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EFFECTIVE LAGRANGIAN DESCRIPTION OF THE
POSSIBLE STRONG SECTOR OF THE STANDARD MODEL

R. Casalbuoni

CERN - Geneva
and
Istituto Nazionale di Fisica Nucleare
Sezione di Firenze

D. Dominici

Istituto Nazionale di Fisica Nucleare
Sezione di Firenze
and
Dipartimento di Fisica dell'Università, Firenze

and

R. Gatto

Département de Physique Théorique
Université de Genève

ABSTRACT

We discuss the effective Lagrangian of the scalar and longitudinal sector of the standard $SU(2) \times U(1)$ model and derive the corresponding Feynman rules. Such a sector becomes strong when the Higgs mass parameter m_H is large. Scalar propagation, in this case, is conveniently described by a degenerate 2×2 matrix. We apply the Feynman rules to calculate scattering amplitudes among longitudinally polarized W and Z, which now satisfy partial wave unitarity also at large m_H . We also calculate production amplitudes among such states and find that they are no longer depressed when m_H is large.

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1. INTRODUCTION

The standard model of weak and electromagnetic interactions is known to develop a strong-interacting sector when the still unknown parameter m_H , which initially coincides with the mass of the physical Higgs particle H , is increased beyond a certain limit. Such a strong-interacting sector includes all the scalar and longitudinal degrees of freedom (i.e., Higgs plus W_L , Z_L , the subscript L standing for longitudinal polarization). Lee, Quigg and Thacker had shown that, for $m_H > \sim 1$ TeV, partial wave unitarity is violated by the lowest perturbative amplitudes of two-body scattering among H, W_L, Z_L ¹⁾. It is the main purpose of the present note to derive from the standard model, through the functional formalism, the set of Feynman rules which allow one to calculate standard model predictions for large M_H .

This will be done by using the results of Refs. 2) and 3) showing that the amplitudes for W_L, Z_L can be evaluated from the corresponding amplitudes for the particles of the scalar sector (the would-be Goldstone bosons), for momenta larger than M_W .

Due to the complexity of the calculation we are able to do this here only in the large- N limit, that is in the approximation where the $O(4)$ scalar Lagrangian of the standard model is replaced with a $O(N)$ Lagrangian and then N is taken equal to 4.

Our main results are shown in Table 1, where the set of Feynman rules (vertices, propagators) are collected, which are equivalent to the effective Lagrangian of the possibly strong sector. These Feynman rules modify those of the standard model, as given for instance in the paper of Veltman⁴⁾, within the sector of H, W_L, Z_L , such as to be now applicable also for large m_H . The demand for such an extension of the set of Feynman rules has been quite lively recently, in fact due to anomalous events at the collider⁵⁾ and some related theoretical speculations⁶⁾, and also due to interest in future prospects at SSC⁷⁾.

2. THE CJT FORMALISM

We are interested in the effective Lagrangian of the bosonic $O(N)$ model defined by the classical action

$$I(\Phi) = \int d^4x \left[\frac{1}{2} (\partial_\mu \Phi)(\partial^\mu \Phi) - \frac{1}{2} m^2 \Phi^2 - \frac{\lambda}{4! N} \Phi^4 \right] \quad (1)$$

where $\Phi^2 = \sum^N (\Phi_a)^2$. To take into account the possibility of two-particle bound states we can use the formalism of Cornwall, Jackiw and Tomboulis (CJT)⁸⁾, where besides the usual sources $J_a(x)$ one also introduces bilocal sources $K_{ab}(x,y)$ in the generating functional $\exp [i W(J,K)]$. In correspondence one has classical fields

$$\varphi_a(x) = \delta W(J,K) / \delta J_a(x) \quad (2)$$

$$G_{ab}(x,y) = \delta W(J,K) / \delta K_{ab}(x,y) \quad (3)$$

The effective action, defined as the double Legendre transform of W takes the form⁸⁾

$$\Gamma(\varphi, G) = I(\varphi) + \frac{i}{2} \text{Tr} \ln D G^{-1} + \frac{i}{2} \text{Tr} (D^{-1} G) + \Gamma_2(\varphi, G) - \frac{i}{2} \text{Tr} \mathbb{1} \quad (4)$$

where D is the free propagator,

$$i D_{ab}^{-1}(\varphi; x, y) = \frac{\delta^2 I(\varphi)}{\delta \varphi_a(x) \delta \varphi_b(y)} \quad (5)$$

and $\Gamma_2(\varphi, G)$ is given by the two-particle irreducible vacuum diagrams calculated for propagators $G_{ab}(x,y)$ and interaction $I_{\text{int}}(\Phi, \phi)$ obtained from the interaction term in Eq. (1) by shifting Φ in $\Phi + \phi$.

