EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN-EP/84-142
22 October 1984

CHANNELING RADIATION FROM 2 TO 55 GeV/c ELECTRONS AND POSITRONS

I. PLANAR CASE

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(Submitted to Nuclear Physics B)

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ABSTRACT

Channeling radiation emitted from 2 - 55 GeV/c electrons and positrons traversing Si and Ge single crystals has been measured for planar directions. The intensity is found to be 30 (Si) to 10 (Ge) times the intensity of normal bremsstrahlung. For channeled electrons, no structure is found in the spectrum, whereas strong and sharp peaks are found for positrons. This peak structure is extremely sharp around a certain "magic" beam momentum between 1 and 10 GeV/c. The spectra are compared to calculated ones based on classical electrodynamics. The agreement is good but for positrons the calculated photon energies are around 5% higher than the measured ones. This discrepancy also exists for practically all previous positron experiments but could be explained in the present investigation by errors in beam momentum calibration. The possibility of subharmonic peaks in the photon spectra is discussed and compared to Monte Carlo calculations. The radiation from {110}, {111}, and {100} planes was measured for different momenta between 1 and 10 GeV/c and for different temperatures. Also the thickness dependence of channeling radiation is presented together with a comparison of photon spectra from Si and Ge crystals. The possibility of making a monoenergetic γ -source in the energy region of 1 - 100 MeV is discussed.

I. Introduction

When a parallel beam of electrons or positrons traverses a crystalline target in directions far from major crystal axes or planes, the well known incoherent bremsstrahlung (IB) is emitted. As the direction of incidence is turned towards planar directions, the projectiles start to feel the crystalline structure, and coherence in the emitted radiation results in the peak structure of coherent bremsstrahlung (CB). The peak energies are given by the lattice constants; and the incident angle with respect to the planes, and the spectra are the same for electrons and positrons. If the projectiles are incident nearly parallel to planar directions, they become trapped in the potential wells due to the planes, resulting in the strong steering effect called channeling. This special motion also gives rise to coherence effects, and the resulting radiation is called channeling radiation (CR). The structure of CR is strongly dependent on the form of the trapping potential but is not directly connected to the lattice parameters. Channeled positrons are pushed away from the atomic planes, whereas channeled electrons are focused around the planes. Since they move in different potentials, their CR spectra will be different. This is in contrast to the CB case.

The main theoretical concepts of channeling were formulated in the classical paper by Lindhard¹⁾. Today, the field is well established and described in many review articles, e.g., Ref. 2. The main aspects of CR were described by Kumakhov³⁾ and coworkers, and a number of review articles already exist⁴⁾. Several experiments have been reported for MeV electrons and positrons⁵⁾; however, little experimental work has been performed in the GeV region where CR effects are very strong. The present paper

reports a continuation of our experiments, where the radiation from 5-55-GeV/c electrons and positrons was studied⁶⁾, and where 5-10-GeV/c positrons showed a very pronounced peak structure, in contrast to electrons in the same energy range. At first it was thought that this difference was due to the electron potential being much more anharmonic than the positron potential. However, a more careful study shows that this is only part of the explanation. Indeed, there is an important interplay between the breakdown of the dipole approximation and the anharmonicity of the positron potential, which gives rise to a narrowing of the peak structure as a specific value of the incident-beam momentum is approached. We refer to the Lorentz factor associated with this beam as γ magic and denote it by γ_{magic} ⁷⁾.

In Sec. II, we review the concepts of channeling and of radiation from a moving charge, which will be needed to understand the results. The discussion is illustrated with data from our first experiment⁶⁾ and includes arguments that classical relativistic mechanics and classical electrodynamics can be used in the GeV region. In Sec. III, the experimental setup is discussed, including various problems associated with channeling at these high energies where the angular resolution must be of the order of microradians. Section IV contains a detailed discussion of the new experimental data and a comparison with our theoretical results. The data include spectra from the {110}, {111}, and {100} planes of silicon for positrons and electrons in the 1-10-GeV/c range. The existence of the predicted γ_{magic} and its importance for a monoenergetic gamma source is discussed. Thickness and temperature effects have been studied, and some data from germanium are

presented. A brief analysis of the motion of planar-channeled particles is given in the appendix, where particular attention is given to the coupling of the transverse and longitudinal motion, which is so important for the understanding of the radiation at the energies under consideration.

The experimental results shown in chapter II are mostly taken from an SPS experiment (WA64), the experimental setup of which can be found in ref.6.

II. Some relevant channeling and radiation concepts

The Lindhard theory of channeling¹⁾ for positive, heavy, nonrelativistic particles is based on a classical view of the motion. At the present velocities, the Bohr condition for a classical orbital description, $\kappa = 2Z_1 Z_2 c / 137v \gg 1$, is not fulfilled for single-scattering events, and such events become less classical as the velocity v increases. Here, Z_1 and Z_2 are the atomic numbers of the projectile and target atom, and c is the velocity of light. However, in the channeling picture, the main features of the motion can be described in a plane perpendicular to the projection of the velocity of the channeled particle onto the crystal planes. For this transverse motion, Lindhard showed that the Bohr condition for a classical description becomes

$$\kappa_{\perp} = 2 \left[\frac{m_{\gamma}}{m_e} Z_1 Z_2^{1/3} \frac{a_0}{d} \right]^{1/2} \gg 1,$$

where m_{γ} is the relativistic mass of the projectile, m_e the electron rest mass, a_0 the Bohr radius, and d a lattice parameter.

