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STATUS REPORT ON LATTICE QCD^{*)}

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A B S T R A C T

A concise and critical review on numerical calculations in lattice QCD is given. The report is on the level of non-experts.

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A concise and critical review on numerical calculations in lattice QCD is given. The report is on the level of non-experts.

1. INTRODUCTION

Let me address the question: Whether and to what extent is QCD the theory of strong interactions? The answer is very vague, because at present-day energies it is difficult to disentangle perturbative and non-perturbative effects. Basic assumptions of QCD are

- confinement
- spontaneous chiral symmetry breaking
- dynamical mass generation

The presupposed behaviour of QCD enters everywhere in model building.

Quantitative non-perturbative methods are needed, such that

- approximations can be controlled
- improved by increasing the computational effort
- QCD could be falsified

From this point of view, lattice QCD is promising, although the quantitative precision is still not accurate enough to allow any falsification. Let me first sketch the basic concepts. For more details the reader should consult the review of Hasenfratz¹

1.1 Continuum gauge theory

Let us consider pure SU(N) Yang-Mills theory in the Euclidean. Vacuum expectation values of gauge invariant operators are calculated by means of a functional integral

$$\langle 0 \rangle = Z^{-1} \int DA_0 \exp\left(-\frac{1}{g^2} \int d^4x F_{\mu\nu}^a F_{\mu\nu}^a\right), \quad (1)$$

which is defined in renormalized perturbation theory. If a sensible final theory exists, it is not allowed to depend on the used regularization scheme. This leads to the renormalization group equation. In case of a mass:

$$\left(\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g}\right) m(g, \mu) = 0. \quad (2)$$

The β -function can be calculated in perturbation theory. The first two terms are universal, i.e., independent of the used regularization scheme. A well-known two-loop calculation² gives

$$\beta(g) = -\beta_0 g^3 - \beta_1 g^5 + O(g^7) \quad (3)$$

$$\text{with } \beta_0 = \frac{11}{3} \left(\frac{N}{16\pi^2}\right) \text{ and } \beta_1 = \frac{34}{3} \left(\frac{N}{16\pi^2}\right)^2.$$

Integrating the renormalization group equation (2) yields

$$m(g, \mu) = \text{const } \mu (\beta_0 g^2)^{-\beta_1 / (2\beta_0)} \exp\left(-\frac{1}{2\beta_0 g^2}\right) [1 + O(g^2)]. \quad (4)$$

From Eq. (4) we read-off: the mass $m(g, \mu)$ is a non-perturbative quantity, it vanishes to all orders in an expansion around $g^2 = 0$. Therefore we are led to the question: what is a non-perturbative definition of the functional integral (1)? (Up to now, the functional integral was defined by perturbation theory.) It is useful to remember: also an ordinary integral $\int_a^b f(x) dx$ has to be defined. For instance, by means of the Riemann integral:

$$\begin{aligned} \int_a^b f(x) dx &= \lim_{\sup\{\epsilon_i\} \rightarrow 0} \sum_{i=1}^N \epsilon_i f(x_i), \quad x_i = x_{i-1} + \epsilon_i, \quad x_N = b, \quad x_0 = a \\ &= \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N f(x_i), \quad x_i = a + \frac{i}{N}(b-a) \end{aligned} \quad (5)$$

The Riemann integral does not depend on the precise way of choosing the intervals ϵ_i as long as $\sup\{\epsilon_i\} \rightarrow 0$. The calculus, allowing to solve many integrals exactly, was of course the great historical progress. Using fast

computers one may, however, well evaluate a Riemann integral numerically on a regular 1d lattice, choosing all ε_i to be equal to $\varepsilon_i = (b-a)/N$. For $N = 11$ this is indicated in Fig. 1.

FIGURE 1
Approximation of the Riemann
integral by $N = 11$ regular
intervals.

For the non-perturbative evaluation of functional integrals, an advanced calculus is missing. But we may immediately try a numerical evaluation on regular lattices. In the limit lattice spacing $a \rightarrow 0$ this provides a definition of the continuum functional integrals. As in the case of the Riemann integral, this limit should not depend on details of the lattice structure. For numerical calculations, a regular lattice is, however, most convenient.

