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INNER BREMSSTRAHLUNG IN LOW ENERGY ANTIPROTON-PROTON ANNIHILATION

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ABSTRACT

Photon bremsstrahlung accompanying antiproton-nucleon annihilation is examined with special regard to the experimental program at LEAR. It is shown that the bremsstrahlung continuum constitutes an important background to the photon lines emitted in radiative transitions between protonium states. Interference between lines and bremsstrahlung can yield interesting information on the protonium atom and the annihilation mechanism, and perhaps even help in the search for baryonium states.

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The study of antiproton-nucleon annihilation at rest and at low incident energies has received a tremendous boost by the successful operation of the LEAR facility at CERN [1]. A number of experiments is directed at investigating photons emitted in the annihilation process. The photon energies of interest range from few keV which is the typical spacing of low-lying levels in the protonium atom, to hundreds of MeV, the expected magnitude of the spacings between hypothetical baryonium states.

We show in this paper that in planning and analyzing these experiments the well known phenomenon of inner bremsstrahlung must be taken into account. Photon bremsstrahlung of charged mesons produced in $\bar{p}N$ -annihilation not only constitutes an important incoherent background to monochromatic photon lines but may also interfere with the latter.

For simplicity, we shall limit the discussion to antiproton-proton annihilation at rest; the generalization to annihilation in flight and in nuclei is straightforward. At rest, the annihilation proceeds from the protonium bound states and leads to various multi-meson final states. Such an annihilation process into n mesons with charges Q_v , four-momenta $p_v = (E_v, \vec{p}_v)$ and masses $m_v = \sqrt{p_v^2}$ is illustrated in Fig. 1a. By convention, the emission of photons from the external charged legs of this diagram is called inner bremsstrahlung and is represented by the diagrams of Fig. 1b. Any other source of photons is symbolized by the structure term depicted in Fig. 1c; it is coherent with inner bremsstrahlung.

The differential width for annihilation of a specific initial protonium state ℓ with mass M into a specific final state of n mesons (Fig. 1a) may be written as

$$d\Gamma_n^\ell = \frac{1}{2M} \left| A_n^\ell \right|^2 d\phi_n \quad (1)$$

where A_n^ℓ is the corresponding invariant annihilation amplitude and $d\phi_n$ is the invariant phase space element of n mesons. The superscript ℓ denotes the angular momentum of the protonium state, while other quantum numbers are not quoted.

Provided the photon energy ω is sufficiently small, the bremsstrahlung diagrams of Fig. 1b may be evaluated in the soft photon limit, i.e. by treating the n-meson final state essentially as a classical charge distribution [2-5]. This approximation is certainly appropriate for photons in the keV region where ω is much smaller than even the smallest meson mass. For a general n-meson annihilation channel, the differential bremsstrahlung width $d\Gamma_{nB}^\ell$ summed over photon polarizations is then given by [6]

$$d\Gamma_{nB}^\ell = \frac{\alpha}{4\pi^2} R_B d\Omega_k \frac{d\omega}{\omega} d\Gamma_n^\ell \quad (2)$$

where

$$R_B = - \sum_{v=1}^n \sum_{w=1}^n Q_v Q_w \frac{(p_v \cdot p_w)}{(p_v \cdot k)(p_w \cdot k)} \quad (3)$$

and $d\Gamma_n^\ell$ is the nonradiative width defined by (1). Here, α is the fine-structure constant, $k = (1, \hat{k})$ with $\hat{k}$ being the unit three-vector in the direction of the photon momentum, and $d\Omega_k$ is the differential solid angle of the photon. All mesons with lifetimes $\tau \gg \omega^{-1}$ contribute to the double sum in eq. (3). For example, for $\omega = 10$ keV, ρ^0 contributes via its decay products π^+ and π^- , whereas η which has the width $\Gamma = 0.9$ keV should be counted as a stable particle (but does not radiate since $Q_\eta = 0$). The inner bremsstrahlung emitted when the η decays is to be considered separately.

Superposed on the bremsstrahlung continuum are monochromatic photon lines originating in radiative transitions between protonium levels. The amplitude for transition and subsequent annihilation may be considered as a special case of the structure term illustrated in Fig. 1c. Thus, the total amplitude for radiative annihilation into a particular n-meson channel is the coherent sum of bremsstrahlung and line emission amplitudes. Correspondingly, to order α the differential radiative annihilation width $d\Gamma_{n\gamma}^\ell$ is given by

$$d\Gamma_{n\gamma}^\ell = d\Gamma_{nB}^\ell + d\Gamma_{nL}^\ell + d\Gamma_{nI}^\ell \quad (4)$$

Here, $d\Gamma_{nB}^{\ell}$ results from eqs. (1-3), $d\Gamma_{nL}^{\ell}$ is the differential width for the radiative protonium transition followed by annihilation into n mesons, and $d\Gamma_{nI}^{\ell}$ denotes the contribution of the interference between line emission and bremsstrahlung.

