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TOWARDS A FINITE $N = 2$ SUSY GUT

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A B S T R A C T

We consider finite, $N = 2$ supersymmetric GUTs based on gauge groups $SU(n)$ and $SO(n)$. As an example, we discuss a semirealistic model based on $SO(12)$. We argue that in finite, $N = 2$ supersymmetric GUTs, gauge symmetry breaking should occur dynamically. We present a heuristic picture in which this is induced by soft, finiteness preserving SUSY breaking terms. The bound states formed cause a very rapid evolution of the $SO(12)$ coupling constant and break $SO(12)$ into $SU(4) \times SU(3)_C \times U(1)$.

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A well-known prototype of a finite field theory is the $N = 4$ supersymmetric Yang-Mills (SYM) theory¹⁾. Theories with $N = 2$ supersymmetry can also be finite provided the particle content is such that one-loop corrections to coupling constant renormalization vanish²⁾⁻⁴⁾. Therefore extended supersymmetries may accommodate a finite grand unified theory (GUT), and perhaps ultimately a theory unifying also gravitation, possibly based on $N = 8$ supergravity⁵⁾. From a phenomenological point of view, one of the most prominent features of extended supersymmetries is the necessary appearance of mirror particles, i.e., particles with V+A weak currents. Making these particles heavier than the current lower bound of 22 GeV has been one of the major obstacles in constructing phenomenologically acceptable models with extended supersymmetries.

Recently, a low energy $N = 2$ model based on the gauge group $SU(4) \times SU(3)_C \times U(1)$ was proposed⁶⁾ where a mechanism for splitting the vector-like theory into low mass scale V-A part and high mass scale ($\sim M_W$) V+A part was found. Evidently such a low energy model cannot be finite, but as was suggested in Ref. 6), it might be the low energy end of a finite GUT. In the present paper we will discuss $N = 2$ supersymmetric GUTs, and we will describe a possible candidate for such a model based on $SO(12)$ gauge symmetry. This model can have the $SU(4) \times SU(3)_C \times U(1)$ symmetric low energy structure suggested in Ref. 6), providing a splitting of fermions from mirror fermions.

One substantial difficulty for $N = 2$ GUTs is, as we will discuss, to trigger the GUT symmetry breaking by the conventional Higgs mechanism. Indeed, in the present paper we do not find a satisfactory spontaneous symmetry breakdown pattern. Instead, we propose a heuristic scenario based on dynamical breaking at the scale of Planck mass M_P .

The ingredients of an $N = 2$ SUSY GUT are the vector multiplet, which consists of one $N = 1$ vector multiplet $V = (V_\mu, \lambda)$ and of one $N = 1$ chiral multiplet $\Phi = (\phi_\phi, \phi)$, all in the adjoint representation of the gauge group G ; the matter hypermultiplet built from one $N = 1$ chiral multiplet $X = (\phi_x, x)$ transforming as R under G , and its mirror counterpart $Y = (\phi_y, y)$, transforming as R^* . In terms of these $N = 1$ multiplets the action can be written as³⁾

$$S = \int d^4x d^4\theta \left[\Phi e^{\partial^V} \Phi + X^\dagger e^{\partial^V} X + Y^\dagger e^{-\partial^V} Y \right] + \frac{1}{64g^2 C_2(G)} \text{Tr} \int d^4x d^2\theta \left[W^\alpha W_\alpha + (gY\Phi X + \text{h.c.}) \right] \quad (1)$$

As is seen from (1), the $N = 2$ theory has Yukawa interactions only of the type $[Y\Phi X]_F$. It is a remarkable feature of the extended SUSY theories that there are no independent Yukawa couplings but all the couplings are related to the gauge coupling. Moreover, there is no generation dependence in the Yukawa terms. To solve the problem of flavours in $N = 2$ theories one may thus need to resort to horizontal symmetries or to non-renormalizable interactions of $N = 2$ supergravity^{7),8)}. We do not address this question here.

It can be shown that $N = 2$ theory is finite in any case beyond one loop, and finite already at one loop whenever²⁾

$$C_2(G) = \sum_i T(R_i) \quad (2)$$

where C_2 and T are the usual Casimir invariants. A complete list of classical groups satisfying the finiteness criteria (2) has been given by Koh and Rajpoot⁴⁾. In addition, it has been shown that the finiteness of the theory is preserved in the presence of a class of soft SUSY breaking terms^{3),6),9)}.