1.2. Lattice QCD

After the heuristic motivation of the previous subsection, let us now recapitulate lattice QCD as introduced by Wilson³

On a 4d hypercubic lattice the (Euclidean) action

$$S = S_G + S_F \quad (6)$$

is given by a gauge (G) and by a fermion (F) part. The gauge part is

$$S_G = - \frac{1}{g^2} \sum_p (\text{Tr } U_p + U_p^\dagger) . \quad (7)$$

Here the sum is over all the (non-oriented) plaquettes p of the lattice. To the links ℓ of the lattice matrices $U(\ell) \in \text{SU}(N)$ of the $\text{SU}(N)$ gauge group are attached, and at each plaquette p , U_p is defined as indicated in Fig. 2.

The fermion part of the action is bilinear in the fermion fields, which are attached to on the sites of the lattice:

Fig. 2

$$U_p = U_1 U_2 U_3^* U_4^*$$

$$S_F = \sum_{x,y} \bar{\Psi}^+(x) \Delta_{xx}(\{U\}) \Psi(y) . \quad (8)$$

I will not specify the matrix $\Delta_{xy}(U)$ here. For problems connected with putting fermions on the lattice, see Refs. 1 and 4. It is important to note that the action (6)-(8) preserves gauge invariance. Vacuum expectation values of gauge invariant operators are calculated with respect to the partition function

$$Z = \int \prod_{\lambda} dU_{\lambda} \prod_i d\bar{\Psi}_i^+ d\Psi_i \exp(-S) = \int \prod_{\lambda} dU_{\lambda} \det[\Delta(\{U\})] \exp(-S_G) \quad (9)$$

1.3. The continuum limit

In the classical continuum limit $a \rightarrow 0$, $g^2 = \text{const}$, $U = e^{iaA_{\mu}}$ the continuum (classical) action is reobtained.

We are interested in the quantum continuum limit. This is (on an infinite lattice) the limit $a(\beta) \rightarrow 0$, where $a(\beta)$ is measured in units of the correlation length

$$\xi = m^{-1} . \quad (10)$$

The correlation length is defined to be the inverse mass gap m of the system. $\beta = 1/g^2$ is introduced to emphasize the analogy with classical statistical mechanics. Equation (9) is just the partition function of a 4d statistical mechanics of gauge and fermion fields. At a critical value $\beta = \beta_c$ such a system may have a (second-order phase transition) and the correlation length ξ is divergent: $\xi \rightarrow \infty$ in units of the lattice spacing $[a]$. The physical scale is set by the correlation length (inverse mass gap) and hence $a(\beta) \rightarrow 0$ for $\beta \rightarrow \beta_c$. For lattice QCD, $\beta_c = \infty$ is argued. The behaviour in going from strong

(g^2 large) to weak (g^2 small) coupling is depicted in Fig. 3. In this figure, the circle indicates a typical physical scale, for instance the proton size.

FIGURE 3

For simplicity, let us now discuss pure gauge theory. Setting $\mu = a^{-1}$, the renormalization group equation (2) predicts quantitatively how the continuum limit is approached:

$$m = \text{const } a^{-1} (\beta_0 g^2)^{-\beta_1/(2\beta_0^2)} \exp\left(-\frac{1}{2\beta_0 g^2}\right) [1+O(g^2)]$$

$$= \text{const } \Lambda_L, \quad \Lambda_L = \text{const } \Lambda_{\text{MOM}}. \quad (11)$$

Neglecting the $O(g^2)$ corrections the behaviour (11) is called asymptotic scaling⁵. Λ_L is a cut-off independent parameter mass and defines the scale. By a one-loop calculation, Λ_L can be related to Λ -scales of other regularization schemes⁶.