As a simple illustrative example we consider the 3P_0 state of protonium which (among its many decay possibilities) either annihilates directly into π^+ and π^- with accompanying inner bremsstrahlung or radiatively decays into the 3S_1 state which then annihilates into π^+ and π^- without bremsstrahlung. The amplitude for the entire process is symbolized in Fig. 1d. The inner bremsstrahlung contribution to $d\Gamma_{\pi^+\pi^-\gamma}^P$ integrated over the 2-pion phase space can be readily derived from eqs. (1-3):

$$d\Gamma_{\pi^+\pi^-\gamma}^P = \frac{\alpha}{\pi^2} \Gamma_{\text{tot}}^P \text{BR}^P(\pi^+\pi^-) \frac{p^2 E^2 \sin^2 \theta}{(E^2 - p^2 \cos^2 \theta)^2} d\Omega_k \frac{d\omega}{\omega} \quad (5)$$

Here, $E = M/2$, $p = (M^2/4 - m^2)^{1/2}$, and m are the energy, the magnitude of the three momentum, and the mass of a charged pion, respectively. Of course, the pions are emitted isotropically back to back, their line of flight subtending an angle θ with respect to the photon direction. Γ_{tot}^P is the total decay width of the 3P_0 state, and $\text{BR}^P(\pi^+\pi^-)$ is the branching ratio for the $\pi^+\pi^-$ channel.

The line width $d\Gamma_{\pi^+\pi^-\gamma}^P$ summed over photon polarizations may be evaluated by making use of gauge invariance and symmetry properties of P- and S-waves. In the narrow width approximation one finds

$$d\Gamma_{\pi^+\pi^-\gamma}^P = \frac{3}{16\pi^2} \frac{\Gamma_r^P \Gamma_{\text{tot}}^S \text{BR}^S(\pi^+\pi^-)}{\omega_0} \sin^2 \theta d\Omega_k \frac{\omega d\omega}{(\omega - \omega_0)^2 + (\Gamma_{\text{tot}}^S/2)^2} \quad (6)$$

where Γ_r^P is the radiative width for the transition $^3P_0 \rightarrow ^3S_1 \gamma$ and Γ_{tot}^S is the total annihilation width of the 3S_1 state. The line maximum is slightly shifted from the position ω_0 by the phase space factor ω in the nominator of eq.(6).

Finally, in the same approximation the interference contribution reads

$$d\Gamma_{\pi^+\pi^-\gamma}^P = N_I \frac{p E \sin^2 \theta}{E^2 - p^2 \cos^2 \theta} d\Omega_k F_I(\omega) d\omega \quad (7)$$

with

$$N_I = \frac{\sqrt{3\alpha}}{2\pi^2} \left[\Gamma_{\text{tot}}^P BR^P(\pi^+\pi^-) \Gamma_r^P \Gamma_{\text{tot}}^S BR^S(\pi^+\pi^-) / \omega_0 \right]^{1/2} \quad (8)$$

and

$$F_I(\omega) = \frac{(\omega - \omega_0) \cos \delta + (\Gamma_{\text{tot}}^S / 2) \sin \delta}{(\omega - \omega_0)^2 + (\Gamma_{\text{tot}}^S / 2)^2} \quad (9)$$

where

$$\delta = \delta_P - \delta_S - \delta_r \quad (10)$$

Here, δ_S and δ_P are the phases of the 3S_1 and 3P_0 annihilation amplitudes respectively, while δ_r is the phase of the radiative $^3P_0 \rightarrow ^3S_1\gamma$ amplitude at $|\omega_0 - \omega| \gg \Gamma_{\text{tot}}^S$.

The angular correlations between one of the pions and the photon are displayed in Fig. 2. Note that these correlations are independent of ω as long as one considers the three contributions to $d\Gamma_{\pi^+\pi^-\gamma}$ separately. The relative normalization is chosen such that the integration over the entire photon solid angle yields equal areas in the three cases. (This is not obvious from Fig. 2 where, for convenience, θ instead of $\cos \theta$ is plotted on the abscissa.) Notice the strong peaking of the bremsstrahlung contribution near $\theta = \gamma^{-1}$ (or $\pi - \gamma^{-1}$) characteristic of rather highly relativistic pions with $\gamma = E/m \simeq 6.7$.