The most pressing open problems of finite $N = 2$ theories are the splitting of the vector-like theory into chiral parts and the breaking of the gauge symmetry. In the present paper we have made a search for a phenomenologically suitable $N = 2$ SUSY GUT based on the symmetry $SO(n)$ or $SU(n)$ with emphasis on these questions. Among theories based on $SU(n)$, we discard those with matter in the adjoint representation (these are the $N = 4$ SYM theories) and those with matter in symmetric tensor representations (these will lead to colour sextets). Then an $SU(5)$ $N = 2$ SUSY GUT has severe difficulties for accommodating u quarks. For $SU(6)$ and $SU(7)$ there is a plethora of possibilities, but these appear to have trouble with giving a mass either to the up or to the down sector, or alternatively, they lead to unacceptable mass relations like $M_U = m_d$ and $M_D = m_u$ (capital letters denote V+A fermions). We illustrate these difficulties briefly in an $SU(6)$ theory.

Consider an $SU(6)$ theory with three matter hypermultiplets in the $(15+15^*)$ representation and the vector supermultiplet in the adjoint 35. The $SU(5)$ decomposition of the Yukawa coupling $[Y\Phi X]_F$ is

$$\begin{aligned}
 Y(15^*) \bar{\phi}(35) X(15) = & F_i(10^*) \phi(5) F_i(5) + f_i(5^*) \phi(5^*) f_i(10) \\
 & + F_i(10^*) \phi(24) f_i(10) + f_i(5^*) \phi(1) F_i(5) \\
 & (i = 1, 2, 3)
 \end{aligned} \tag{3}$$

where f_i denotes ordinary V-A fermions and F_i corresponding V+A fermions. Note that the supermultiplet $X(15) = F(5) + f(10)$ contains both V-A and V+A fermions, as does the mirror supermultiplet $Y(15^*) = f(5^*) + F(10)$. The first term in (3) gives mass to D_i and L_i , the second to d_i and λ_i . Choosing $\langle \phi(5) \rangle \gg \langle \phi(5^*) \rangle$ the former can be made heavier than the latter, as required. However, the up quarks get masses only through the fF mixing terms which would spoil their chiral nature. It appears that realistic finite $N = 2$ SUSY GUTs are rather difficult to construct if based on $SU(n)^*$.

Let us move to $SO(n)$ GUTs to see if the V-A/V+A mass splitting is easier to be arranged there. Let us recall how fermions and mirror fermions were separated in conventional non-finite models, e.g., $SO(11)$, $SO(14)$ or $SO(15)$, where the Higgs sector was not constrained¹¹⁾. In the $SO(10)$ level fermions belong to $\phi_+(16)$ and mirror fermions to $\phi_-(16^*)$, where the subscript refers to the $SO(10)$ chiral projection [i.e., $\phi_{\pm} = \frac{1}{2}(1 \pm \gamma_{11})\phi$]. Assume that at the $SO(11)$ level we have Higgses both in a vector representation 11 and in an adjoint 55, both of which contain 10 of $SO(10)$, a representation responsible for fermion masses. Since the two 10's transform as $\phi^T C \gamma_i \psi$ and $\phi^T C \gamma_{11} \gamma_i \psi$, respectively, the mass Lagrangian can be made $SO(10)$ chiral by suitably choosing the magnitudes and the relative sign of the vev's $\langle 11 \rangle$ and $\langle 55 \rangle$.