For mass ratios, we expect

$$\frac{m_1}{m_2} = \text{const}_{12} + O\left(\frac{a^2}{\xi^2} \ln \frac{a}{\xi}\right). \quad (12)$$

This behaviour is called scaling (in general)⁵. The corrections to scaling are exponentially small in the coupling g^2 . Therefore we expect first scaling and then asymptotic scaling (see Fig. 3). Symanzik improved actions⁷ improve scaling, for instance

$$O\left(\frac{a^2}{\xi^2} \ln \frac{a}{\xi}\right) \rightarrow O\left(\frac{a^4}{\xi^4} \ln \frac{a}{\xi}\right),$$

but not necessarily asymptotic scaling.

Rotation and Lorentz invariance are broken by the lattice regularization. These symmetries have to become restored in the continuum limit. It is encouraging that this already happens at intermediate couplings accessible to nowadays numerical calculations.

Lang and Rebbi⁸ demonstrated restoration of rotation invariance for SU(2) lattice gauge theory (Fig. 4). Recently, a similar result was obtained using the SU(3) gauge group (Hasenfratz et al.⁸).

FIGURE 4
Equipotential lines for SU(2) lattice gauge theory
at $\beta = 2.00$ and $\beta = 2.25$.

Lorentz invariance was investigated by Billoire, Meyer, Panagiotakopoulos and myself^{9*)} and independently by Schierholz and Teper¹¹. Figure 5 demonstrates restoration of Lorentz invariance for low-lying momenta $\vec{k}$. The SU(2) gauge group is used. In Ref. 11, similar results are also obtained in case of the SU(3) gauge group.

1.4. The Monte Carlo (MC) method

As already outlined we have to rely on numerical evaluations of the partition function (9). As the Boltzmann factor e^{-S} varies over several orders of magnitude, a naive direct evaluation (Riemann sum) is completely inefficient. The MC method (for a review, see Binder¹²) sets up a Markov process (Metropolis, heat bath, etc.) such that

$$p_{eq}(\{U, \dots\}) \rightarrow \text{const } e^{-S(\{U, \dots\})}$$

*) These results were first presented in my Cargèse lectures¹⁰. For related work, see also Ref. 5.

FIGURE 5
"Effective" masses $m(\vec{K}, t) = \sqrt{E(\vec{K}, t)^2 - \vec{K}^2}$ for several momenta $\vec{K}$ and measured at different distances⁹.

in the asymptotic limit. In that way "importance sampling" is achieved, and vacuum expectation values with respect to the partition function (9) become arithmetic averages:

$$\langle O \rangle = \bar{O} = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N O(\{U, \dots\}_i)$$

1.5. The fermion determinant

A major problem is the inclusion of fermions into MC simulation. This has to do with the fact that fermions are elements of a Grassmann algebra. Integrating out the bilinear fermions, one is left with the determinant $\det[\Delta(\{U\})]$ in Eq. (9). $\Delta(\{U\})$ is a huge, but sparse, matrix depending on all the gauge fields $\{U\}$. Several proposals have been made for evaluating inverse elements of this matrix.

The use of pseudofermions has mainly been advertized by the Italian group¹³. The basic idea is to exploit the equation

$$(\Delta^{-1})_{iK} = \int \prod_j (d\phi_j^* d\phi_j) \phi_i^* \phi_K e^{-\sum_{x,y} \phi_x^* \Delta_{xy} \phi_y}$$

by means of a MC calculation with auxiliary boson field ϕ_j, ϕ_j^* , called "pseudofermions".

The hopping expansion (³, Erice lectures) is useful for Wilson fermions. One writes

$$\Delta = 1 - \kappa B(\{U\}),$$

and a systematic expansion

$$(\Delta^{-1})_{ij} = \sum_{\lambda} c_{\lambda}(\{U\}; ij) \kappa^{\lambda}$$

for inverse matrix elements exists. It has been put forward by the Hasenfratz group¹⁴ and was recently used¹⁵ for including the fermion determinant.

Stochastic methods were proposed by Foerster and myself¹⁶ and later also by Kuti¹⁷. The idea is to expand the determinant $\Delta(\{U\})$ in terms of random walks and to simulate these random walks by MC techniques. For some recent work, see Ref. 18.

