose boundary is space-time. The gauged version of the action (1.1) was obtained by several groups^{1), 4-6)}; here we reproduce it for self consistency of the paper and we refer to the original papers for more details^{*)}.

$$\begin{aligned}\Gamma_{\text{eff}}(U, A_\mu^L, A_\mu^R) &= \frac{f_\pi^2}{4} \int d^4x \text{Tr}[(\partial_\mu U + A_\mu^L U - U A_\mu^R)(\partial_\mu U^\dagger + A_\mu^R U^\dagger - U^\dagger A_\mu^L) \\ &\quad - \frac{1}{2g^2} (F_{\mu\nu}^L)^2 - \frac{1}{2g^2} (F_{\mu\nu}^R)^2] - i W_{\text{WZ}}(U, A_\mu^L, A_\mu^R)\end{aligned}\quad (1.2)$$

$$W_{\text{WZ}}(U, A_\mu^L, A_\mu^R) = W_c(U, A_\mu^L, A_\mu^R) - W_c(\mathbb{1}, A_\mu^L, A_\mu^R) \quad (1.3)$$

*) We use the notations of Ref.5).

$$\begin{aligned}
 W_c(U, A_\mu^L, A_\mu^R) = & W_{WZ} + \frac{-N_c}{48\pi^2} \int d^4x \epsilon_{\mu\nu\rho\sigma} \text{Tr} \left\{ -(A_\mu^R R_\nu R_\rho R_\sigma + A_\mu^L L_\nu L_\rho L_\sigma) \right. \\
 & + \frac{1}{2} (A_\mu^R R_\nu A_\rho^R R_\sigma - A_\mu^L L_\nu A_\rho^L L_\sigma) - A_\mu^R U^\dagger A_\nu^L U R_\rho R_\sigma + A_\mu^L U A_\nu^R U^\dagger L_\rho L_\sigma \\
 & + \left[\partial_\mu A_\nu^R U^\dagger A_\rho^L U R_\sigma + \partial_\mu A_\nu^L U A_\rho^R U^\dagger L_\sigma \right. \\
 & + (A_\mu^R \partial_\nu A_\rho^R + \partial_\mu A_\nu^R A_\rho^R + A_\mu^R A_\nu^R A_\rho^R + A_\mu^R U^\dagger A_\nu^L U A_\rho^R) R_\sigma \\
 & + (A_\mu^L \partial_\nu A_\rho^L + \partial_\mu A_\nu^L A_\rho^L + A_\mu^L A_\nu^L A_\rho^L + A_\mu^L U A_\nu^R U^\dagger A_\rho^L) L_\sigma \left. \right] \\
 & + \left[A_\mu^R U^\dagger A_\nu^L \partial_\rho A_\sigma^L U + A_\mu^R U^\dagger \partial_\nu A_\rho^L A_\sigma^L U + \partial_\mu A_\nu^R A_\rho^R U^\dagger A_\sigma^L U \right. \\
 & + A_\mu^R \partial_\nu A_\rho^R U^\dagger A_\sigma^L U + A_\mu^R A_\nu^R A_\rho^R U^\dagger A_\sigma^L U + A_\mu^R U^\dagger A_\nu^L A_\rho^L A_\sigma^L U \\
 & \left. + \frac{1}{2} A_\mu^R U^\dagger A_\nu^L U A_\rho^R U^\dagger A_\sigma^L U \right] \left. \right\} \quad (1.4)
 \end{aligned}$$

The anomalous part of (1.2), i.e., W_{WZ} is locally invariant under the following $U(N)_V$ subgroup of $U(N)_R \otimes U(N)_L$

$$W_{WZ}(U, A_\mu^L, A_\mu^R) = W_{WZ}(a U a^\dagger, a(\partial_\mu + A_\mu^L) a^\dagger, a(\partial_\mu + A_\mu^R) a^\dagger) \quad (1.5)$$

$$a \equiv a(x) \in U(N)$$

The purpose of the paper is to study the Lagrangian (1.3) and its associated currents. We will use the following notations:

$$U = \exp i\varphi, \quad \varphi = \frac{\pi^a \lambda_a}{f_\pi}, \quad \lambda_a \lambda_b = 2 \delta_{ab} \quad (1.6)$$

$$R_\mu = U^{-1} \partial_\mu U, \quad L_\mu = \partial_\mu U U^{-1} \quad (1.7)$$

