SYMMETRY BREAKING IN GAUGED SUPERSYMMETRIC SIGMA-MODELS

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ABSTRACT

We discuss several aspects of gauged supersymmetric sigma-models: off-shell formulation, supersymmetry breaking and breaking of internal symmetry. Some specific examples are given.

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Supersymmetric non-linear Lagrangians /1-9/ have several unusual features. In global supersymmetry, the scalar fields must take values in a Kähler manifold /1,4/. There are interesting possibilities for spontaneous supersymmetry breaking /4,5,6/, that do not exist in the linear models. And gauging of internal symmetries introduces scalar potentials that exhibit intricate patterns of internal symmetry breaking /6/.

The Lagrangian of the supersymmetric non-linear sigma-model /1,2/ is in components:

$$\begin{aligned} \mathcal{L} = & -g_{\alpha\beta} \left(\partial_\mu \bar{z}^\beta \partial_\mu z^\alpha + \bar{\psi}_L^\beta \not{\partial} \psi_L^\alpha - \bar{H}^\beta H^\alpha \right) \\ & - \left(g_{\alpha\beta, \gamma} H^\alpha \bar{\psi}_L^\beta \psi_R^\gamma + g_{\alpha\beta, \gamma} \bar{H}^\beta \bar{\psi}_R^\alpha \psi_L^\gamma \right) \\ & - \left(g_{\alpha\beta, \gamma} \partial_\mu z^\gamma - g_{\alpha\beta, \gamma} \partial_\mu \bar{z}^\gamma \right) \bar{\psi}_L^\beta \not{\partial}_\mu \psi_L^\alpha \\ & + g_{\alpha\beta, \gamma} \delta \bar{\psi}_R^\alpha \psi_L^\gamma \bar{\psi}_L^\beta \psi_R^\delta. \end{aligned} \quad (1)$$

The scalar fields $(z^\alpha, \bar{z}^\alpha)$; $\alpha, \underline{\alpha} = 1, \dots, N$, take values in a Kähler manifold with (hermitean) metric $g_{\alpha\beta}(z, \bar{z})$. This metric may admit a set of holomorphic Killing vectors $R^{i\alpha}(z)$, $\bar{R}^{i\underline{\alpha}}(\bar{z})$ ($i=1, \dots, n$), which are the solutions of the Killing equation

$$R^i_{\beta, \alpha} + \bar{R}^i_{\alpha, \beta} = 0. \quad (2)$$

Here $R^i_{\beta} = g_{\alpha\beta} R^{i\alpha}$, and the comma denotes ordinary differentiation with respect to $z, \bar{z}$. Eq. (2) is the necessary and sufficient condition for an infinitesimal transformation

$$\delta z^\alpha = \xi_i R^{i\alpha}(z), \quad \delta \bar{z}^\alpha = \xi_i \bar{R}^{i\alpha}(\bar{z}) \quad (3)$$

to leave the metric $g_{\alpha\beta}$ invariant. Clearly, the bosonic terms in the Lagrangian (1) are then invariant under such transformations. The transformations (3) can be extended to full scalar (chiral) multiplets $\phi^\alpha = (z^\alpha, \psi_L^\alpha, H^\alpha)$ in the following way:

$$\begin{aligned}
 \delta z^\alpha &= \xi_i R^{i\alpha}(z), \\
 \delta \psi_L^\alpha &= \xi_i R^{i\alpha}{}_{,\beta} \psi_L^\beta, \\
 \delta H^\alpha &= \xi_i (R^{i\alpha}{}_{,\beta} H^\beta - R^{i\alpha}{}_{,\beta\gamma} \bar{\psi}_R^\beta \psi_L^\gamma), \quad \psi_R^\alpha = C \bar{\psi}_L^{\alpha T}
 \end{aligned}
 \tag{4}$$

The form of these transformations is uniquely determined by requiring the right hand side to transform as a scalar multiplet under the supersymmetry transformations:

$$\begin{aligned}
 \delta z^\alpha &= \bar{\epsilon}_R \psi_L^\alpha, \\
 \delta \psi_L^\alpha &= \frac{1}{2} (\not{\partial} z^\alpha \epsilon_R + H^\alpha \epsilon_L), \\
 \delta H^\alpha &= \bar{\epsilon}_L \not{\partial} \psi_L^\alpha.
 \end{aligned}
 \tag{5}$$