Most promising, at present, is the microcanonical approach¹⁹. The energy is kept fixed and if the system is ergodic, averaging over the energy shell can be replaced by averaging over time

$$\langle O[q] \rangle = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T dt O[q(t)],$$

where $\{q_n(t)\}$ is the solution of the canonical equations of motion. β is then calculated via the equipartition theorem. The application of this approach to lattice gauge theories is investigated in Ref. 20. Polonyi and Wyld suggested to bosonize the path integral of lattice field theories with fermion fields. Using then the microcanonical formulation the determinant can be included and the number of instructions to the computer to obtain a new configuration is proportional to the lattice volume. A major problem of the microcanonical approach can, however, be lack of ergodicity.

Finally, the usefulness of Langevin's equation²¹ remains to be explored for the fermion determinant problem.

1.6. Results

As all the methods for including the fermion determinant encounter problems, most calculations are done in the "quenched approximation"²², which simply consists in replacing

$$\det \Delta \rightarrow 1.$$

In the forthcoming sections, I will summarize results on the following topics:

2. String tension;
MC renormalization group (MCRG)
3. Glueballs
4. Finite temperature phase transition
5. Chiral symmetry breaking
6. Hadronic spectrum

All these results rely on fits to (asymptotic) scaling at moderate β values on rather small sized lattices. This establishes a new type of phenomenology and one may speak about the "lattice model". The relation to very large β (continuum limit) and implied possible systematic errors are not (yet) under sufficient control. Some final conclusions are presented in Section 7.

2. STRING TENSION AND MCRG

2.1. String tension

In units of Λ_L the development of the string tension,

$$\sqrt{K} = C \Lambda_L,$$

estimates may be sketched as follows:

SU(2) gauge group:

1980 :	78±12	Creutz ^{23 24}
1981 :	~78	Bhanot and Rebbi,... ²⁵
1983 :	~76/~56	Stack; Gutbrod and Montvay ²⁶
1984 :	50± 3	Billoire and Marinari ²⁷

SU(3) gauge group:

1980 :	215±70	Creutz ²⁴
1981/83 :	~160	Pietarinen,... ²⁸
1983 :	~145	Gutbrod et al. ²⁹
1983/84 :	~ 90	Parisi et al., ..., Barkai et al.,... ³⁰

Figure 6 reproduces results from the original paper by Creutz²³. Dots are MC data.

One recognizes a rapid cross-over from the lowest order SC expansion ($-\ln \beta/4$) to asymptotic scaling. The work by Creutz was noticed with great excitement and stimulated the tremendous amount of subsequent work, because it suddenly seemed feasible to reach the (asymptotic) scaling region on rather small sized lattices.

FIGURE 6
SU(2) string tension estimate of
Creutz²³.

The full curve in Fig. 6 is the highest available order SC expansion, due to Münster³¹. The asymptotic scaling estimate is just tangent to the SC curve, suggesting a simple way of estimating continuum results from high order SC expansions.

Despite this combined evidence the estimate $\sqrt{K} \approx 78 \Lambda_L$ did not survive later investigations. The first evidence for a significantly lower string tension came from SU(3) investigations by Gutbrod et al.²⁹ and Parisi et al.³⁰. Considering the whole potential²⁶ the reason is found to be due to Coulomb term contributions to the Creutz ratios²⁴. A particularly clear-cut method for avoiding these contributions is to calculate momentum zero correlations of Polyakov loops²⁷.

We like now to set the Λ_L scale in MeV units. The charmonium potential suggests

$$\sqrt{K} \approx 420 \text{ MeV.} \quad (13a)$$

Relying on this we may use the best available SU(3) results $\sqrt{K} \approx 90 \Lambda_L$ to obtain

$$\Lambda_L \approx 4.7 \text{ MeV.} \quad (13b)$$

At least up to $\beta \gtrsim 6$ the SU(3) string tension results exhibit violations of asymptotic scaling. Therefore the reliability of (13b), extracted from the

largest available β -values, may be doubted. We may, however, believe in scaling of mass ratios and use directly Eq. (13b) at appropriate β -values.