In the finite theories, however, this mechanism does not work. This follows from the fact that it is only the adjoint representation of the $SO(n)$ group ($n > 11$) that has Yukawa couplings to fermions. The symmetry of the adjoint representation dictates that for each light 16 there is a light 16^* . Let us demonstrate this by taking $SO(12)$ as an example. Let X denote the $SO(12)$ spinor 64, consisting of two V-A families $f_1^X, f_2^X \sim 16$ and of two V+A families $F_1^X, F_2^X \sim 16^*$. Y is the corresponding mirror spinor with the contents $f_1^Y, f_2^Y \sim 16$ and $F_1^Y, F_2^Y \sim 16^*$. The mass Lagrangian will, from (1), be proportional to

*) Similar conclusions were reached also in Ref. 10).

$$g [Y^T C \gamma_i \gamma_{11} X \langle \phi^{i,11} \rangle + Y^T C \gamma_i \gamma_{12} X \langle \phi^{i,12} \rangle + \quad (4) \\ + Y^T C \gamma_{11} \gamma_{12} X \langle \phi^{11,12} \rangle] + h.c.$$

where $\phi^{i,a}$ ($a=11,12$) are 10's of $SO(10)$, whereas $\phi^{11,12}$ is an $SO(10)$ singlet. The last term is of the type $16 \cdot 16^*$ and hence undesired, and we must choose $\langle \phi^{11,12} \rangle = 0$. The first two terms decompose as follows

$$g [f_1^{yT} C \gamma_i f_1^x - F_1^{yT} C \gamma_i F_1^x - f_2^{yT} C \gamma_i f_2^x + F_2^{yT} C \gamma_i F_2^x] \langle \phi^{i,11} \rangle \\ + g [-f_1^{yT} C \gamma_i f_1^x + F_1^{yT} C \gamma_i F_1^x - f_2^{yT} C \gamma_i f_2^x + F_2^{yT} C \gamma_i F_2^x] \langle \phi^{i,12} \rangle \quad (5) \\ + h.c.$$

One could make, e.g., the combination $f_1^x + f_1^y$ to form a light V-A family by choosing $\langle \phi^{i,11} \rangle = \langle \phi^{i,12} \rangle$, but as can be seen from (5), there would then also be a light V+A family $F_1^x + F_1^y$. Going from $SO(12)$ to $SO(13)$ or to higher rank groups will not bring any improvement. In $SO(13)$, for example, there would be one more $SO(10)$ vector 10 in the decomposition $\phi^{i,13}$, giving rise to the term

$$g Y^T C \gamma_i \gamma_{13} X \langle \phi^{i,13} \rangle + h.c. \\ = -g [f_1^{yT} C \gamma_i f_2^x + F_1^{yT} C \gamma_i F_2^x + f_2^{yT} C \gamma_i f_1^x \\ + F_2^{yT} C \gamma_i F_1^x] + h.c. \quad (6)$$

but this still keeps the V-A and the V+A families pairwise degenerate.

We conclude that the V-A/V+A splitting cannot be made at the $SO(10)$ level, but the gauge breaking should instead go to another direction. In the case of $SO(12)$, which we will examine in what follows, a suitable breaking pattern turns out to be

$$SO(12) \rightarrow SU(4) \times SU(3) \times U(1) \rightarrow SU(2)_L \times SU(3)_C \times U(1)_Y \quad (7)$$

Under $SU(4) \times SU(3)_C \times U(1)$ each spinor 32 decomposes to the following multiplets

$$\begin{aligned} (4, 1, 1) &= (\nu, e, N, E)_L \\ (4, 3, -\frac{1}{3}) &= (u_i, d_i, U_i, D_i)_L \\ (4^*, 1, -1) &= (\nu^c, e^c, N^c, E^c)_L \\ (4^*, 1, \frac{1}{3}) &= (u_i^c, d_i^c, U_i^c, D_i^c)_L \end{aligned} \quad (8)$$

As was shown by del Aguila et al.⁶⁾, certain non-diagonal neutral members of the adjoint 15 $\subset \Phi$ will make the V+A part of (8) heavy while leaving the V-A part massless.

Let us now focus on the finite $SO(12)$ GUT. The $N = 1$ supermultiplet content is

$$\begin{aligned} X, X' &\sim 32 \quad ; \quad Y, Y' \sim 32 \\ V_X, V_X' &\sim 12 \quad ; \quad V_Y, V_Y' \sim 12 \end{aligned}$$

where Y denotes the mirror multiplet. Note that the spinor multiplets X, X' and their mirror counterparts Y and Y' transform identically because the spinor representation 32 of $SO(12)$ is self-adjoint^{*)}. We will assign one generation of matter to the spinors X' and Y' . The rest of the supermultiplets will be decoupled from the low energy phenomenology.