Because the transformations (4) commute with the supersymmetry transformations (5), the Lagrangian (1) is invariant under both transformations simultaneously. Representing invariances of the Kähler manifold, the transformations (4) must have a Lie-algebra structure; that is, the commutator of two such transformations is again a Killing vector:

$$R^{i\alpha}{}_{,\beta} R^{j\beta} - R^{j\alpha}{}_{,\beta} R^{i\beta} = f^{ijk} R^{k\alpha}. \tag{6}$$

The f^{ijk} are the structure constants of the algebra. A special feature of Kähler manifolds is that their Killing vectors can locally be written as the gradient of some real scalar functions, the Killing potentials $\tilde{M}^i(z, \bar{z})$:

$$R^i_{\underline{\alpha}} = -i \tilde{M}^i_{,\underline{\alpha}}, \quad \bar{R}^i_{\alpha} = i \tilde{M}^i_{,\alpha}. \tag{7}$$

However, these equations determine the potentials $\tilde{M}^i$ only up to arbitrary constants of integration:

$$M^i = \tilde{M}^i + c^i \tag{8}$$

We can choose these constants by requiring the M^i to transform homogeneously according to the adjoint representation of the

algebra (6):

$$\delta M^i = \xi_j (M^i{}_{,\alpha} R^{ja} + M^i{}_{,\underline{\alpha}} \bar{R}^{ja}) = f^{ijk} \xi_j M^k. \quad (9)$$

Eq.(9) is obtained directly by integration of (6), using eq.(7). In case the algebra (6) is semi-simple, this equation determines all the constants c^i unambiguously. Only if the algebra contains U(1) factors, the corresponding potentials still admit arbitrary constants /4/.

We now turn to the possibility of realizing the transformations (4), or a closed sub-algebra, locally. For each independent local Killing vector $R^{ia}(z)$ we introduce a vector multiplet $V^i = (A_\mu^i, \lambda^i, D^i)$, in the adjoint representation of the algebra of the (local) Killing vectors. Since we work in the Wess-Zumino gauge, the supersymmetry transformations close only modulo a gauge transformation, needed to restore the Wess-Zumino gauge. The supersymmetry variations of the vector multiplet are

$$\begin{aligned} \delta A_\mu^i &= \frac{1}{2} \bar{\epsilon} \gamma_\mu \lambda^i, \\ \delta \lambda^i &= \frac{1}{2} (-\sigma_{\mu\nu} F_{\mu\nu}^i + i D \gamma_5) \epsilon, \\ \delta D^i &= \frac{i}{2} \bar{\epsilon} \gamma_5 \not{D} \lambda^i, \end{aligned} \quad (10)$$

where

$$\begin{aligned} F_{\mu\nu}^i &= \partial_{[\mu} A_{\nu]}^i - g f^{ijk} A_\mu^j A_\nu^k, \\ D_\mu \lambda^i &= \partial_\mu \lambda^i - g f^{ijk} A_\mu^j \lambda^k. \end{aligned}$$

In order to obtain uniform closure of the supersymmetry transformations on all fields, we must then adjust the supersymmetry transformations (5) in the following way:

$$\begin{aligned} \delta z^\alpha &= \bar{\epsilon}_R \psi_L^\alpha, \\ \delta \psi_L^\alpha &= \frac{1}{2} (\not{D} z^\alpha \epsilon_R + H^\alpha \epsilon_L), \\ \delta H^\alpha &= \bar{\epsilon}_L (\not{D} \psi_L^\alpha - g \lambda_R^i R^{ia}(z)), \quad \lambda_R^i = \frac{1}{2} (1 - \gamma_5) \lambda^i. \end{aligned} \quad (11)$$

In these expressions, the non-linear covariant derivatives are

defined by:

$$\begin{aligned} \mathcal{D}_\mu z^\alpha &= \partial_\mu z^\alpha - g A_\mu^i R^{i\alpha}(z), \\ \mathcal{D}_\mu \psi_L^\alpha &= \partial_\mu \psi_L^\alpha - g A_\mu^i R^{i\alpha}{}_\beta \psi_L^\beta. \end{aligned} \quad (12)$$

The supersymmetry transformations (11) now close off-shell modulo a (non-linear) gauge transformation. We can then use standard multiplet calculus rules to construct an off-shell invariant Lagrangian. Namely, we construct the multiplet corresponding to the Kähler potential, and take the highest component /1/.