2.2. MCRG

MCRG calculations for lattice gauge theories were initiated by Wilson³². For an introduction to the method, see Ref. 1. Recently, Hasenfratz et al.³³ investigated SU(3) lattice gauge theory. Let us define $\Delta\beta$ to be the shift in the fundamental plaquette coupling of the standard Wilson action corresponding to a change of scale by a factor of two. Figure 7 compares results as obtained by the "improved ratio method"³³ with $\Delta\beta$ estimates as obtained from

FIGURE 7
The average shift $\Delta\beta$ as a function of β obtained from the analysis of one-loop improved ratios, from the string tension and from the finite temperature phase transition T_c .

the string tension of Barkai et al.³⁰ and from the finite temperature phase transition T_c ^{34 35}. All three $\Delta\beta$ estimates show a similar behaviour. Asymptotic scaling $\Delta\beta \approx 0.6$ is clearly violated, but within the errors scaling would be consistent.

3. GLUEBALLS

Glueballs are the mass spectrum states of pure lattice gauge theories. For a detailed review of lattice glueball calculations, see Ref. 10. An update is given in Ref. 36. All calculations agree in finding

$$m(0^{++}) = c \Lambda_L,$$

to be the lowest glueball (mass gap). Only for this state an unambiguous cross-over from SC to [asymptotic(?)] scaling is seen. SU(2) and SU(3) mass gap estimates developed as follows (for more complete references, see the reviews^{10 36}):

SU(2) gauge group:

1980 : 280± 90 Berg³⁷; Bhanot and Rebbi²⁵
 1981 : 140± 60 Münster, SC³⁸
 1981/82 : ≈200/≈170 Berg, Billoire, Rebbi,...³⁹
 1984 : 190±30 Berg, Billoire, Meyer, Panagiotakopoulos⁹

SU(3) gauge group:

1981 : ≈670 Münster, SC³⁸
 1981 : 720±100 Hamber and Parisi²²
 1982 : 350± 50 Berg and Billoire⁴⁰
 1982 : ≈310 Münster³⁸, Seo⁴¹, SC
 1983 : 280± 40 Berg and Billoire⁴²
 1984 : 260± 20 De Forcrand, Schierholz, Teper⁴³

The first (1980) mass gap estimates were done for the SU(2) gauge group. As one was very optimistic at that time, no sophisticated method was used and these estimates simply rely on plaquette-plaquette correlations. Correspondingly the (estimated) error was very high. Within these errors there is still consistency with nowadays best estimates, which rely on a MC variational method (MCV) proposed by Wilson.

MC calculations are often considered as a "black box". SU(3) mass gap estimates are a good example that errors in complicated analytic calculations are as difficult to trace back. The first estimate relied on the SC series of Ref. 38. The first MCV calculation⁴⁰ had (by reasons of statistics) to rely on correlations at distance $t = 1$. Already at this distance a clear signal of scaling was found and the mass gap estimate was significantly lower than the previous SC one. Stimulated by this result, errors in the SC calculation were discovered^{38 41} and consistency with MC was reobtained.

Asymptotic scaling violations?

In contrast to results reported in the previous section, MC data for $m(0^{++})$ are (still) consistent with asymptotic scaling. As one may see from the MCV high statistics SU(2) analysis⁹, this could be due to the fact that results at larger distances are not precise enough. The improved SU(3) analysis of Ref. 43 relies on a source method. In fact, at the largest considered β -value ($\beta = 5.9$), asymptotic scaling may be violated. Astonishing, however, that at lower β -values, asymptotic scaling is confirmed with a high precision.