Since $\kappa_{\perp}$ is proportional to the square root of the relativistic mass, channeling becomes more classical as the incident momentum increases. This is the opposite of the single-scattering-event case, and for 1-GeV/c positrons or electrons, $\kappa_{\perp}$ is already large. The extent of quantal corrections to the classical description depends on the number of quantum states in the planar-potential well, and also this number is proportional to $(m_{\gamma})^{1/2}$. For 5-GeV/c electrons and positrons channeled along the (110) planes in silicon, there are approximately 70 and 100 quantum states, respectively. This suggests that classical relativistic mechanics and classical electrodynamics can be used for studying the radiation emitted by GeV/c electrons and positrons, and it will become clear

that this gives a detailed understanding of the experimental data.

The nonrelativistic expressions for axial- and planar-channeling critical angles can be used for relativistic particles if the energy term $\frac{1}{2}mv^2$ is replaced by $\frac{1}{2}pv$ for incident momentum p . The planar critical angle is then given by

$$\psi_p = \left(\frac{4Z_1 Z_2 e^2 N_d C a_p}{pv} \right)^{1/3}, \quad (1)$$

where C is a constant of the order of $\sqrt{3}$, N_d represents the number of atoms per unit area of the plane, d_p is the distance between planes, and $a = a_0 Z_2^{-1/3}$ is a screening length in the atomic potential. The derivation of Eq.(1) uses Lindhard's standard potential. For particles incident on a crystal with an angle to a plane less than or of the order of ψ_p , channeling effects are important, and for 5-GeV/c positrons along the $\{110\}$ planes of silicon, $\psi_p = 72 \mu\text{rad}$. The axial critical angle ψ_1 is larger by about a factor of three and given by

$$\psi_1 = \sqrt{4Z_1 Z_2 e^2 / pvd}, \quad (1')$$

where d is the atomic spacing along the considered axis.

A typical planar-channeling trajectory is illustrated in Fig. 1. To a good approximation, it is given by

$$\vec{r}(t) = x(t)\vec{e}_x + [\bar{\beta}_z ct + z_p(t)]\vec{e}_z, \quad (2)$$

where $x(t)$ is determined from $m\gamma_0 \ddot{x} + U'(x) = 0$; γ_0 is the initial value of γ , and $U(x)$ is a continuum potential; U is periodic, with period d_p , symmetric in x and is chosen such that $U(0) = 0$. The x motion conserves transverse energy, $E_\perp = \frac{1}{2}m\gamma_0 \dot{x}^2 + U(x)$, and for $E_\perp < U(d_p/2) = U_0$, the x motion is periodic with period $T(E_\perp)$. The z motion is determined by the x motion and consists of an average z -velocity term $\bar{\beta}_z c$, which depends on $E_\perp$ and a periodic

term with period $\frac{1}{2}T(E_{\perp})$. A discussion of the dynamics for $E_{\perp} < U$, is given in the appendix. Particle motion for $E_{\perp} > U$, gives a significant contribution to the experimental data. However, this over-the-barrier motion is a simple extension of the ideas for $E_{\perp} < U$, and will not be discussed in any detail.

Because of the regular motion in Fig. 1, it is to be expected that coherence effects will be important for the radiation. Since the largest contribution to the radiation intensity comes from the largest accelerations, we examine the emission of photons in the ϑ direction from the particle at the extreme positions. If $\vartheta = 0$, then, for emission at the first position to interfere constructively with emission at the second and subsequent positions, the distance between the particle at an extreme position and the wave front generated at the previous position must be an integral number of light wavelengths. Since the wavelength is $2\pi c/\omega$, where ω is the frequency of the radiation, and $T(E_{\perp})$ is the time for the particle to go through one cycle, it follows that $cT - \bar{\beta}_z cT = 2\pi nc/\omega$. For $\vartheta > 0$, the same argument gives $cT - \bar{\beta}_z cT \cos\vartheta = 2\pi nc/\omega$, therefore

$$\omega(E_{\perp}, \gamma_0, \vartheta, n) = \frac{2\pi n}{T(E_{\perp})[1 - \bar{\beta}_z(E_{\perp})\cos\vartheta]} \quad (3)$$

Although this is a simple classical argument, Eq. (3) is correct. This simple expression is important because it explains many features of the photon-energy spectra. The maximum frequency $\omega_m(E_{\perp}, \gamma_0, n)$ is emitted in the forward direction, thus $\omega_m = \omega(E_{\perp}, \gamma_0, 0, n)$.

The ideas used in deriving Eq. (3) can also be used to derive the analogous formula for the peak energies in coherent

bremsstrahlung if the emitted photon momentum is negligible compared to the projectile momentum. In this case, $\bar{\beta}_z$ and T must be replaced by the average total velocity and the time to go from one plane to the next, respectively. In general, the CB-peak energies must be derived using conservation of energy and momentum. Here the change in particle momentum must equal the sum of the emitted photon momentum and N times a reciprocal lattice vector. For a particles moving at an angle θ with respect to planes with a spacing d_p , this gives

$$\frac{N\omega}{m\gamma c^2} = \left(1 + \frac{mc^2}{Nc} \frac{d_p}{4\pi n} \frac{1}{\gamma\theta}\right)^{-1} \quad (4)$$

for emission in the forward direction. If θ is small so that the second term on the right-hand side is large, then

$$\omega \approx \frac{4\pi n c \gamma^2 \theta}{d_p}, \quad (5)$$

which is the formula that would be obtained by the argument leading to Eq. (3). An important feature of Eq. (5) is that the CB-peak energies vary linearly with θ as the incident beam is tilted away from a planar direction. This feature is observed in the experimental spectra.