The photon energy spectrum is illustrated in Fig. 3 taking $\sin^2 \delta = \cos^2 \delta = 1/2$ and $\Gamma_{\text{tot}}^S = \omega_0/10$. The latter assumption corresponds to the expected width of the protonium K-lines. Fig. 3a exhibits the bremsstrahlung continuum, the "monochromatic line", and the interference term, separately. They are put equal to each other at $\omega = \omega_0$ which is realistic as shown later in eq. (13). As can be seen from eq. (9) the interference contribution consists of an absorptive part (the term proportional to $\sin \delta$ in eq. (9)) which has been plotted with the negative sign, for clarity, and a dispersive part (the term proportional to $\cos \delta$ in eq. (9)) which disappears upon integration over an energy interval centered at ω_0 . Their relative normalization is fixed by the choice of $\sin^2 \delta$. The resulting total photon energy spectrum depends on the values of the parameters entering eqs. (5-10) and also on the experimentally accepted range in θ (see Fig. 2). Fig. 3b presents four possible spectra obtained by just adding with equal weights the distributions of Fig. 3a for $\delta = \pm 45^\circ$ and $\delta = \pm 135^\circ$. In view of our ignorance of the actual value of the phase δ , we illustrate in Fig. 3b the richness of possible spectral shapes due to the interference between radiative protonium transitions and bremsstrahlung. If the absorptive interference term turned out to be larger in absolute magnitude than the line contribution and if the phase δ were about -90° , one would even observe a dip instead of a peak at ω_0 , an amusing and not impossible case.

In order to investigate the relative contributions of bremsstrahlung, line, and interference, it is convenient to integrate eqs. (5-7) over the full solid angle Ω_k of the photon and over an energy interval from $\omega_0 - \Delta\omega$ to $\omega_0 + \Delta\omega$. One obtains the following proportions:

$$\begin{aligned} \Gamma_{\pi^+\pi^-B}^P &: \Gamma_{\pi^+\pi^-L}^P : \Gamma_{\pi^+\pi^-I}^P = \\ &= \alpha \frac{\Gamma_{\text{tot}}^P \text{BR}^P(\pi^+\pi^-)}{\Gamma_r^P \text{BR}^S(\pi^+\pi^-)} \left(\frac{1+\beta^2}{2\beta} \ln \frac{1+\beta}{1-\beta} - 1 \right) \ln \frac{\omega_0 + \Delta\omega}{\omega_0 - \Delta\omega} \left(\arctg \frac{2\Delta\omega}{\Gamma_{\text{tot}}^S} \right)^{-1} : \\ &: 1 : \left[12 \alpha \frac{\Gamma_{\text{tot}}^P \Gamma_{\text{tot}}^S \text{BR}^P(\pi^+\pi^-)}{\Gamma_r^P \omega_0 \text{BR}^S(\pi^+\pi^-)} \right]^{1/2} \left(\frac{1}{\beta} - \frac{1-\beta^2}{2\beta^2} \ln \frac{1+\beta}{1-\beta} \right) \sin \delta \quad (11) \end{aligned}$$

Here $\beta = p/E$ is the pion velocity.

Theoretical estimates [7] yield $\Gamma_{\text{tot}}^P/\Gamma_r^P \approx 290$ and $\Gamma_{\text{tot}}^S/\omega_0 \approx 0.1$. These predictions are consistent with experimental data [8]. Less is known about the ratio $\text{BR}^P(\pi^+\pi^-)/\text{BR}^S(\pi^+\pi^-)$. However, plausible arguments [9] indicate that the annihilation of the 3P_0 state into $\pi^+\pi^-$ with relative angular momentum $\ell_{\pi^+\pi^-} = 0$ is favoured over the $\ell_{\pi^+\pi^-} = 1$ annihilation from the 3S_1 state. Again, these ideas are indirectly supported by experimental data [9,10]. Taking the above ratio of branching ratios to be 1 is therefore conservative in the sense that it probably overestimates the line contribution with respect to the contributions of interference and bremsstrahlung. Finally, the phase δ is unknown and we arbitrarily take $\sin^2\delta = 0.5$. With this input and for $\Delta\omega = 2\Gamma_{\text{tot}}^S$, an experimentally reasonable energy bin, eq. (11) gives

$$\Gamma_{\pi^+\pi^-B}^P : \Gamma_{\pi^+\pi^-L}^P : \left| \Gamma_{\pi^+\pi^-I}^P \right| \approx 2.7 : 1 : 1.1 \quad (12)$$

which underlines the crucial role of bremsstrahlung in such experiments. Note that the corresponding relative normalization of the energy spectra at $\omega = \omega_0$ amounts to

$$\frac{d\Gamma_{\pi^+\pi^-B}^P}{d\omega} : \frac{d\Gamma_{\pi^+\pi^-L}^P}{d\omega} : \left| \frac{d\Gamma_{\pi^+\pi^-I}^P}{d\omega} \right| \approx 0.9 : 1 : 1.1 \quad (13)$$

justifying our assumption in Fig. 3.