Other glueball states

The irreducible representations of the cubic group may be used to classify glueball states according to spin J , parity P and C-parity C ⁴⁴. In Ref. 42, the irreducible representations of the cubic group were constructed on Wilson

loops up to length 8. In the continuum limit one obtains 16 states:

$$0^{PC}, 1^{PC}, 2^{PC} \text{ and } 3^{PC} \text{ with } P = \pm, C = \pm.$$

In MCV calculations, the lowest state above the mass gap is 2^{++} . One finds^{10 42 43}:

$$m(2^{++}) \gtrsim 2m(0^{++}).$$

Of phenomenological interest⁴⁵ is the 0^{-+} state. MCV calculations give^{10 42}

$$m(0^{-+}) \gg m(2^{++}).$$

This indicates that either the $\chi(1440)$ is no glueball or that the present lattice glueball calculations are very remote from continuum QCD.

The glueball situation may be summarized in two figures. Figure 8 compares MCV results with high order SC expansions (for details, see Ref. 10). Only for the 0^{++} state one finds a β -region where MC results are significantly different from the SC series expansion, and consistent with the (asymptotic) scaling RG estimate.

Figure 9 exhibits the common problem of all nowadays MC calculations on QCD (non-Abelian gauge theories). We have a β region up to $\beta < \beta_{\max}$ (for

FIGURE 8

FIGURE 9

Continuum limit extrapolations for several glueballs.

glueballs $\beta_{\max} \sim 5.9 \rightarrow 6.0$) under control. Then we have to extrapolate all the way to the continuum limit $\beta \rightarrow \infty$. Only if these extrapolations follow with high precision the (asymptotic) scaling curves (broken lines), our results are continuum results. If deviations (dotted lines) are still possible, the continuum limit is essentially unpredictable.

4. FINITE TEMPERATURE PHASE TRANSITION

Using SC methods the finite temperature phase transition was first investigated by Polyakov and Susskind⁴⁶. A finite temperature is introduced by taking (at fixed β) the extension N_L of the Euclidean lattice in time direction to be finite, while sending $N_X = N_Y = N_Z \rightarrow \infty$. An order parameter is the vacuum expectation value $\langle L \rangle$ of the Polyakov loop

$$L = \text{Tr} \prod_{\vec{x}} U(\vec{x}, t), \quad (14)$$

which is closed by means of the periodic boundary conditions in time direction. The critical temperature T_c separates the confinement phase ($\langle L \rangle = 0$) from a supposed plasma phase of free quarks and gluons ($\langle L \rangle \neq 0$). For non-Abelian gauge theories (without quarks) the first MC investigations (gauge group SU(2) and using $\langle L \rangle$ as order parameter) were carried out by McLerran and Svetitsky and by Kuti, Polonyi and Szlachányi⁴⁷. For the gauge group SU(3), Fig. 10 (from a recent paper⁴⁸) shows how $\bar{L} = \langle L \rangle$ behaves as an order parameter.

Alternatively, one may study singularities in thermodynamic variables like the average plaquette action, the gluonic energy density, ... This approach was mainly put forward by the Bielefeld group⁴⁹. For SU(3) the phase transition is 1.-order as exhibited, for the gluonic energy density, in Fig. 11 (from Ref. 48).

Recently, Kennedy et al⁵⁰ did a high precision SU(3) study of the distribution of Polyakov loops in the complex plane. In Fig. 12, one realizes clearly the three complex roots corresponding to the centre of the SU(3) gauge group. The histograms on the right indicate the phase transition. The critical temperature T_c has the dimension of a mass. In the continuum limit (asymptotic) scaling is expected. A conventional estimate is⁵⁰

$$T_c \approx 55 \Lambda_L, \quad (15a)$$

FIGURE 10
 $\langle L \rangle$ above and below T_c^{48} .

FIGURE 11
SU(3) gluonic energy density⁴⁸.

FIGURE 12
The graphs on the left show the distribution of the Polyakov loop P in phase space. The top row corresponds to $\beta < \beta_c$ (confined), the middle row is at β_c ,

and one finds violations of asymptotic scaling as already mentioned in Section 2.2. Setting the scale by the string tension [Eq. (13)], this amounts to

$$T_c \approx 260 \text{ MeV.} \quad (15b)$$

By other ways of setting the scale, this may eventually be reduced by a factor of two (see next section).