For a channeled particle, the total energy density radiated at frequency ω into a solid angle (ϑ, φ) can be calculated from

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2 \omega^2}{4\pi^2 c} \left| \int_0^\tau (\vec{k} \times \vec{\beta}(t)) \exp[i\omega(t - \vec{k} \cdot \vec{r}(t)/c)] dt \right|^2, \quad (6)$$

once $\vec{r}(t)$ in Eq. (2) is determined. Here $\vec{\beta}(t) = \dot{\vec{r}}(t)/c$, $\vec{k} = \cos\varphi \sin\vartheta \hat{e}_x + \sin\varphi \sin\vartheta \hat{e}_y + \cos\vartheta \hat{e}_z$ is the direction of observation, and τ is the time the particle spends in the crystal. (This is Eq. (14.67) of Ref. 8, which is obtained from Eq. (14.65) by ignoring the boundary terms after an integration by parts.) Note that the unit of I in Eq. (6) is energy. Multiplying Eq. (6) by

$\sin\theta d\theta d\phi$ and integrating over solid angle ($0 \leq \theta \leq \pi$, $0 \leq \phi \leq 2\pi$) gives $dI/d\omega$, where $(dI/d\omega)_{\Delta\omega}$ is the energy radiated by the particle into the frequency interval $(\omega, \omega + \Delta\omega)$. Therefore $P_{CR}(\mathcal{H}_\omega, \Delta\omega) = (1/\mathcal{H}_\omega)(dI/d\omega)_{\Delta\omega}$ can be interpreted as the probability of emitting a photon in the energy range $(\mathcal{H}_\omega, \mathcal{H}_\omega + \mathcal{H}_{\Delta\omega})$ for a given positron or electron with trajectory given by $\vec{r}(t)$, $0 \leq t \leq \tau$. Experimentally, the average value of P_{CR} over the distribution of transverse energies is measured.

For IB, the Bethe-Heitler formula gives

$$\frac{dI}{d\omega} = 4Z_1^2 \frac{e^2}{c} N r_e^2 \ell F(s), \quad (7)$$

where $r_e = e^2/m_e c^2$ is the classical electron radius, ℓ is the crystal thickness, $s = \mathcal{H}_\omega/m\gamma c^2$ is the ratio of the emitted photon to the projectile energy, and $F(s) = [1 + (1-s)^2 - \frac{2}{3}(1-s)] \ln 183Z^{-1/3} + \frac{1}{9}(1-s)$, in the complete screening limit⁹⁾. As in the CR case, we introduce $P_{IB}(\mathcal{H}_\omega, \Delta\omega) = (1/\mathcal{H}_\omega)(dI/d\omega)_{\Delta\omega}$ from Eq. (7), which is the probability that a positron or electron undergoing random scattering off lattice atoms in the crystal will emit a photon in the energy range $(\mathcal{H}_\omega, \mathcal{H}_\omega + \mathcal{H}_{\Delta\omega})$.

In Fig. 2, experimental photon-counting spectra for IB and CR are presented for the case of 10-GeV/c positrons incident on a perfect silicon crystal of thickness 100 μm . The abscissa is the photon energy $E_\gamma = \mathcal{H}_\omega$. The ordinate is the number of photons emitted per positron in the photon-energy range $(\mathcal{H}_\omega, \mathcal{H}_\omega + \mathcal{H}_{\Delta\omega})$, where $\mathcal{H}_{\Delta\omega} = 2.3$ MeV and corresponds to the probabilities P_{CR} (open circles) and P_{IB} (solid circles) discussed above. For CR, the incident angles were in $(-\psi_p, \psi_p)$, where ψ_p is the critical angle defined in Eq. (1). There is a large enhancement of CR over IB in the energy region shown in the figure. The first harmonic is clearly evident in

the peak/shoulder structure near 55 MeV, and there is a slight indication of the second harmonic near 105 MeV. These energies are consistent with Eq. (3) for $\theta = 0$ and $n = 1, 2$. There is also an indication of a shoulder near 20 MeV which will be discussed later.

The number of photons emitted and the total energy radiated can be determined from the experimental data of Fig. 2 and can also be calculated from Eqs. (6) and (7). In fact, from Eq. (7), $P_{IB}(\hbar\omega, \Delta\hbar\omega) = 1.34 \times 10^{-3} \Delta\hbar\omega/\hbar\omega$, which is in good agreement with the solid line in Fig. 2. In IB, the number of photons/positron is 0.01, and the energy radiated per positron is 11 MeV. For CR, the number of photons/positron is 0.09, and the total energy radiated is in the range 3-6 MeV, depending on E_L , as can be calculated from the formulas in Sec. IV. Thus CR from positrons creates less radiated energy, more low-energy photons, and less high-energy photons than IB from positrons. The latter is to be expected because channeling of positrons significantly decreases the probability of a close encounter with a nucleus. Crudely speaking, the channeling process takes photons from the high-energy region and puts them in the low-energy region.

Figure 3 shows the photon intensity from positrons as a function of photon energy and angle with respect to the {110} planes. The data corresponds to the case of Fig. 2, but the photon spectra have been normalized by the Bethe-Heitler formula. For angles near zero, the large enhancement of the CR at low photon energies and the expected drop below IB for large photon energies are apparent. The two red areas come from the first and second harmonics of the CR, which will be discussed later. The CB arms, going

out nearly linearly in angle, beautifully highlight the figure. In this case, Eq. (4) with $d = 1.92 \text{ \AA}$ and $n = 1$ becomes $\hbar\omega = 10^4 (1+2.0/\theta)^{-1} \text{ MeV}$ for θ in mrad. For $\theta = 6\psi_p = 0.3 \text{ mrad}$, $\hbar\omega \approx 1300 \text{ MeV}$, in reasonable agreement with the figure. $\square$.