$$\tilde{R}_\mu = U^{-1} \mathcal{D}_\mu U, \quad \tilde{L}_\mu = \mathcal{D}_\mu U U^{-1} \quad (1.8)$$

$$V_\mu = \frac{1}{2}(A_\mu^R + A_\mu^L), \quad V_\mu = \frac{g}{2i} \lambda^a V_\mu^a, \dots \quad (1.9)$$

$$A_\mu = \frac{1}{2}(A_\mu^R - A_\mu^L) \quad (1.10)$$

$$F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu + [V_\mu, V_\nu]$$

$$D_\mu = \partial_\mu + [V_\mu,] \quad (1.11)$$

In terms of V_μ and A_μ , the invariance in Eq. (1.5) can be expressed as

$$W_{WZ}(U, V_\mu, A_\mu) = W_{WZ}(a U a^\dagger, a(\partial_\mu + V_\mu)a^\dagger, a A_\mu a^\dagger) \quad (1.12)$$

We will also use the following summation formula extensively later in making the small field expansions.

$$U^{-1} \partial_\mu U = \left(\frac{1 - \exp(-iT\varphi)}{T\varphi} \right)_{ab} \lambda^a \partial_\mu \varphi^b \quad (1.13)$$

$$U^{-1} (V^a \lambda^a) U = (\exp(-iT\varphi))_{ab} \lambda^a V^b \quad (1.14)$$

with the definition

$$(T\varphi)_{ab} = \sum_{n=1}^{N^2-1} 2i f_{abn} \varphi^n, \quad [\lambda^a, \lambda^b] = 2i f_{abc} \lambda^c \quad (1.15)$$

It is worth noticing that the formula (1.13) is still true when we substitute ordinary derivatives by covariant ones.

2. PURE VECTOR CASE

In order to simplify the problem described in the introduction, we have assumed first that $A_{\mu}^L = A_{\mu}^R$ is the Lagrangian (1.13). This corresponds to the gauging of the purely vectorial $U(N)_V$, which is the diagonal subgroup of the full $U(N)_L \otimes U(N)_R$ and contains the very important electromagnetic case.

We have computed the vector current taking derivatives directly from (1.3). After a long algebra, the current can be put in a form manifestly covariant under the vector gauge transformations (1.5)

$$J_{\mu} = \frac{\delta W_{WZ}}{\delta V_{\mu}} (U, A_{\mu}^L = A_{\mu}^R = V_{\mu}) \quad (2.1)$$

$$= \frac{-N_c}{48\pi^2} \epsilon_{\mu\nu\rho\sigma} \left[-\tilde{R}_{\nu} \tilde{R}_{\rho} \tilde{R}_{\sigma} - \tilde{L}_{\nu} \tilde{L}_{\rho} \tilde{L}_{\sigma} + \{F_{\nu\rho}, \tilde{R}_{\sigma} + \tilde{L}_{\sigma}\} \right. \\ \left. + \frac{1}{2} \{U^{-1} F_{\nu\rho} U, \tilde{R}_{\sigma}\} + \frac{1}{2} \{U F_{\nu\rho} U^{-1}, \tilde{L}_{\sigma}\} \right] \quad (2.2)$$

where we used the notation (1.8). Taking the trace of J_{μ} gives the baryonic current B_{μ} .

$$B_{\mu} = \frac{1}{3} \text{Tr} J_{\mu} \quad (2.3)$$

$$= \frac{(N_c/3)}{48\pi^2} \epsilon_{\mu\nu\rho\sigma} \text{Tr} [2 \tilde{R}_{\nu} \tilde{R}_{\rho} \tilde{R}_{\sigma} - 3 F_{\nu\rho} (\tilde{L}_{\sigma} + \tilde{R}_{\sigma})] \quad (2.4)$$

$$= \frac{(N_c/3)}{24\pi^2} \epsilon_{\mu\nu\rho\sigma} \text{Tr} [R_{\nu} R_{\rho} R_{\sigma} - 3 \partial_{\nu} (V_{\rho} R_{\sigma} + V_{\rho} L_{\sigma} + V_{\rho} U^{\dagger} V_{\sigma} U)] \quad (2.5)$$

The formula (2.4) follows directly from (2.2) and is clearly invariant under (1.5); the second one (2.5), obtained after use of Eqs (1.8) and (1.11), exhibits the conservation of the current ($\partial^{\mu} B_{\mu} = 0$) due to the antisymmetric symbol ϵ .