A crucial question is the influence of fermions on the finite temperature phase transition. Whether there is no, a 2.- or a 1.-order phase transition is of importance for a variety of problems reaching from heavy ion collisions⁵¹ up to cosmology⁵². As was pointed out by Banks and Ukawa and by DeGrand and DeTar⁵³, the influence of fermions on the 4d gauge system is similar to turning on a magnetic field in a 3d spin system. In the 3d spin system, the 2.-order phase transition disappears. In the gauge system we will encounter a subtle quantitative problem, as was pointed out by Hasenfratz et al.⁵⁴. Figure 13, obtained by using MC and the hopping expansion, illustrates this. An increasing magnetic field corresponds to

FIGURE 13
Decrease of ΔL for
decreasing quark
masses⁵⁴

decreasing quark masses. Unfortunately, the hopping expansion becomes unreliable for small quark masses and therefore the nature of the phase transition in the zero mass limit remains unclear.

The work of Fischler and Roskies⁵⁵ exhibits the problem very clearly: these authors included the fermion determinant on a small 4^4 lattice exactly for zero-quark mass. Figure 14 shows that the first order phase transition disappears in this limit. The physical limit is, however, $V \rightarrow \infty$ and then quark masses $\rightarrow 0$. The quantitative problem is whether some envelope like behaviour will develop in the physical limit or not.

FIGURE 14
Average plaquette action⁵⁵

A more quantitative analysis has been attempted by a number of authors⁵⁶⁻⁵⁹. The main problem is of course the extrapolation to very small quark masses. Celik et al.⁵⁶ use the hopping expansion. Figures 15 and 16 illustrate the change in the gluonic energy density. There is no signal

FIGURE 15

Gluon energy density ϵ_G/T^4 in fourth order hopping parameter expansion as function of $6/g^2$ and T/Λ_L , for $\kappa = 0.20$ (ordered, disordered start), $\kappa = 0.15$ (o ordered, • disordered start) and $\kappa = 0$ (ordered, disordered start; here the T/Λ_L scale does not apply). The dashed line is the ideal gas limit.

anymore for a 1.-order phase transition. It remains a "rapid cross-over", which is suspected to be a 2.-order phase transition. The estimate of the critical temperature becomes

FIGURE 16

(a) Gluon energy density ϵ_G/T^4 with quark loops, as function of the number of lattice interactions, in bins of 200, for $\alpha = 0.15$, at the deconfinement point, (b) The corresponding gluon energy density ϵ_G/T^4 for the pure Yang-Mills system.

$$T_c \approx 130 \Lambda_L (\approx 600 \text{ MeV?}). \quad (16a)$$

This is too high to be seen in a heavy ion collider. A similar order of magnitude

$$T_c \approx 100 \Lambda_L (\approx 470 \text{ MeV?}) \quad (16b)$$

is obtained by Polonyi et al.⁵⁷, who do a microcanonical²⁰ simulation. Figure 17 exhibits their results for the gluonic and the fermionic internal energy.

FIGURE 17

Results from a microcanonical simulation⁵⁷.

Pseudofermions¹³ are used in Refs. 58^{*)}, 59. References 56-58 agree in a "rapid cross-over", only Ref. 59 finds some evidence for a 1.-order phase transition in the presence of quarks. Further clarification is desirable

5. CHIRAL SYMMETRY BREAKING

Chiral symmetry breaking is of central importance for the understanding of QCD (see, for instance, Ref. 60). One has

$$\langle \bar{\psi}\psi \rangle = n_f \alpha^{-4/11} a(\beta)^3 \langle \psi\psi \rangle_{\text{inv.}}, \quad \alpha = \frac{g^2}{4\pi} \quad (17)$$

and $\langle \bar{\psi}\psi \rangle_{\text{inv}}$ has the dimension $[m^3]$. In the quenched approximation, Hamber and Parisi²² observed asymptotic scaling for this quantity (see Fig. 18). Their estimate had the order of magnitude