3. THE BOUND STATES AT $N \rightarrow \infty$

We shall use the limit $N \rightarrow \infty$. In this limit only the diagram in Fig.1 contributes to Γ_2 . Since this diagram is expressed only in terms of $G_{ab}(x, x)$, one sees that the only possible composites in the limit are $N(N+1)/2$ Lorentz scalars.

We thus introduce local fields⁹⁾

$$\chi(x) = m^2 + \frac{\lambda}{6N} (\varphi^2(x) + \text{tr } G(x, x)) \quad (6)$$

$$\hat{\chi}_{ab}(x) = \frac{\lambda}{3N} (\varphi_a(x)\varphi_b(x) + G_{ab}(x, x)) - \delta_{ab} (\chi(x) - m^2) / 2N \quad (7)$$

which transform respectively as $\underline{1}$ and $[N(N+1)/2]-1$ of $O(N)$. Using the calculations of Ref. 7), one obtains for the effective Lagrangian

$$\Gamma(\varphi, \chi, \hat{\chi}_{ab}) = \int d^4x \left[\frac{1}{2} (\partial_\mu \varphi) (\partial^\mu \varphi) + \frac{\lambda}{12N} \varphi^4 + \frac{3N}{2\lambda} \chi^2 - \frac{3N}{\lambda} m^2 \chi - \frac{1}{2} \varphi^2 \chi + \frac{3N}{4\lambda} \hat{\chi}_{ab} \hat{\chi}_{ab} - \frac{1}{2} \varphi_a \varphi_b \hat{\chi}_{ab} \right] + \frac{i}{2} \text{Tr} \ln [\square + \chi] \delta_{ab} + \hat{\chi}_{ab} \quad (8)$$

Expanding the $\text{Tr} \ln$ in Eq. (8), one sees that in the $N \rightarrow \infty$ limit the traceless part of the argument disappears. Therefore $\hat{\chi}_{ab}$ can be eliminated from Eq. (8) by use of its equation of motion

$$\hat{\chi}_{ab} = \frac{\lambda}{3N} (\varphi_a \varphi_b - \frac{1}{N} \delta_{ab} \varphi^2) \quad (9)$$

leading to the action

$$\Gamma(\varphi, \chi) = \int d^4x \left[\frac{1}{2} (\partial_\mu \varphi) (\partial^\mu \varphi) + \frac{3N}{2\lambda} \chi^2 - \frac{3N}{\lambda} m^2 \chi - \frac{1}{2} \varphi^2 \chi \right] + \frac{i}{2} \text{Tr} \ln [\square + \chi] \quad (10)$$

This form of Γ coincides with the action of Coleman, Jackiw and Politzer¹⁰⁾, derived in absence of the fields $\hat{\chi}_{ab}$.

4. THE EFFECTIVE ACTION

We have thus shown that in the $N \rightarrow \infty$ limit the only expected bound state is the $O(N)$ singlet corresponding to χ . The effective action of Eq. (10) is

renormalized by adding counterterms $c_1\chi$ and $c_2\chi^2$, where by regularizing with a cut-off Λ and for a renormalization point M ,

$$c_1 = \frac{N}{32\pi^2} \Lambda^2, \quad c_2 = -\frac{N}{64\pi^2} \ln \Lambda^2/M^2 \quad (11)$$

The effective potential V can be obtained from Eq. (10) by choosing constant fields ϕ and χ and using Eq. (11):

$$V(\varphi, \chi) = -\frac{3N}{2\lambda} \chi^2 + \frac{3Nm^2}{\lambda} \chi + \frac{1}{2} \varphi^2 \chi - \frac{N}{128\pi^2} \chi^2 + \frac{N}{64\pi^2} \chi^2 \ln(\chi/M^2) \quad (12)$$

The extremum of V for $m^2 < 0$ is given by

$$\langle \chi \rangle = 0, \quad \langle \varphi^2 \rangle = v^2 = -\frac{6Nm^2}{\lambda} \quad (13)$$

Introducing $H = \phi_N - v$, $w_i = \phi_i$ ($i = 1, \dots, N-1$) and $\sigma = \chi/m_H$, where $m_H^2 = -2m^2$, the effective action of Eq. (10) becomes

$$\Gamma = \int d^4x \left[\frac{1}{2} \partial_\mu H \partial^\mu H + \frac{1}{2} \partial_\mu w_i \partial^\mu w_i + \frac{1}{2} v^2 \sigma^2 - v m_H H \sigma - \frac{1}{2} m_H (H^2 + w_i w_i) \sigma \right] - \frac{N}{2} \sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n} (m_H)^n \int \prod_i^n d^4x_i V_n(x_1, \dots, x_n) \sigma(x_1) \dots \sigma(x_n) \quad (14)$$

where

$$V_n(x_1, \dots, x_n) = \int \prod_i^n \left(\frac{d^4 p_i}{(2\pi)^4} \right) (2\pi)^4 \delta^4 \left(\sum_i^n p_i \right) \cdot e^{i \sum_i^n p_i x_i} V_n(p_1, \dots, p_n) \quad (15)$$

and

$$V_2(p) = \frac{1}{16\pi^2} \left[1 - \ln(-p^2/M^2) \right] \quad (16)$$

$$V_n(p_1, \dots, p_n) = -i(-1)^n \int \frac{d^4 k}{(2\pi)^4} \prod_{i=1}^n \frac{1}{(p_i + \dots + p_{i+k})^2}, (n \geq 3) \quad (17)$$