We can also write down B_μ in terms of the φ field itself making use of the formulas (1.13) and (1.14):

$$B_\mu = \frac{N_c/3}{24\pi^2} \epsilon_{\mu\nu\rho\sigma} \left\{ -3 F_{\nu\rho}^a D_\sigma \varphi^b \left(\frac{1 - \cos T\varphi}{T\varphi} \right)_{ab} \right. \\ \left. + i D_\nu \varphi^a D_\rho \varphi^b D_\sigma \varphi^c f_{a'b'c'} \left(\frac{1 - \exp i T\varphi}{T\varphi} \right)_{a'a} \left(\frac{1 - \exp i T\varphi}{T\varphi} \right)_{b'b} \left(\frac{1 - \exp i T\varphi}{T\varphi} \right)_{c'c} \right\} \quad (2.6)$$

So far we have succeeded in our goal at the level of the baryonic current. Let us turn now to the action (2.4); the situation is not so simple; however, considering a small field expansion of the matrix U [see (1.6)] we have obtained following first few terms:

$$W_c = \frac{-iN_c}{48\pi^2} \epsilon_{\mu\nu\rho\sigma} \int d^4x \left[\frac{1}{5} \text{Tr}(\varphi D_\mu \varphi D_\nu \varphi D_\rho \varphi D_\sigma \varphi) + \frac{3}{2} \text{Tr} \varphi F_{\mu\nu} F_{\rho\sigma} \right. \\ \left. + \frac{1}{4} \text{Tr} F_{\mu\nu} \{ [\varphi, D_\rho \varphi], D_\sigma \varphi \} \right. \\ \left. + \text{Tr} F_{\mu\nu} \left(\frac{5}{2} [D_\rho \varphi^4, D_\sigma \varphi] - [D_\rho \varphi^3, D_\sigma \varphi] - 2 \{ \varphi^3, [D_\rho \varphi, D_\sigma \varphi] \} \right. \right. \\ \left. \left. + 3 \{ \varphi^2, [D_\rho \varphi^2, D_\sigma \varphi] \} + \frac{3}{2} \{ \varphi, [D_\rho \varphi^2, D_\sigma \varphi^2] \} \right. \right. \\ \left. \left. - 4 \{ \varphi, [D_\rho \varphi^3, D_\sigma \varphi] \} \right) + O(F_\pi^{-2}) \right] \quad (2.7)$$

All the terms in this formula are written in a conventional way; including the first term of the Wess-Zumino part. There is a lot of freedom in writing (2.7) due to partial integrations; as an example, the $3-\pi$ terms can be written in full generality

$$\int d^4x \epsilon_{\mu\nu\rho\sigma} \text{Tr} F_{\mu\nu} \left(\left(\frac{1}{2} - \alpha - \beta \right) D_\rho \varphi D_\sigma \varphi \varphi - \left(\frac{1}{2} + \beta \right) D_\rho \varphi \varphi D_\sigma \varphi \right. \\ \left. + \alpha \varphi D_\rho \varphi D_\sigma \varphi + \frac{\beta}{2} (F_{\rho\sigma} \varphi^3 - \varphi F_{\rho\sigma} \varphi^2) \right) \quad (2.8)$$

it is easy to check that the coefficients of α and β in (2.8) are identically zero (using partial integrations) and that (2.7) corresponds to the choice $\alpha = 1/4$, $\beta = 0$ for which (anti) commutators appear explicitly.

Let us stress that all the terms missing in (2.7) contain at least seven pseudoscalar fields so that it can be considered satisfactory for many phenomenological applications.

Next, we will concentrate on the important electromagnetic case corresponding to

$$A_\mu^L = A_\mu^R = -i a_\mu Q^{em}. \quad (2.9)$$

In this case, the action W_c reduces to

$$W_c(U, a_\mu) = W_{WZW} - \frac{i N_c}{48\pi^2} \int d^4x \epsilon_{\mu\nu\rho\sigma} a_\mu \left[\text{Tr } Q(R_\nu R_\rho R_\sigma + L_\nu L_\rho L_\sigma) + i f_{\mu\nu} a_\rho G_\sigma \right] \quad (2.10)$$