Recently, the K-lines originating in the radiative transitions of protonium P-levels to the $n = 1$ S-level have been observed [8]. Not surprisingly, the experimental search for the lines was first successful when an anticoincidence with charged mesons was required, thus strongly favouring neutral annihilation channels which are not accompanied by bremsstrahlung. For charged annihilation channels, a careful subtraction of the bremsstrahlung contribution to the photon energy spectrum as well as attention to interference effects appear to be mandatory.

For experiments which completely reconstruct the charged mesons in coincidence with the photon, the bremsstrahlung contribution can be directly computed from eqs. (1-3). For other experiments this calculation is still feasible [6], provided that the inclusive spectra of charged mesons and the two-meson correlations are known; triple or higher meson correlations are irrelevant. Incidentally, inner bremsstrahlung may even play an experimentally useful role by providing an exactly calculable photon continuum for normalization purposes.

In contrast, no simple general expression analogous to eq. (2) can be given for the differential line width $d\Gamma_{nL}^k$ and the interference term $d\Gamma_{nI}^k$. However, one can obtain expressions similar to eqs. (6) and (7) for other experimentally interesting cases, typically at the cost of an increased number of parameters. The latter are not predictable without a detailed understanding of the annihilation process and not all of them can be determined by independent measurements. This poses a problem in the study of weak and relatively broad lines which are most seriously distorted by interference effects, as shown for the case of the 3P_0 protonium K-lines in Fig. 3. On the other hand, an experimental apparatus, such as the ASTERIX spectrometer, which completely reconstructs charged mesons in coincidence with the photons may be able to isolate cases where interference is important but still describable in terms of only a few parameters, in analogy to our example $^3P_0 \rightarrow \pi^+\pi^-\gamma$. A detailed study of such cases could yield interesting information on otherwise hardly measurable quantities such as relative phases of annihilation amplitudes. The alternative is to suppress bremsstrahlung altogether by triggering on neutral annihilation channels or, in the case of $\bar{p}$ -annihilation in nuclei, on channels without fast charged particles.

Another aspect of antiproton-proton physics for which inner bremsstrahlung may play a role is the search for monochromatic γ -lines indicative of the formation of baryonium states. Although the soft photon approximation is somewhat questionable for photon energies in the 100 MeV range, expected for baryonium transitions, it suffices for a qualitative discussion. A particularly interesting situation occurs if the amplitude for such a γ -transition is so weak that $\Gamma_r^P/\Gamma_{tot}^P \ll \alpha \Gamma_{tot}^S/\omega_0$ (in the

present context, the superscript S indicates the bound baryonium state). For a suitable annihilation channel such that $BR^S(n\pi)$ is reasonably large and $BR^P(n\pi) \gtrsim BR^S(n\pi)$, bremsstrahlung, through its interference with the line emission, may lead to a large enhancement of the effect of a weak γ -transition on the γ -energy spectrum. This enhancement is especially welcome in view of the large π^0 background that one usually has to fight.

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FIGURE CAPTIONS

Fig. 1 Diagrams for various annihilation processes of protonium states:

- (a) hadronic annihilation into n mesons;
- (b) annihilation with inner bremsstrahlung;
- (c) structure term contributing to radiative annihilation;
- (d) bremsstrahlung and radiative protonium transition contributing to ${}^3P_0 \rightarrow \pi^+\pi^-\gamma$.

Fig. 2 Angular correlation between the photon and one of the pions in the annihilation channel ${}^3P_0 \rightarrow \pi^+\pi^-\gamma$ for photon energies in the region of the ${}^3P_0 \rightarrow {}^3S_1$ K-line.

Fig. 3 Photon energy spectrum from ${}^3P_0 \rightarrow \pi^+\pi^-\gamma$ in the vicinity of the K-line:

- (a) the individual contributions of the K-line, the bremsstrahlung and the interference between them as described in the text.
- (b) the total photon energy spectrum for four values of δ .

Fig . 1

Fig . 2

Fig. 3