As mentioned before, the conventional spontaneous gauge symmetry breaking will not work in this model, since the finiteness condition (2) allows only for low order chiral multiplets; already one adjoint supermultiplet alone saturates (2). One is therefore forced to look for alternative mechanisms. One

*) Indeed, one could consider a spinor 32 which contains sixteen ordinary and sixteen mirror states¹²⁾. In that case the finiteness condition (2) would be modified accordingly.

possibility would be dynamical breaking by bound states. In the following we will describe and discuss this idea.

Let us assume that supersymmetry, or more precisely one of the two supersymmetries, gets broken at the Planck scale by finiteness preserving mass terms

$$M_X [XX]_F + M_Y [YY]_F \quad (10)$$

where $M_X, M_Y = O(M_P)$. The basic condition for the bound state picture to work is that the finite $SO(12)$ is a strongly interacting theory at the scale $\Lambda_{12} \sim M_P$. This might be fulfilled naturally, especially if $SO(12)$ is to be unified with gravity. Then

$$g_{12}(M_P) \simeq G_N(M_P) = 1 \quad (11)$$

where G_N is the coupling strength of gravity. As in QCD, one would expect the bound states to be $SO(12)$ singlets with radius of confinement $O(1/\Lambda_{12})$. However, about the Planck time the radius of the Universe was smaller than this, and therefore one could expect that at the beginning bound states were formed at least temporarily in every available channel. After the radius got much bigger than $1/\Lambda_{12}$, these would have coalesced into $SO(12)$ singlets. Besides breaking the gauge symmetry, the temporal gauge non-singlet bound states could be responsible for the rapid evolution of the gauge coupling necessary for the scenario to work. Since we do not expect that $N = 2$ SUSY could be broken dynamically, the bound states also would probably obey the finiteness conditions except for the bound states formed from the SUSY breaking multiplets (10). Let us hence consider the system of bound states built from X and Y alone. Because (10) preserves the reflection symmetry $X \rightarrow X, Y \rightarrow -Y, \Phi \rightarrow -\Phi$ of the Lagrangian (1), we assume that this symmetry is also obeyed by the bound states. In effect, we will assume that gauge non-singlet bound states will be formed only in XX and YY channels. The spinor product in $SO(12)$ decomposes as follows:

$$32 \times 32 = (1 + 495)_A + (66 + 462)_S \quad (12)$$

Indeed, the symmetric bound states are gauge non-singlets. The bound state spectrum will thus consist of two adjoint fields $\Sigma_1(XX), \Sigma_2(YY) \sim 66$, and of two order-6 tensors $T_1(XX), T_2(YY) \sim 462$. As (10) conserves one supersymmetry, we take the interactions of the bound states to be described by an effectively $N = 1$ supersymmetric superpotential:

$$f_{\text{eff}} = \sum_{i,j=1}^2 \left[M_{ij} T_R \Sigma_i \Sigma_j + \lambda_{ij} \Sigma_i^2 \Sigma_j^2 \right. \\ \left. + \mu_i T_i^2 + \lambda_i T_i^4 + T_i^2 \xi_{ij} T_R \Sigma_i \Sigma_j + \dots \right] \quad (13)$$

Because we are here dealing with scales of the order of Planck mass, non-renormalizable terms are also included into the superpotential.

As usual in global SUSY models, the effective superpotential (13) will lead to a potential with several SUSY conserving degenerate minima with different gauge symmetry breaking patterns. Among these a suitable choice is obtained when the vev's of Σ_1 and Σ_2 lie in the direction $\sigma_{14} + \sigma_{25} + \sigma_{36}$, where σ_{AB} ($A, B=1, \dots, 12$) are the generators of $SO(12)$, i.e.,

$$\langle \Sigma_i \rangle = \frac{s_i}{\sqrt{6}} \begin{pmatrix} 0 & -1 & | & 0 \\ 1 & 0 & | & 0 \\ \hline 0 & 0 & | & 0 \end{pmatrix} \quad (14)$$

where the sub-blocks are 6×6 matrices. When $s_1 \neq 0$, the $SO(12)$ is broken to $SO(6) \times SU(3) \times U(1) \simeq SU(4) \times SU(3) \times U(1)$. Since $\langle T_1 \rangle$ are negligible at this stage, the first of equations

$$\frac{\partial f_{\text{eff}}}{\partial T_i} = 0 \\ \frac{\partial f_{\text{eff}}}{\partial s_i} = 0 \quad (15)$$

is automatically satisfied. It is also straightforward to check that the second equation has a solution $s_1, s_2 \neq 0$ for a natural range of parameters and, accordingly, that the breaking $SO(12) \rightarrow SU(4) \times SU(3) \times U(1)$ indeed occurs, while supersymmetry remains unbroken.