$$f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu. \quad (2.11)$$

$$G_\sigma = \text{Tr} [Q^2(L_\sigma + R_\sigma) + \frac{1}{2}(UQ U^{-1}Q L_\sigma + U^{-1}Q U Q R_\sigma)]$$

where G_σ is a gauge invariant quantity. The electromagnetic current can be written in several interesting ways: First, taking the trace of J_μ in (2.2) with the charge matrix Q gives a manifestly invariant form:

$$\begin{aligned} J_\mu^{em} &= \text{Tr} (Q J_\mu) \\ &= \frac{N_c}{48\pi^2} \epsilon_{\mu\nu\rho\sigma} \text{Tr} [Q (\tilde{L}_\nu \tilde{L}_\rho \tilde{L}_\sigma + \tilde{R}_\nu \tilde{R}_\rho \tilde{R}_\sigma) \\ &\quad - 2 F_{\nu\rho} Q (\tilde{L}_\sigma + \tilde{R}_\sigma) - \frac{1}{2} F_{\nu\rho} \{ \tilde{L}_\sigma, U^{-1} Q U \} - \frac{1}{2} F_{\nu\rho} \{ \tilde{R}_\sigma, U Q U^{-1} \}] \end{aligned} \quad (2.12)$$

Using (2.13) directly leads to another equivalent expression

$$\begin{aligned} J_\mu^{em} &= \frac{N_c}{48\pi^2} \epsilon_{\mu\nu\rho\sigma} \text{Tr} [Q (L_\nu L_\rho L_\sigma + R_\nu R_\rho R_\sigma) + 2i \partial_\nu (a_\rho G_\sigma)] \\ &\quad + \frac{N_c}{48\pi^2} \epsilon_{\mu\nu\rho\sigma} 2i (\partial_\nu a_\rho) G_\sigma. \end{aligned} \quad (2.13)$$

where we have split explicitly the current into two gauge invariant parts; the first of which is automatically conserved. The conservation of the total electromagnetic current is in general guaranteed only after use of the equations of motion.

So far, there are two conserved and vector gauge invariant anomalous currents: the baryonic current (2.5) and a second one obtained in subtracting from J_μ^{em} the second line of Eq. (2.13).

At least, in order to exhibit the difficulties met in finding simple expressions in the physical π -field language, we just write explicitly the quantity G_σ [see (2.11)] in such a form:

$$G_\sigma = 2i \mathcal{D}_\sigma \varphi^b \left[\left(\frac{\sin T\varphi}{T\varphi} \right)_{ab} \hbar (\lambda_a Q^2 + \frac{Q_0}{2} \lambda_a Q) + K_b \right] \quad (2.14)$$

$$K_b = \frac{Q_c}{2} \frac{Q_d}{2} d_{db'c'} \left\{ \left(\frac{\sin T\varphi}{T\varphi} \right)_{b'b} (\cos T\varphi)_{c'c} - \left(\frac{1 - \cos T\varphi}{T\varphi} \right)_{b'b} (\sin T\varphi)_{c'c} \right\} \quad (2.15)$$

This shows that, even in the Abelian limit, it is not a trivial matter to obtain a simple formula in terms of the physical field π , not to speak of the same problem with the action (2.10) itself.

3. VECTOR CASE IN SU(2)

Now, we want to specialize the two flavour case because, when restricted to SU(2), the quantities above (2.10), (2.11), (2.12) simplify drastically; moreover this case is important since it is related to the pion physics⁹⁾.

Using the quaternionic parametrisation of U in SU(2) and the representation

$$Q^{em} = e \left(\frac{1}{6} \mathbb{1} + \frac{1}{2} \sigma_3 \right) = e \operatorname{diag} \left(\frac{2}{3}, -\frac{1}{3} \right) \quad (3.1)$$

for the charge matrix, one can prove the following relations:

$$\begin{aligned} G_\sigma &= \frac{1}{2} \hbar Q (L_\sigma + R_\sigma) \\ &= ie \left(\varphi_{3,\sigma} + \frac{\cos |\varphi| \sin |\varphi| - |\varphi|}{|\varphi|^3} (|\varphi|^2 \varphi_{3,\sigma} - \varphi_3 (\vec{\varphi} \cdot \vec{\partial}_\sigma \vec{\varphi})) \right) \end{aligned} \quad (3.2)$$

$$T_L(L_\nu L_\rho L_\sigma \sigma_3) = T_L(R_\nu R_\rho R_\sigma \sigma_3) = 0 \quad (3.3)$$

$$\begin{aligned} T_L(L_\nu L_\rho L_\sigma) &= T_L(R_\nu R_\rho R_\sigma) \\ &= 3\partial_\nu \left[\frac{|\varphi| - \cos|\varphi| \sin|\varphi|}{|\varphi|^3} (\vec{\varphi} \cdot \partial_\rho \vec{\varphi} \wedge \partial_\sigma \vec{\varphi}) \right] \end{aligned} \quad (3.4)$$