5. FEYNMAN RULES

The Eqs. (14)-(17) give the complete finite expression of the effective action of our problem. By performing the appropriate functional derivatives, one can read from Eq. (14) all the one-particle irreducible Green's function of the theory. In Table 1 we have summarized the Feynman rules for our effective Lagrangian (14).

6. PHYSICAL INTERPRETATION

If in the action Γ in Eq. (14), one keeps at most linear terms in m_H all terms containing the V_n can be neglected. The field σ can then be eliminated from its equation of motion in this limit

$$v^2 \sigma - v m_H H - \frac{1}{2} m_H (H^2 + w_i w_i) = 0 \quad (18)$$

giving for Γ

$$\begin{aligned} \Gamma = \int d^4x \left[\frac{1}{2} \partial_\mu H \partial^\mu H - \frac{1}{2} \partial_\mu w_i \partial^\mu w_i - \frac{m_H^2}{8v^2} (H^2 + w_i w_i)^2 - \right. \\ \left. - \frac{m_H^2}{2v} (H^2 + w_i w_i) - \frac{m_H^2}{2} H^2 \right] \quad (19) \end{aligned}$$

which is the classical action for the Higgs sector.

This elimination of the field σ that we have here performed for small m_H/v becomes unpractical for larger m_H/v . On the other hand, from the $H - \sigma$ propagator matrix of Table 1, one verifies immediately that there is only one complex pole μ^2 given by the equation

$$\mu^2 [1 + \Delta(\mu^2)] - m_H^2 = 0 \quad (20)$$

The existence and location of this pole can also be derived by summation of ladder diagrams¹¹⁾. The pole residuum is a 2×2 matrix. The matrix is degenerate and its rank is one. The location of the pole in the complex μ^2 - plane evolves from

$$\mu^2 = m_H^2 \left(1 - \frac{i}{8\pi} \frac{m_H^2}{v^2} + O(m_H^4/v^4) \right) \quad (21)$$

for small m_H/v , to

$$\mu^2 = \frac{16\pi v^2}{3} \left[-i + \frac{16\pi}{3} \frac{v^2}{m_H^2} \left(1 - \frac{2i}{3\pi} \right) + O(v^4/m_H^4) \right] \quad (22)$$

for large m_H/v . We have verified that Eq. (22) is accurate to 3%, for $m_H > 2.8$ TeV. In Eqs. (21) and (22) the renormalization point has been taken equal to $|\mu^2|$. Of course, one could have chosen any other re-normalization point and the pole position would have remained unaffected.

7. TWO-BODY AMPLITUDES AMONG LONGITUDINALLY POLARIZED VECTOR BOSONS

As the simplest application, we calculate the two-body amplitudes among longitudinally polarized $W^\pm$ and Z . On the basis of the result of Refs. 2) and 3), then amplitudes are identical for c.m. energy $\sqrt{s} \gg M_W$ to those among the corresponding scalars $W^\pm$ and Z , which can now be evaluated by using the Feynman rules of Table 1.

We obtain

$$T(W_L^+ W_L^- \rightarrow W_L^+ W_L^-) = -\sqrt{2} G_F m_H^2 \frac{s}{s[1+\Delta(s)] - m_H^2} + (s \leftrightarrow t) \quad (23)$$

$$T(Z_L Z_L \rightarrow W_L^+ W_L^-) = -\sqrt{2} G_F m_H^2 \frac{s}{s[1+\Delta(s)] - m_H^2} \quad (24)$$

$$T(Z_L Z_L \rightarrow Z_L Z_L) = -\sqrt{2} G_F m_H^2 \frac{s}{s[1+\Delta(s)] - m_H^2} + (s \leftrightarrow t, s \leftrightarrow u) \quad (25)$$

The perturbative limits of these amplitudes ($\Delta(s) \rightarrow 0$) coincide with the expression of Lee, Quigg and Thacker¹⁾, which had been shown by the authors to violate partial-wave unitarity for $M_H > \sim 1$ TeV. On the other hand, the full expressions (23)-(25) are easily seen to be fully consistent with unitarity.