Combining with the supersymmetric Yang-Mills action, we find

$$\mathcal{L} = \mathcal{L}_{YM} + \mathcal{L}_{matter} \quad (13)$$

with

$$\mathcal{L}_{YM} = -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2} \bar{\lambda}^i \not{D} \lambda^i + \frac{1}{2} D^{i2} + g \alpha D^0, \quad (14)$$

and $\mathcal{L}_{matter}$ is given by eq. (1), where one must replace the derivatives $\partial_\mu z^\alpha, \partial_\mu \psi_L^\alpha$ by the covariant derivatives (12), and add some Yukawa- and scalar-couplings:

$$\begin{aligned} \mathcal{L}_{matter} &= \mathcal{L}_c(\partial_\mu \rightarrow \mathcal{D}_\mu) + 2g(R^i{}_\alpha \bar{\psi}_L^\alpha \lambda_R^i + \bar{R}^i{}_\alpha \bar{\lambda}_R^i \psi_L^\alpha) \\ &\quad + g D^i M^i. \end{aligned} \quad (15)$$

In eq. (14) we have also added a Fayet-Iliopoulos term with parameter α in case there is a U(1) vector multiplet, with auxiliary component D^0 . The potentials M^i must be defined so as to transform homogeneously, as in eq.(9), in order for the last term in (15) to respect the internal symmetry. Eqs.(13-15) represent the complete result. We can proceed to eliminate the auxiliary fields H^α, D^i :

$$H^\alpha = \Gamma^\alpha{}_{\beta\gamma} \bar{\psi}_R^\beta \psi_L^\gamma, \quad D^i = -g(M^i + \alpha \delta^{i0}). \quad (16)$$

Then we reobtain the Lagrangian of ref. (4):

$$\begin{aligned}
 \mathcal{L} = & -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2} \bar{\lambda}^i \not{D} \lambda^i - g_{\alpha\beta} (\mathcal{D}_\mu \bar{z}^\beta \mathcal{D}_\mu z^\alpha + \bar{\psi}_L^\beta \not{D} \psi_L^\alpha) \\
 & - (g_{\alpha\beta, \gamma} \mathcal{D}_\mu z^\gamma - g_{\alpha\beta, \bar{\gamma}} \mathcal{D}_\mu \bar{z}^{\bar{\gamma}}) \bar{\psi}_L^\beta \gamma_\mu \psi_L^\alpha \\
 & + \frac{1}{2} R_{\alpha\beta\gamma\delta} \bar{\psi}_L^\beta \gamma_\mu \psi_L^\alpha \bar{\psi}_L^\delta \gamma^\mu \psi_L^\gamma - \frac{1}{2} g^2 (M^i + \alpha \delta^{0i})^2 \\
 & + 2g (R^i_{\alpha} \bar{\psi}_L^\alpha \lambda_R^i + \bar{R}^{i\alpha} \bar{\lambda}_R^i \psi_L^\alpha).
 \end{aligned} \tag{17}$$

The most interesting terms in this expression are the following.

1. The scalar potential is $V_{sc} = \frac{1}{2} g^2 (M^i + \alpha \delta^{0i})^2$.

Since this is a sum of squares, it is positive definite.

Moreover, it vanishes only if each of the terms vanishes simultaneously: $M^i + \alpha \delta^{0i} = 0$, for all i . However, because of the presence of the constants c^i , this is often not possible.

In that case supersymmetry is spontaneously broken. Also, for the same reason, the minimum of the full potential may occur at non-zero vacuum expectation value of the scalars, signalling the spontaneous break-down of the internal symmetry.