In Fig. 4, the experimental CR spectra normalized to IB for the case of 10-GeV/c positrons (a) and electrons (b) channeled along the $\{110\}$ planes of silicon are shown. The incident angles were in the interval $(-\psi_p, \psi_p)$, and the angular resolution of the system was about ψ_p . The positron spectra can be obtained from Fig. 2 by dividing the CR by the IB, or from Fig. 3 by averaging over $(-\psi_p, \psi_p)$. Note the large enhancement over IB caused by the coherent radiation from the regular channeling motion. Also note that the positron and electron spectra are qualitatively different. This may be surprising to those unfamiliar with channeling but familiar with CB, where the spectra are the same. However, the channeling trajectories depend on the planar continuum potentials, and these are quite different for the two particles. For positrons, the potential is approximately harmonic, giving rise to a peak near 50 MeV, in agreement with Eq. (3). The smaller peak near 100 MeV is the second harmonic of the radiation. Note that the first and second harmonics are clearer here than in the counting spectra of Fig. 2. In contrast, the potential for electrons is very anharmonic, which causes a large spread in the values of ω_m . The experimental spectrum is composed of channeled particles of different transverse energies, and this explains the very broad electron spectra.

The difference in electron and positron spectra can also be understood from a simple quantum-mechanical picture. At 10 GeV/c, there are on the order of 100 quantum states in the

planar-potential well. Since to a good approximation positrons feel a harmonic potential, with the resulting equidistant energy levels, the photon spectrum is expected to show a peak structure. The electrons, on the other hand, see an anharmonic potential, with unequal distance between energy levels, so for a population of many states, which is inevitable in an experiment, a structureless channeling-radiation spectrum is expected.

In Fig. 5, CR spectra from the $\{110\}$ planes of silicon for 5, 10, 20, and 55-GeV/c positrons are shown. The data are taken from Ref. 6, and the 10-GeV/c case is repeated from the previous figures. Once again, there is a large enhancement over the IB in the energy region shown. The peak values are in reasonable agreement with Eq. (3) for $\vartheta = 0$ and for 5, 10, and 20 GeV/c; however, the peaks still seem to broaden, and by 55 GeV/c, there is little structure in the spectra. To understand this, Eq. (3) was studied, and the results are shown in Fig. 6, where the Molière and Doyle-Turner potentials were used. Here the variation of ω_m with $E_{\perp}$ is rather small for 5 and 10 GeV/c, moderate for 20 GeV/c, and quite large for 55 GeV/c. This, combined with the increased importance of higher harmonics and linewidth as the beam momentum increases, explains the transition from a sharp peak at 5 GeV/c to a broad spectrum at 55 GeV/c. Furthermore, Fig. 6 suggests that $\omega_m(E, \gamma_0, 1)$ becomes approximately independent of $E_{\perp}$ for a value of γ_0 , which is near the 5-GeV/c case, but this becomes clear from Fig. 14. This seemed a likely explanation of the sharp peak structure near 5 GeV/c and opened the possibility of a monoenergetic photon source in the range of 1-10 MeV. To investigate this 'magic' γ value, γ_{magic} , and the possibility of a monoenergetic

source, a new experiment was designed, with better angular resolution in the 1-10-GeV/c range. The experiment and the results are presented in the following two sections.

III. Experimental setup

1. General layout

The experimental apparatus was installed in the t_7 beam of the CERN 28-GeV/c proton synchrotron. A schematic layout of the experiment can be seen in Fig. 7.

The beam was a high-intensity ($\sim 10^6/\text{cm}^2/\text{sec}$) secondary, nonseparated beam with a momentum adjustable between 2 and 10 GeV/c. The beam divergence was ± 1 mrad, the momentum spread was normally $< 1\%$, and momentum slits were used for reduction of intensity to limits acceptable for detectors (drift chambers, solid-state detectors, and photon detectors). In the positive polarity at 5 GeV/c, the beam contained approximately 30% protons, 60% pions, and 8% positrons, but the positron content varied from 50% at 2 GeV/c to 0.5% at 10 GeV/c, so the 10-GeV/c experiment was time-consuming and required high long-term stability of the entire setup. The electron content was somewhat higher than the positron content.

Particle identification was made with threshold Cerenkov counters, but the rather low electron/positron content and the low counter efficiency for the higher momenta meant that correct particle identification was inaccurate, and a lead-glass array placed at the end of the experiment was used as an extra e^+/e^- identification.

Scintillation counters SC1 and SC3 in anticoincidence with SC2 were used to define a usable fraction of the beam with diameter 15 mm at the target, which was only half the opening of the goniometer. The incoming- and outgoing-particle trajectories were measured by a set of three position-sensitive drift chambers DC1, DC2, and DC3. The beam lines were evacuated to a

pressure below 10^{-2} torr to eliminate multiple scattering and radiation background from air tubes. The goniometer allowed the adjustment of the crystal axes or planes to the center of the beam cone. A suitable liquid-nitrogen cooling system could maintain the targets at a nearly constant temperature of ~ 90 K. The entire target box was evacuated to below 10^{-6} torr and for most experiments was completely thermally isolated to reduce long-term temperature variation to less than 1 degree.

The beam was bent 10 mrad by BM1 right after the first drift chamber to prevent background bremsstrahlung created upstream from hitting the target area and photon detectors. The influence of this bend on the incident angular resolution was negligible (less than 5 μ rad). The exit charged particles were swept out of the photon beam by a 100-mrad bending magnet BM2. The emitted radiation was detected by a 10"×12" NaI detector.

The final drift chamber, DC3, was placed in front of the Cerenkov and lead-glass array to ensure that the beam was correctly steered into these.

As the beam passed through DC2 and the nearby mylar windows, an incoherent bremsstrahlung background appeared. This no-target background was measured and found equivalent to IB from 190- μ m amorphous silicon. This is in good agreement with the IB estimates based on knowledge of the material in DC2, air gaps, and mylar windows. This radiation background is subtracted from the present data.