FIGURE 18
 $\langle \bar{\psi}\psi \rangle^{1/3}$ from Hamber and Parisi²².

$$\langle \bar{\psi}\psi \rangle_{\text{inv.}}^{1/3} \approx 1.1 \Lambda_{\text{mom}}. \quad (18)$$

With $\Lambda_{\text{mom}} = 83.5 \Lambda_L^6$ and a quark mass estimate of Gasser and Leutwyler⁶⁰ this implies

$$\Lambda_L \approx 2.4 \text{ MeV}. \quad (19)$$

The thus estimated scale differs by a factor of two from Eq. (13) and correspondingly glueball masses and deconfinement temperature T_c may reduce by a factor two in MeV units. Both methods (string tension and chiral condensate) are not very reliable for setting the scale, because both quantities are not directly measurable. Hadron spectroscopy seems to be more direct, but has its

*) Ref. 58 concentrates on chiral symmetry breaking. See next section.

own problems (see next section).

The estimate of Hamber and Parisi²² is based on an extrapolation from rather high quark masses to zero quark mass. Barbour et al.⁶¹ did an interesting study by calculating the full quark mass dependence on finite lattices of different sizes. By increasing the lattice size clearly an envelope-like behaviour develops, demonstrating chiral symmetry breaking at the considered β -values. For $\beta = 5.7$, see Fig. 19.

FIGURE 19
Quark mass dependence of the chiral condensate on different lattices⁶¹.

Studying several β -values the estimate (18) is found with an increased numerical precision.

At a critical temperature T_F the chiral symmetry becomes restored. Kogut et al.⁶² pointed out that in the quenched approximation

$$T_F \approx T_C . \quad (20)$$

This relation is not well understood. For some analytic work in the SC region, see Ref. 63.

Finally the influence of the fermion determinant has to be investigated. Gava et al.⁵⁸ use pseudofermions and mainly consider the SC region. The abrupt transition at T_F is changed into a "rapid cross-over"

In their microcanonical simulation, Polonyi et al.⁵⁷ find still

$$T_F \approx 105 \Lambda_L \approx T_C , \quad (21)$$

as is indicated in Fig. 20. An extension of this work is in progress.

FIGURE 20
Zero mass extrapolation⁵⁷ for $\langle \bar{\psi}\psi \rangle$
and the Polyakov loop WL.

6. HADRONIC SPECTRUM

Spectrum calculations are mainly done with Wilson fermions (Erice lectures,³) and with Kogut-Susskind fermions⁶⁴. A number of large groups at Brookhaven, CERN, DESY, Edinburgh, Rome, SACLAY and other places did work in the quenched approximation. I can only give a very incomplete summary.

With Wilson fermions, propagators may be calculated using the hopping expansion or some numerical matrix inversion. The work relying on the hopping expansion may be traced back from Ref. 65; for the other work, see Ref. 66. A general tendency of these calculations is a too high proton/rho mass ratio and a too low Δ -P mass difference. As an example, I collect results from a rather optimistic paper by Lipps et al.⁶⁶.

$\beta = 6.0$:

$$a^{-1} = (2.2 \pm 0.1) \text{ GeV} \quad [a \approx 0.09 \text{ fermi}]$$

$$\text{Scale: } \Lambda_L \approx 5 \text{ MeV}$$

$$K_S = 0.1548 \pm 0.002$$

$$m_{\Omega^-} = (1.65 \pm .15) \text{ GeV}$$

$$K_u = 0.1566 \pm 0.002$$

$$m_\rho = (0.82 \pm 0.03) \text{ GeV}; \quad m_K = (0.50 \pm 0.03) \text{ GeV}$$

$$m_{K^*} = (0.93 \pm 0.04) \text{ GeV}$$

$$m_P = (1.25 \pm 0.15) \text{ GeV}; \quad m_\Delta = (1.35 \pm 0.20) \text{ GeV.}$$

(22a)

Some calculations with staggered fermions⁶⁴ are collected in Ref. 67. The following results are from the paper of Gilchrist et al.⁶⁷.

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