2. The gauge couplings in the scalar kinetic terms then give a mass to the vector fields of broken symmetries through the Brout-Englert-Higgs mechanism.
3. Simultaneously, vacuum expectation values for the Yukawa couplings make some or all of the chiral spinors ψ_L^α combine with the gaugino's λ_R^i , corresponding to broken generators, into massive Dirac fermions. Of course, if supersymmetry is spontaneously broken, at least one spinor must become a Goldstone fermion and remain massless. Usually this is the gaugino of a $U(1)$ gauge multiplet.

Examples of these mechanisms have been presented in the literature /4,5,6/. Here we wish to exhibit some interesting and illustrative cases, that bring out in particular the role of the integration constants c^i , and shed light on some of the criteria for symmetry breaking also found in ref./6/.

We consider the Grassmannian σ -models /5,6,7/, defined on the coset space $U(N+M)/U(N) \times U(M) \cong SU(N+M)/SU(N) \times SU(M) \times U(1)$. We can parametrize this Kähler manifold by writing an arbitrary element Z of $U(N+M)$ in the form /7/

$$Z = \begin{pmatrix} e^{iL} \frac{1}{\sqrt{1+f^2\phi\phi^\dagger}} & e^{iL} \frac{f}{\sqrt{1+f^2\phi\phi^\dagger}} \phi \\ -e^{iK} \frac{f}{\sqrt{1+f^2\phi^\dagger\phi}} \phi^\dagger & e^{iK} \frac{1}{\sqrt{1+f^2\phi^\dagger\phi}} \end{pmatrix}, \quad (18)$$

where ϕ_i^α ; $i=1,\dots,N$, $\alpha=1,\dots,M$; is a general complex $N \times M$ matrix, while e^{iL} , e^{iK} represent $U(N)$, $U(M)$ phase factors; f is a dimensionful constant, included to make Z dimensionless. On the coset space, all elements differing only by these phase factors are identified. Hence one may choose the gauge $L=K=0$. The independent scalar fields ϕ , $\phi^\dagger$ are now the complex co-ordinates of the manifold. They correspond to the Goldstone bosons of a spontaneous break-down of $SU(N+M)$ to $SU(N) \times SU(M) \times U(1)$. The metric $g_{\alpha\beta}$ on this manifold reads

$$g_{\alpha\beta} = \left(\frac{1}{1+f^2\phi^\dagger\phi} \right)_\alpha \left(\frac{1}{1+f^2\phi\phi^\dagger} \right)_\beta. \quad (19)$$

The Killing vectors correspond to right multiplication of Z by an element of $SU(N+M)$; infinitesimally:

$$Z' = ZU, \quad U = 1 + i\xi = \begin{pmatrix} 1 + i\Lambda & \epsilon \\ -\epsilon^\dagger & 1 + i\bar{\Lambda} \end{pmatrix}, \quad (20)$$

where Λ , $\bar{\Lambda}$ are hermitean; this implies

$$\delta\phi = \frac{1}{f}\epsilon + f\phi\epsilon^\dagger\phi - i\Lambda\phi + i\phi\bar{\Lambda} = \xi \cdot \mathbb{R} \quad (21)$$

Explicitly, the Killing vectors are

$$\mathbb{R}_i^\alpha = \begin{pmatrix} -i\delta_i^j \phi_k^\alpha & \frac{i}{f} \delta_i^j \delta_\beta^\alpha \\ -if\phi_i^\gamma \phi_k^\alpha & i\delta_\beta^\alpha \phi_i^\gamma \end{pmatrix}. \quad (22)$$

Finally, the corresponding Killing potentials can be collected in the form

$$\underline{\underline{M}} = \begin{pmatrix} \left[\frac{\phi\phi^\dagger}{1+f^2\phi\phi^\dagger} - \frac{1}{2f^2} \mathbb{1}_N \right]_k^j & -\frac{1}{f} \left[\frac{1}{1+f^2\phi\phi^\dagger} \phi \right]_k^\gamma \\ -\frac{1}{f} \left[\frac{1}{1+f^2\phi^\dagger\phi} \phi^\dagger \right]_\beta^j & \left[-\frac{\phi^\dagger\phi}{1+f^2\phi^\dagger\phi} + \frac{1}{2f^2} \mathbb{1}_M \right]_\beta^\gamma \end{pmatrix} \quad (23)$$

It may readily be checked that $g_{\beta}^{i\alpha} \underline{\underline{M}}_{i\beta}^{\gamma} = -i \frac{\delta}{\delta \phi_{\alpha}^{\gamma}} \underline{\underline{M}}$, eq.(7). The expressions (22),(23) are all we need to discuss the properties of the Lagrangian (17) for these models. We consider three special cases.