For each accepted event, the on-line data-acquisition system stored information from drift chambers, detectors, Cerenkov counters, and lead-glass array on magnetic tape. This system

contained a small on-line computer supervising it and which filled various spectra based on a fraction of the data. In this way, the correct behaviour and long-term stability of the entire system was ensured.

2. Angular resolution of detection system

The extremely good angular resolution needed for this type of channeling experiment, where planar critical angles are 20-30 μrad , required extra investigation of the drift-chamber resolution together with mechanical stability. The procedure in these investigations was first to measure the actual position resolution of the drift chambers and then to use some of the well established channeling effects to measure the overall angular resolution of the entire system for short- and long-term operation. The long-term stability was ensured by the on-line computer, which for each burst analyzed some events, the results of which were plotted on a display. In these on-line programs, the incident angular distributions were plotted for particles fulfilling certain scattering-angle or energy-loss criteria, or emitting radiation of a certain energy.

In most experiments using drift chambers, the drift velocity v is assumed constant over the entire drift space. The value $v = 52.1 \mu\text{m/nsec}$, as used by nearly all groups, was found by the Charpak group in their original detailed investigation of drift chambers¹⁰⁾. In one of our earlier CERN experiments (PS164), the drift velocity was measured and found to be linearly dependent on the distance x from the wire¹¹⁾. The gas mixture was the original 'magic' one (67.2% argon, 30.3% isobutane, and 2.5% methylal), but the drift field was rather high,

namely 1.6 kV/cm. Based on these investigations, the expression for v was found to be $v = 56.22 \mu\text{m/nsec} - 0.09x \mu\text{m/nsec mm}$. This expression for v has been used in all analyses so far. In the present experiment, the drift velocity was measured again, and for this, an extra drift chamber, DC4, was placed 175 mm behind DC2 and could be moved sideways with a position accuracy of $\sim 10 \mu\text{m}$. By means of DC2, a narrow particle beam was defined. The drift time in DC4 was measured by use of a time-to-amplitude converter, and thereby the local and average drift velocity across DC4 was found. With a drift field of ~ 1.6 kV/cm, perfect agreement was obtained with the measurement from Ref. 11, shown in Fig. 8a. For a drift field of 1.4 kV/cm, the variation of mean drift velocity for the present experiment is shown in Fig. 8b, and the linear fit is given as $v(x) = [55.34 - 0.14x/\text{mm}] \mu\text{m/nsec}$. The drift velocity is lower than in Fig. 8a, which means that it has not saturated at these drift fields.

In a setup containing three drift chambers (like the present) but with no target, the influence of actual drift velocity on position resolution is normally found by track reconstruction. However, the disadvantage is that this type of measurement does not include the target and the mechanical stability of the entire setup. The high-energy channeling effects, on the other hand, offer an excellent on-line tool for resolution and stability measurements. Figure 9 shows the intensity distribution in incident-angle space of 10-GeV/c protons transmitted through a 1-mm thick $\langle 110 \rangle$ germanium crystal with scattering angles less than 0.1 mrad. In Fig. 9a, a linearly varying drift velocity $v = [56.22 - 0.09x/\text{mm}] \mu\text{m/nsec}$ was used, whereas

the constant drift velocity $v = 52.10 \text{ } \mu\text{m/sec}$ was used in Fig. 9b. The anode wires in the incident beam are indicated in the figure. Here it is clear that planar channeling is very sensitive to drift velocity. Since the width of the planar-channeling effects is proportional to the distance between the planes d_p and amounts to less than $30 \text{ } \mu\text{rad}$ for the faintest planes in Fig. 9a, a measurement of the actual widths gives a lower limit of the overall angular resolution. In Fig. 10 is shown an azimuthal scan, centered in the $\langle 110 \rangle$ axis, of Fig. 9a and with a radius between $3\psi_1$ and $4\psi_1$. The high-order planes with a width of $30 \text{ } \mu\text{rad}$ are clearly resolved, giving an angular resolution around $15 \text{ } \mu\text{rad}$, in good agreement with calculations, using a position resolution of $100 \text{ } \mu\text{m}$ on DC1 and DC2, 15 m apart. In Fig. 11a is shown a coloured version of Fig. 9a, in which the incident beam was divided up in $(25 \times 25) \text{ } \mu\text{rad}^2$ boxes. From such a picture, a qualitative overall measure of intensity variations is obtained for planes and axes. Here it is clearly seen how the planar effects gradually disappear close to the axial region, where the circle indicates the axial critical channeling angle ψ_1 . The sharp contrasts in colours also show that the above angular resolution of $15 \text{ } \mu\text{rad}$ is in agreement with this experiment. Figure 11b once again shows the intensity distribution in incident-angle space but here for 5 GeV/c positrons transmitted through a $200\text{-}\mu\text{m}$ thick $\langle 110 \rangle$ germanium crystal, and emitting bremsstrahlung in the energy region of $10\text{-}40 \text{ MeV}$, which are the photon energies expected from certain axial-channelled positrons in germanium. Also here a clear overall view of intensity variations is obtained. Such spectra were used in the on-line analysis for

checking long-term stability of the entire setup in planar- and axial-channeling radiation experiments.