8. MULTIPLE PRODUCTION OF W AND Z

As a more complex application, we have calculated the production processes $2W_L \rightarrow 4W_L$, $2W_L \rightarrow 2Z_L + 2W_L$, and so on. The Feynman graphs are shown in Fig.2. We shall not report here the lengthy expressions. It is, however, easy to verify that in the limit of large s the leading production amplitudes [which arise from graphs a) and b) in Fig.2] differ from the elastic two-body amplitudes, Eqs. (23)-(25) only by kinematical factors of the type $1/s$ or $1/s(\ln s)^2$, without appearance of any coupling constant factors.

9. CONCLUSIONS

The Feynman rules of Table 1 refer to what we think is the most convenient formal description of the possibly strong longitudinal and scalar sector of the standard model. Such a description is unusual from the point of view of elementary perturbation theory. Two scalars H and σ can propagate and interact, but their propagation is described by a 2×2 matrix propagator possessing one single complex pole with a degenerate residuum. The pole position evolves from m_H [see Eq. (21)] when m_H/v is small ($v = 250$ GeV) but then diverges from m_H , becoming pure imaginary for infinite m_H/v [Eq. (22)]. The notion of a physical Higgs particle thus becomes void of physical meaning at large m_H .

From the result of Refs. 2) and 3), the amplitudes for W_L, Z_L can be calculated from the corresponding amplitudes for the scalar fields w, z , for momenta much larger than M_w . The Feynman rules of Table 1, are thus in such a limit the rules for the scalar and longitudinal sector of the standard $SU(2) \times U(1)$ model and can be used to calculate any physical process.

We have applied such rules to calculate the two-body amplitudes among W_L, Z_L . Such amplitudes coincide with their perturbative limits of Ref. 1) for small m_H , but they do no longer violate unitarity when m_H becomes large.

We have also calculated production processes (see Fig.2) finding that at large s they are not intrinsically depressed with respect to scattering processes. This confirms qualitatively the results of Chanowitz and Gaillard¹²⁾ who extrapolated perturbation theory for a preliminary indication.

Within the $N \rightarrow \infty$ limit the above results are complete. In particular we find no strong interacting propagator pole at $J = 1$ or higher in this limit. Such poles may however arise at higher order in $1/N$. They may be of great

physical interest, perhaps also in connection with the recent experimental results⁵⁾, as they would modify the vector boson propagator into a matrix propagator with sensible deviations from perturbation theory especially in the neighbourhood of such singularities.

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FIGURE CAPTIONS

- Fig. 1 Diagram contributing to Γ_2 for $N \rightarrow \infty$.
- Fig. 2 Diagrams for multiple W,Z production.

Table 1

Feynman rules for the possibly strong sector of the standard model (longitudinal and scalar sector).

$$\begin{array}{ccc}
 \text{H} \text{ --- } \text{H} & i D_{HH}(q^2) = i \frac{1 + \Delta(q^2)}{q^2(1 + \Delta(q^2)) - m_H^2} & \text{Higgs-propagator}
 \end{array}$$

$$\begin{array}{ccc}
 \sigma \text{ --- } \sigma & i D_{\sigma\sigma}(q^2) = i \left(\frac{q}{v} \right)^2 \frac{1}{q^2(1 + \Delta(q^2)) - m_H^2} & \sigma\text{-propagator}
 \end{array}$$

$$\begin{array}{ccc}
 \text{H} \text{ --- } \sigma & i D_{H\sigma}(q^2) = i \left(\frac{m_H}{v} \right) \frac{1}{q^2(1 + \Delta(q^2)) - m_H^2} & \text{H-}\sigma\text{-propagator}
 \end{array}$$

$$D_{\sigma H}(q^2) = D_{H\sigma}(q^2)$$

$$\Delta(q^2) = \frac{N}{32\pi^2} \left(\frac{m_H}{v} \right)^2 \left[1 - \ln \frac{-q^2}{M^2} \right]$$

$$-i m_H$$

$$-i m_H$$

Fig. 1

a)

b)

c)

Fig. 2

Table 1 (continued)

$$-i N(m_H)^3 V_3(p_1, p_2, p_3)$$

$$V_3(p_1, p_2, p_3) = \frac{1}{8\pi^2 p_3^2} \int_0^1 dy \frac{1}{ay + b(1-y) - y(1-y)} \ln \frac{y(1-y)}{ay + b(1-y)}$$

$$a = p_1^2/p_3^2, \quad b = p_2^2/p_3^2$$

$$-i \frac{N}{2} (n-1)! (m_H)^N V_n(p_1, \dots, p_n)$$

$$V_n(p_1, \dots, p_n) = -i (-1)^n \int \frac{d^4 k}{(2\pi)^4} \prod_{i=1}^n \frac{1}{(p_i + \dots + p_i + k)^2}$$

$$n \geq 3$$