1. If the full group $SU(N+M)$ is gauged, the scalar potential becomes

$$V_{sc} = \frac{1}{2} g^2 \text{Tr} \underline{\underline{M}}^2 = g^2 \frac{M+N}{8f^4}, \quad (24)$$

a positive constant /4,5/. This results from the presence of the integration constants on the diagonal in $\underline{\underline{M}}$, chosen such that $\underline{\underline{M}}$ transforms homogeneously: $\delta \underline{\underline{M}} = -i [\underline{\underline{\xi}}, \underline{\underline{M}}]$.

Thus supersymmetry is broken. At the same time, the vector bosons corresponding to the broken generators of $SU(N+M)$ become massive by absorbing the scalars. This is evident in the unitary gauge $\phi = 0$. The chiral fermions ψ_L^{α} all combine with the gaugino's of the broken generators to become massive Dirac spinors /4,5/. Because of the massive vectors and the absence of physical scalars, supersymmetry is not manifest in the unitary gauge, but of course the spectrum of helicity states preserves the supersymmetry, even if the masses are split.

2. If one gauges the full stability group $SU(N) \times SU(M) \times U(1)$, we have the freedom to add a Fayet-Iliopoulos term for the $U(1)$ factor. This can be fine-tuned such that it cancels exactly the contribution of the constants in $\underline{\underline{M}}$. Namely, one can now write the potential in the form

$$V_{sc} = \frac{1}{2f^4} \left\{ g_1^2 \text{Tr} \left(\frac{f^2 \phi \phi^\dagger}{1+f^2 \phi \phi^\dagger} - \left(\frac{1}{2} - \alpha \right) \right)^2 + g_2^2 \text{Tr} \left(\frac{f^2 \phi^\dagger \phi}{1+f^2 \phi^\dagger \phi} - \left(\frac{1}{2} - \alpha \right) \right)^2 \right\}. \quad (25)$$

Hence, for $\alpha = \frac{1}{2}$, the potential has its minimum at $\phi = 0$. Then supersymmetry and internal symmetry remain unbroken. However, if we do not add (or fine-tune) α , then again the potential has non-trivial minima (fig.1); for $\alpha = 0$, the minimum is reached for

$$\phi = \frac{1}{f} (e^{iL}; 0), \quad M \geq N. \quad (26)$$

For $N \neq M$, supersymmetry is broken; also, the internal symmetry is broken to $SU(N) \times SU(M-N) \times U(1)$, for $N < M$, or $SU(N)$, for $N = M$. Of the $2NM$ real spin-0 degrees of freedom, $2MN - N^2$ become Goldstone bosons of this symmetry breaking and subsequently become absorbed in generating mass for the vector fields. The remaining N^2 scalars receive masses of order g^2/f^2 . This is possible, because gauging a proper subgroup of $SU(N+M)$ breaks the global $SU(N+M)$ symmetry explicitly by terms of $O(g)$ in the Lagrangian. Hence the scalar fields ϕ now are pseudo-Goldstone bosons, rather than true Goldstone bosons. Finally, their fermion partners ψ_L again become massive, but some gaugino components remain as massless spinors (either chiral or Majorana) in the spectrum of states.