3. Targets

The targets were discs of high-purity monocrystalline silicon or germanium with a diameter of 30 mm and a thickness between 0.1 and 1 mm. They were cut with the $\langle 110 \rangle$ axis perpendicular to the surface, and being thin, the crystals were very easy to bend; however, such bending could be avoided by using high-purity crystals (for germanium the impurity concentration was $\lesssim 5 \times 10^{13} / \text{cm}^3$), and glueing them at one single point to a copper ring, which could be mounted in the goniometer. The mounted crystals were tested for bending at room temperature with a double-crystal x-ray spectrometer, which also gave the orientation of the crystalline planes. From the rocking curve, a very accurate measurement of the bending was obtained, and this ensured that the crystals were bent no more than 10 μrad . A final check of the bending was obtained, after the crystal was mounted and aligned in the goniometer, by locating the planes for various small spots across the crystal by use of the channeling-radiation effect. A bending here will result in different incident angle coordinates for the same set of planes.

4. Photon detector

The photons were detected with a 10"×12" NaI detector¹²⁾. The NaI crystal was surrounded by scintillators to discriminate against charged-particle background and to increase the energy resolution. An outer shielding with paraffine blocks and Cd plates slowed down and stopped fast neutrons originating from

other beam lines. The energy resolution at 50 MeV was 4%, increasing to 8% at 5 MeV and 100 MeV. The linearity of the detector and electronics was checked with a precision pulser and found to be within 10^{-4} between 5 and 100 MeV. Significant parts of the shower would escape above 100 MeV, reducing the energy resolution and destroying the linearity. Photons below 4 MeV were discriminated against, to avoid the background from thermal neutrons and the Th calibration source. The acceptance angle was much larger than $1/\gamma$, and we had no means to collimate the photon beam; hence we detected the emitted radiation integrated over the full solid angle.

5. Data taking and data analysis

A fast trigger was generated each time a particle passed correctly through the three scintillators and the data-acquisition system was idle. Two fast triggers were required to be separated by at least five microseconds, otherwise they were rejected. When more than one hit was registered, the entire event was also rejected. The surviving triggers, giving a signal in the photon detector or a preset fraction of the total beam, generated a read signal.

The information for a single event was registered in pattern units, ADC's and TDC's, mounted in a CAMAC crate and stored on magtape for subsequent analysis. The data were analyzed in the computer centers at Aarhus and CERN.

IV. Discussion of new data and comparison with theory

1. 7-GeV/c e^+ and e^-

In Fig. 12, the measured radiation spectra (circles) for 7-GeV/c positrons and electrons transmitted along the {110} plane in a 105- μ m thick silicon crystal are shown. The angular resolution of the detection system was approximately $1/5 \psi_p$, and positrons and electrons with incident angle greater than ψ_p were excluded. The spectra are normalized to the IB for the same thickness of amorphous foil, as discussed in Sec. II. As before, there is a large increase over this incoherent radiation for both electrons and positrons in the photon-energy range of 10-120 MeV. As expected, the electron spectrum has very little structure, whereas the positron spectrum has distinct first- and second-harmonic peaks near 40 and 75 MeV. The fact that the second harmonic is more evident here than in Fig. 4 is due to the increased angular resolution in the experimental setup.

Angular spectra are shown in the upper corner of each figure. These are obtained from the data, as organized in Fig. 3, by selecting a photon energy or a small energy range and then scanning in angle. In the positron case of Fig. 12a, an energy range of 28-35 MeV around the first harmonic is used, and the angular scan is in steps of $1/5 \psi_p$. In the electron case of Fig. 12b, the energy range is 19-38 MeV whereas the angular scan again is in steps of $1/5 \psi_p$. As before, the spectra are normalized to the IB. In both cases, channeling radiation sets in very abruptly for incident angles near ψ_p . For positrons, there is a dip in the middle of the peaks, whereas for electrons, the intensity is flat near its maximum at $\psi = 0$. The planar-continuum potential

for positrons and electrons differ only in sign; therefore, both potentials are flat near the mid point between two atomic planes and steep near the atomic planes. In perfect alignment ($\psi=0$), most of the positrons move only in the flat portion of the potential and thus have small accelerations and emit very little radiation. As ψ increases, they begin to move into the steep portion of the potential and the radiation increases. The intensity begins to decrease for $\psi \approx \psi_p$ as some of the positrons enter the CB and IB regions. The electrons oscillate about the planes, hence all channeled electrons, even for $\psi=0$, move in the steep part of the potential. Thus no dip for electrons is to be expected. The dip in the positron angular spectra indicates that the angular resolution is approximately $1/5 \psi_p$, and its existence was used as a check on the stability of the experimental setup. It should be noticed that the experimental critical angles here are the same for electrons and positrons. In many experiments and classical channeling calculations, critical angles for electrons are much narrower than those for positrons²⁾.

The solid curves in Fig.12 are the result of calculations based on Eqs.(2) and (6). In order to understand the calculations and the structure of the spectra more details of the theory are necessary. To compute the radiation, it is convenient to expand the term in the integrand of Eq.(6) with period T in a Fourier series and with the definition $\tilde{I} \equiv \lim_{\ell \rightarrow \infty} I/\ell$ as $\ell \rightarrow \infty$, where ℓ is the crystal thickness, the result is

$$\frac{d^2 \tilde{I}}{d\omega d\Omega} = \frac{e^2 \omega^2}{2\pi c^2} \sum_{n=1}^{\infty} \frac{I_n}{(1 - \cos \theta \frac{\omega}{z})} \delta(\omega - \frac{2\pi n}{T (1 - \cos \theta \frac{\omega}{z})}) \quad (8)$$

Therefore, $\frac{d^2 I}{d\omega d\Omega}$ is the photon frequency, solid-angle energy density per unit crystal thickness in the limit of a large number of oscillations in the crystal. Here,

$$I_n = |\vec{a}_n|^2 - |\vec{k} \cdot \vec{a}_n|^2 \quad (9)$$

and

$$\vec{a}_n = \frac{1}{T} \int_0^T \beta(t) \exp(-i\frac{\omega}{c}[\cos\varphi \sin\vartheta x(t) + \cos\vartheta z_p(t)]) e^{i\frac{2\pi n t}{T}} dt$$