Let us now track down the evolution of the vacuum from the Planck era onwards. As the Universe cools in the $SO(12)$ phase, supersymmetry will be broken by the soft terms (10). The non-singlet bound state spectrum of the

strongly interacting $SO(12)$ theory will then be described by the superpotential (13). This leads to a very rapid evolution of the $SO(12)$ coupling constant α_{12} through changes in the renormalization scale. With the fields Σ_1, Σ_2, T_1 and T_2 the $N = 1$ SUSY renormalization group equations yield, together with the boundary condition (11),

$$\alpha_{12}^{-1}(\mu) = 1 + \frac{121}{\pi} \ln \frac{M_P}{\mu} \quad (16)$$

which is valid for $\mu \gtrsim M_{1j}$ [see Eq. (13)]. Therefore, even though the range in μ is very short, the evolution of the $SO(12)$ gauge coupling is extremely rapid. For instance, for $\mu \approx \frac{1}{2}M_P$ we find $\alpha_{12}(\mu) \approx 1/28$. This suggests that the process of confinement of the $SO(12)$ theory will never be completed and that below $\mu \approx \mu_c$ the particle spectrum consists of superheavy non-singlet states. The degeneracy between the $SO(12)$ and the $SU(4) \times SU(3) \times U(1)$ vacua can then be lifted by finite temperature effects. This happens if the light degrees of freedom of the $SO(12)$ phase have been transformed to superheavy bound states, while the $SU(4) \times SU(3) \times U(1)$ contains also light fields. As the finite temperature corrections to effective potential are proportional to $-N(T)T^4$, where $N(T) = N_B + \frac{7}{8}N_F$ is the number of light degrees of freedom, these will lower the $SU(4) \times SU(3) \times U(1)$ phase below the symmetric one.

In the resulting effective $SU(4) \times SU(3) \times U(1)$ theory, which is no more finite, ordinary fermions can be split from mirror fermions by a $SU(2)$ breaking vev of the adjoint field in the vector multiplet, as in Ref. 6). Of course, to have a phenomenologically acceptable model, the full theory must contain other soft terms besides those given in (10). We expect, e.g., that there are mass terms $[V_X V_X]_F$, $[V_Y V_Y]_F$, $[V_X \tilde{V}_X]_F$, etc., for the fields in 12. If the scale of these other SUSY breaking soft terms is somewhat lower than those in (10), the picture described above will remain unchanged. However, we will leave the details for further study.

Let us finish by remarking that in $N = 2$ SUSY GUTs the family problem is difficult to solve, and to accommodate the known families one must enlarge the gauge group. For $SO(n)$, there are no solutions to the finiteness condition (2) beyond $SO(14)$. A possible interesting candidate for family unification would be E_7 with three fundamental representations and containing three $SO(12)$ spinors.

In conclusion, we have investigated a finite $N = 2$ SUSY GUT based on the gauge group $SO(12)$. We argued that the full theory is strongly interacting at

the scale of M_p , and that gauge non-singlet bound states are formed from the fields participating in the soft breakdown of $N = 2$ to $N = 1$. This leads to a very rapid decrease of the $SO(12)$ gauge coupling constant. The phase transition $SO(12) \rightarrow SU(4) \times SU(3) \times U(1)$ becomes possible because of finite temperature effects. Because the whole region of strong coupling dynamics is very unclear at present, our arguments should, however, be considered somewhat heuristic. We would like to stress that, as a result of our present analysis, we feel that such a dynamical scenario is necessary if one is to build a realistic phenomenological $N = 2$ SUSY GUT model. Another possibility would be to break gauge symmetries by $N = 2$ supergravity effects as discussed in Ref. 8).

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