The two conserved currents described in the preceding section coincide in SU(2) and take the following form [to be contrasted with Ref.8)]

$$J_\mu^{em} = \frac{e}{2} B_\mu + \frac{e}{2} T_L J_\mu \sigma_3 \quad (3.5)$$

$$= \frac{e}{2} B_\mu + \frac{i(N_c/3)}{64\pi^2} \epsilon_{\mu\nu\rho\sigma} e f_{\nu\rho} T_L \sigma_3 (L_\sigma + R_\sigma)$$

$$\begin{aligned} B_\mu &= \frac{-N_c/3}{8\pi^2} \epsilon_{\mu\nu\rho\sigma} \partial_\nu \left[a_\rho \varphi_{3,\sigma} + \frac{|\varphi| - \cos|\varphi| \sin|\varphi|}{|\varphi|^3} (\vec{\varphi} \cdot \partial_\rho \vec{\varphi} \wedge \partial_\sigma \vec{\varphi} \right. \\ &\quad \left. - a_\rho (|\varphi|^2 \varphi_{3,\sigma} - \varphi_3 (\vec{\varphi} \cdot \partial_\sigma \vec{\varphi})) \right] \end{aligned} \quad (3.6)$$

As a consequence, the non-conserved part of the anomalous electromagnetic current is exactly an isovector contribution; this extra piece is the one which, together with the non anomalous part, assures the conservation of the electromagnetic current.

In SU(2) again, the action (2.10) takes the following form:

$$W_{wz}(\pi, q_\mu) = \frac{i N_c}{48\pi^2} \int d^4x \epsilon_{\mu\nu\rho\sigma} e f_{\mu\nu} \left\{ e f_{\rho\sigma} \frac{\pi^0}{2f_\pi} + \frac{\vec{\pi} \cdot \vec{D}_\rho \vec{\pi} \wedge \vec{D}_\sigma \vec{\pi}}{f_\pi^3} \left(\frac{m_\pi^2 |\gamma| - 2|\gamma|}{4|\gamma|^3} \right) \right\} \quad (3.7)$$

This formula describes all the anomalous photon-pion interactions; the first term is nothing but the standard Abelian anomaly describing the decay $\pi^0 \rightarrow \gamma\gamma$; all other processes (like $\gamma\gamma \rightarrow \pi\pi\pi$)⁹⁾ are included in the second term. It can also be checked that the development of (3.7) in powers of $1/f_\pi$ agrees with the formula (2.7).

4. VECTOR - AXIAL VECTOR CASE IN SU(2)

In order to understand the anomalous processes involving pseudoscalar mesons with their (pseudo) vector partners, an elegant idea was suggested in the Ref.[6]. It consists in considering the vector and axial fields V_μ and A_μ appearing through the gauging as the vector and pseudovector mesons respectively. Phenomenological aspects of this model are treated in Ref[6] and [10]; further investigations are performed by the present authors in Ref [11]. However, a few formal properties of the corresponding action can be derived in the spirit of the first part of this paper.

We will restrict ourselves to the gauging of $SU(2)_L \otimes SU(2)_R \otimes U(1)_V$ with the notations

$$V_\mu = \frac{g}{2i} (\omega_\mu + \vec{V}_\mu \cdot \vec{\sigma}) \quad (4.1)$$

$$A_\mu = \frac{g}{2i} \vec{A}_\mu \cdot \vec{\sigma} \quad (4.2)$$

The isosinglet part corresponds to the physical ω_μ but the interpretation of $\vec{V}_\mu$ and $\vec{A}_\mu$ as the $\vec{\rho}_\mu$ and $\vec{A}_1$ is a lot more subtle as discussed in Refs. 6), 11).