Note that I_n is dimensionless. (10)

It is difficult to decipher the qualitative properties of the radiation from Eqs. (8 - 10) although an attempt has been made to write the formulas in an instructive way. The first thing to note is that the photon energy $\hbar\omega$, solid angle Ω , and energy density of Eq.(8) is nonzero when ω and ϑ are related by

$$\omega = \frac{2\pi n}{T(1 - \bar{\beta}_z \cos\vartheta)} \quad (11)$$

This is identical to Eq.(3) which was derived in a simple way. Thus, an observer at an angle ϑ will see a line spectrum with fundamental frequency given by Eq.(11) with $n = 1$. It is seen that the maximum frequency is in the forward, $\vartheta = 0$, direction. In the appendix it is shown that

$$1 - \bar{\beta}_z = \frac{1}{2\gamma_0^2} \left(1 + \frac{2\gamma_0}{mc^2} K_{\perp}(E_{\perp}) \right), \quad (12)$$

where $\gamma_0 = pc/mc^2$, and $K_{\perp} = \frac{1}{2m\gamma_0} \frac{1}{T} \int_0^T \dot{x}^2 dt$ is the average kinetic energy of the x motion. The average kinetic energy is $\leq E_{\perp}$ and is independent of γ_0 .

The general formula can now be written

$$\omega(E_{\perp}, \gamma_0, \vartheta, n) = \frac{4\pi\gamma_0^2 n}{T(E_{\perp}) \left(1 + \frac{2\gamma_0}{mc^2} K_{\perp}(E_{\perp}) + (\gamma_0 \vartheta)^2\right)} \quad (13)$$

for small ϑ . The photon energy $\hbar\omega$ as a function of ϑ is seen to decrease very fast in units of γ_0^{-1} . Consider the $\vartheta = 0$ case which gives the maximum radiated energy, $\omega_m = \omega(E_{\perp}, \gamma_0, 0, 1)$. T is proportional to $\gamma_0^{\frac{1}{2}}$, and $K_{\perp}$ is independent of γ_0 ; hence, ω changes from a $\gamma_0^{3/2}$ behaviour to $\gamma_0^{\frac{1}{2}}$ as γ_0 increases. Also $T(E_{\perp})K_{\perp}(E_{\perp})$ is a measure of the available phase space for values of the transverse energy $\leq E_{\perp}$. For positrons, T is a decreasing function of $E_{\perp}$ over most of the $E_{\perp}$ domain, and since the denominator in Eq.(13) is a linear combination of T and $TK_{\perp}$, the practical independence of ω_m on $E_{\perp}$ shown in Fig.6 can be understood. For electrons, however, T is monotonically increasing with $E_{\perp}$; hence it is not possible for Eq.(13) to become independent of $E_{\perp}$. The term $2\gamma_0 \frac{K_{\perp}}{mc^2}$ in the denominator is due to the coupling of the transverse and longitudinal motion (see Eq.(A3b) in the appendix). This becomes important when $\gamma_0 U_0 / mc^2 \sim 1$, which means $\gamma_0 \sim 2 \cdot 10^4$ for positrons in the silicon {110} plane.

To get a feeling of how the radiated energy is distributed as a function of ϑ , the integral of Eq.(8) with respect to ω is written

$$\frac{dI}{d\Omega} = \frac{2\pi e^2}{(Tc)^2 (1 - \cos\vartheta \bar{\beta}_z)^3} \sum_{n=1}^{\infty} n^2 I_n. \quad (14)$$

From Eqs.(9) - (10), it is evident that $|\vec{a}_n| \leq 1$ and $0 \leq I_n \leq 1$. Furthermore I_n tends to decrease with ϑ ; therefore the behaviour of Eq.(14) is dominated by the $(1 - \cos\vartheta \bar{\beta}_z)^{-3}$ factor which is

strongly peaked in the forward ($\vartheta = 0$) direction. It drops off very fast with increasing $\gamma_0 \vartheta$, and at $\gamma_0 \vartheta = 2$, it has decreased by a factor of 80. Thus most of the intensity is radiated in the forward direction, where the photon energies are the largest.

The total power radiated by a channeling trajectory can be calculated from the Liénard result⁸⁾,

$$\text{Power} = \frac{2}{3} \frac{e^2}{m^2 c^3} \gamma^2 (U'(x))^2 (1 - \beta_x^2) . \quad (15)$$

This result is exact; furthermore, the β_x^2 term can be ignored. Since $x(t)$ is periodic, the average power is

$$\langle P \rangle = \frac{4\sqrt{2}}{3} \frac{e^2}{(mc^2)^{3/2}} \gamma_0^{5/2} \frac{1}{T(E_\perp)} \int_0^{x_0} (U'(x))^2 [U(x_0) - U(x)]^{-1/2} dx, \quad (16)$$

where $E_\perp = U(x_0)$. These results are somewhat surprising as they imply that while the z motion is important for the shape of the spectra, it does not contribute to the average power radiated. Note that $\langle P \rangle$ increases like γ_0^2 if radiation damping is to be ignored.