3. Finally, consider gauging a subgroup $SU(M) \times U(1)$ of the stability group. Results then depend on whether $M > N$ or $M \leq N$. For $M > N$, if we do not add a separate Fayet-Iliopoulos term, supersymmetry and internal symmetry are again broken. On the other hand, if $M \leq N$ supersymmetry is never broken, although internal symmetry may still be broken spontaneously. To see this, consider the potential (with $\alpha = 0$):

$$V_{sc} = \frac{g^2}{2f^4} \text{Tr} \left(\frac{f^2 \phi^\dagger \phi}{1 + f^2 \phi^\dagger \phi} - \frac{1}{2} 1_M \right)^2. \quad (27)$$

(For convenience we have here chosen the $SU(M)$ and $U(1)$ coupling constants equal; this is not necessary, but qualitatively the results are not affected by this choice). The potential is sketched in fig.2. The minimum occurs for

$$\phi = \begin{cases} \frac{1}{f} \begin{pmatrix} e^{iL} \\ \vdots \\ 0 \end{pmatrix}, & M > N; \\ \frac{1}{f} \begin{pmatrix} e^{iK} \\ \vdots \\ 0 \end{pmatrix}, & M \leq N. \end{cases} \quad (28)$$

The global internal symmetry is then broken to:

$$\begin{aligned} &SU(N) \times SU(M-N) \times U(1), \text{ for } M > N; & SU(N), \text{ for } M = N; \\ &SU(N-M) \times SU(M) \times U(1), \text{ for } M < N. \end{aligned}$$

For $M > N$, with broken supersymmetry, the pattern of masses is similar to that when the full stability group is gauged. The only surviving local symmetry is now $SU(M-N) \times U(1)$, hence again $2MN - N^2$ vectors become massive, while N^2 massive physical scalars remain.

For $M=N$, these numbers are equal, hence the number of massive vectors and scalars are equal and fit exactly into N^2 massive vector multiplets of supersymmetry, with as many massive Dirac vectors. Finally, for $M < N$, all M^2 vectors become massive, together with the appropriate number of spinors and scalars to form massive vector multiplets. The remaining scalars are real Goldstone bosons of the broken global symmetries (not gauged), and hence remain massless. Since supersymmetry is not broken, also their spinor partners remain massless.

Our results are seen to qualify certain statements in the literature that supersymmetry and internal symmetries are not broken if one gauges a symmetry contained in (or equal to) the stability group H of a Kähler manifold G/H . Certainly, our results involve the gauging of the $U(1)$ factor in the stability group in an essential way. This may seem somewhat artificial. On the other hand, if one gauges a group larger than the stability group, this $U(1)$ generally becomes part of a non-Abelian group (the stability group being maximal). Then its inclusion in the gauge group and its contribution to the scalar potential are unavoidable, while no additional Fayet-Iliopoulos term is allowed. In that case, both supersymmetry and internal symmetry are broken, exactly as in the examples above. The same is true for other Kähler manifolds as well /10/.

One may worry about some of the symmetries being anomalous /9/. This is indeed the case in the pure σ -models. However, exactly as in gauge theories with fermions in complex representations, here also the anomalies may be cancelled by coupling more fermions to the model in such representations so as to make the co-efficient of the anomaly vanish /7,11/. Since this can be done in a supersymmetric way /12/, we do not believe this problem to be fundamental. Moreover, we note that in most (semi-)realistic models additional fermions are necessary for other reasons as well, for example to complete the full family structure of quarks and leptons.

An interesting feature of the models we have presented is that one obtains a hierarchy in symmetry breaking. The scalars

representing the (pseudo-) Goldstone bosons of G/H , which may effectively be bound states of some more fundamental theory at a scale $\Lambda^2 = 1/f^2$ (where the global symmetry G gets broken), can induce spontaneous breaking of H in the presence of weakly coupled vector fields via a Higgs type of potential at a scale $\sim g^2 \Lambda^2$. We have found the same mechanism to work for example in the case of particular gaugings of $E_6/SO(10) \times U(1)$. We will report about these results elsewhere /10/.

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Figure captions.

Fig.1: Scalar potential for the Grassmannian σ -model with gauged $SU(N) \times SU(M) \times U(1)$ (with $M \geq N$), for $\alpha = 0$.

Fig.2: Scalar potential for the Grassmannian σ -model with gauged $SU(M) \times U(1)$ and $\alpha = 0$; broken line: $M > N$; continuous line: $M \leq N$.

Fig.1

Fig.2