Fortunately enough, due to the absence of anomaly in SU(2), the effective action is very simple; involving only the coupling of the singlet ω_μ to the singlet current [there is no WZ term in SU(2)]

$$W_{WZ}(\pi, \vec{V}_\mu, \omega_\mu, \vec{A}_\mu) = -ig \int d^4x \omega_\mu J_\mu^W \quad (4.3)$$

$$\begin{aligned} J_\mu^W &= \frac{1}{2} \text{Tr} \left(\frac{\delta W_{WZ}}{\delta A_\mu^R} + \frac{\delta W_{WZ}}{\delta A_\mu^L} \right) \\ &= \frac{N_c}{48\pi^2} \epsilon_{\mu\nu\sigma\tau} \text{Tr} \left[R_\nu R_\sigma R_\tau - 3 \partial_\nu (A_\sigma^R R_\tau + A_\sigma^L L_\tau + U A_\sigma^R U^\dagger A_\tau^L - A_\sigma^R A_\tau^L) \right] \end{aligned} \quad (4.4)$$

The current (4.4) is trivially conserved and can be put in a gauge invariant form after some algebra:

$$\begin{aligned} J_\mu^W &= \frac{N_c}{48\pi^2} \epsilon_{\mu\nu\sigma\tau} \text{Tr} \left[\tilde{R}_\nu \tilde{R}_\sigma \tilde{R}_\tau - \frac{3}{2} F_{\nu\sigma} (\tilde{L}_\tau + \tilde{R}_\tau) \right. \\ &\quad \left. + 3 \partial_\nu (A_\sigma (\tilde{L}_\tau - \tilde{R}_\tau) + U A_\sigma U^{-1} A_\tau) \right] \end{aligned} \quad (4.5)$$

It is worth emphasizing that Eqs (4.4) and (4.5) for the singlet current in the presence of vector and axial sources are actually more general: they hold for arbitrary U(N) groups.

In SU(2) again, using for U the quaternionic parametrization, we managed to obtain the action (4.3) in a conventional form analogous to (3.7). The result is

$$\begin{aligned} W_{WZ} &= \frac{i N_c g}{16\pi^2} \epsilon_{\mu\nu\sigma\tau} \int d^4x \partial_\mu \omega_\nu \left\{ g \vec{\psi} \cdot \vec{F}_{\sigma\tau} + \frac{\sin 2|\varphi| - 2|\varphi|}{2|\varphi|^3} (\vec{\psi} \cdot \vec{D}_\sigma \vec{\psi} \wedge \vec{D}_\tau \vec{\psi}) \right. \\ &\quad \left. + 2g (\vec{\psi} \cdot \vec{A}_\sigma \wedge \vec{D}_\tau \vec{\psi}) \left(\frac{\sin |\varphi|}{|\varphi|} \right)^2 - g^2 (\vec{\psi} \cdot \vec{A}_\sigma \wedge \vec{A}_\tau) \frac{\sin 2|\varphi|}{2|\varphi|} \right\} \end{aligned} \quad (4.6)$$

The relevance of this result for the understanding of the $\pi^+ \pi^- \omega \rightarrow \vec{A}_1$ anomalous interactions is discussed in Ref. 11).

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REFERENCES

- 1) E. Witten. Nucl.Phys. B223 (1983) 422, 433;
H. Adkins, C. Nappi and E. Witten, Nucl.Phys. B228 (1983) 552.
- 2) J. Wess and B. Zumino, Phys.Lett. 37B (1971) 95.
- 3) H. Skyrme, Proc.Roy.Soc.London. A260 (1961) 127.
- 4) K.C. Chou, H.Y. Guo, K.Wu and X.C. Song;
Academia Sinica 11P-83-027 and 11P-83-032 (1983).
- 5) N.K.Pak and P.Rossi, to appear in Nucl.Phys.B.
- 6) Ö. Kaymakçalan, S. Rajeev and J. Schechter, Syracuse Univ.Prep. SU-4222-278 (1983).
- 7) P. Di Vecchia, B.Durhuus and J.L. Petersen. Nordita prep. NBI-HE-84.02 (1984).
- 8) C. Callan and E. Witten, Nucl.Phys. B239 (1984) 161.
- 9) N. Pak and P.Rossi, CERN preprint TH3964 (1984).
- 10) H.Gomm, Ö. Kaymakçalan and J. Schechter, Syracuse Univ.Prep. SU-4222-283 (1984).
- 11) Y. Brihaye, N.K. Pak and P. Rossi, in preparation.