The calculations shown in Fig.12 are based on Eqs.(8) - (10). The dotted lines are calculations of the first three harmonics, i.e., the $n = 1, 2$, and 3 terms of Eq.(8). The dashed lines represent the first two harmonics from those particles which have transverse energy greater than the potential barrier ("over barrier particles"). The solid line is the sum of the first three harmonics and the two over barrier terms. In the calculation, a trajectory $\vec{r}(t)$ is determined for a fixed transverse energy by numerically integrating the x equation with initial conditions $x(0) = x_0$, $\dot{x}(0) = 0$, where

$E_{\perp} = U(x_0)$. This determines $x(t)$, $\dot{x}(t)$, $z(t)$, $\dot{z}(t)$, and Eq.(8) is then integrated over solid angle (ϑ, φ) . The ϑ integration can be done analytically because of the delta function, but the φ integration must be done numerically. The radiation spectrum for each transverse energy is thus obtained. The distribution in transverse energy is obtained by assuming an incident beam uniform in space, $|x| \leq d_p/2$, and angle, $|\psi| \leq \psi_p$. The calculated spectrum is then obtained by integrating over this transverse energy distribution. The calculations use a Doyle-Turner thermally averaged potential but do not include the effects of nuclear and electronic multiple scattering.

The overall agreement is fair, and the fact that the intensity in the calculated spectra are too high is to be expected since multiple scattering is not included (see also sect.IV.8). The calculated ω_m value is too high, which is a trend for other energies, other planes, and other crystals. For electrons, the agreement is perhaps unexpectedly good.

2. Dependence of channeling radiation on positron energy:
 a magic γ -value

In Fig.13 are shown the experimental spectra (circles) for 2, 4, 5, 7, and 10 GeV/c positrons channeled along the {110} planes in a 0.105 mm silicon crystal at room temperature. The 7-GeV/c data are repeated from Fig.12a. The major peaks are due to the first harmonic and are at 7, 19, 26, 37, and 57 MeV for beam momenta of 2, 4, 5, 7, and 10 GeV/c, respectively. The second harmonic is almost absent at 2 GeV/c but very pronounced at 10 GeV/c, showing the in-

creased importance of ultrarelativistic effects as the dipole approximation breaks down. The experimental spectra (circles) contain all particles incident within ψ_p of the $\{110\}$ planes. The solid lines are the calculated spectra, and they include the contributions from the first three harmonics and the first two "over the barrier" terms. The calculation takes into account the incident-angle distribution of the beam and the experimental angular resolution by using these to define a transverse-energy distribution. The Doyle-Turner potential is used but multiple-scattering and finite crystal-thickness (linewidth) effects are not included. The overall agreement between measured and calculated spectra is good especially for the high energies. However, the steep edge of the peaks in the calculated spectra is always at an energy about 5% higher than the experimental edge. Changes in the potential needed to explain this difference are larger than seems reasonable. The calculated intensities are larger than the experimental ones, but this is most likely explained in terms of multiple scattering. These discrepancies are discussed further in Sec.IV.9.

For a more detailed understanding of the spectra, the manner, in which the individual positron trajectories contribute to the photon spectra, must be understood. The radiation from each positron depends only on its transverse energy, and for each $E_{\perp}$, the photon-energy density per unit length $\frac{d\tilde{I}}{d\omega}$ for the first harmonic is zero for $\omega > \omega_m(E_{\perp}, \gamma_0, 1)$ and is peaked at ω_m . If these spectra are superimposed, then the dependence of ω_m on $E_{\perp}$ causes a broad peak structure with width determined by the range of $\omega_m(E_{\perp})$. For the case of a harmonic potential $V(x) = V_0 x^2$, the period $T(E_{\perp}, \gamma_0) = 2\pi(m_e \gamma_0 / 2V_0)^{\frac{1}{2}}$ is independent of $E_{\perp}$ and

$K_{\perp} \propto E_{\perp}$. Hence ω_m is independent of $E_{\perp}$ in the dipole limit and decreases monotonically with $E_{\perp}$ (see Eq.(13)) when deviations from the dipole approximation become significant. For more realistic potentials, the period decreases with transverse energy, whereas $TK_{\perp}$ increases and ω_m becomes practically independent of $E_{\perp}$ at a certain value of γ denoted by γ_{magic} . Thus there is a beam momentum, at which the maximum radiated frequency is practically independent of $E_{\perp}$. In Fig.14, $K\omega_m$ vs. $E_{\perp}$ is shown for the {110} plane using the Moliere and the Doyle-Turner potentials, and ω_m is seen to be independent of $E_{\perp}$ for $m\gamma c^2 \sim 5$ GeV. For $\gamma < \gamma_{\text{magic}}$, ω_m decreases with $E_{\perp}$, and for $\gamma > \gamma_{\text{magic}}$, ω_m increases. Therefore below γ_{magic} , the high-energy side of the first harmonic peak should be quite steep, with the intensity of the composite peak increasing as γ approaches γ_{magic} . Above γ_{magic} , the high-energy side should broaden with a resultant relative decrease in the peak intensity. The term relative is used here because the peak intensity depends on γ_0^{α} , where $\alpha \sim \frac{1}{2}$. For example, the dipole approximation of Eq.(13) shows that $\alpha = \frac{1}{2}$, whereas the Bethe-Heitler formula (Eq.7) shows the IB to be approximately independent of γ_0 in the 1 - 10 GeV/c range.

These qualitative trends in peak intensity and high-side steepness are seen in calculated spectra of Fig.13, where the maximum steepness and peak height are seen at ~ 5 GeV/c. The 2 GeV/c experimental data are influenced by multiple scattering and do not have the predicted steepness, but maximum steepness does occur at about 5 GeV/c as predicted. This can be seen from the relative steepness of the high-energy side of the first harmonics shown in the figures.

From Fig.14 it is clear that the very narrow energy region for γ_{magic} predicted from the Moliere potential is not found for a more realistic potential like the thermally averaged Doyle-Turner potential. From this there is a broader range of incident energies (4 - 7 GeV/c for {110} in silicon), for which ω_m is practically independent of transverse energies. This is also in good agreement with the experimental results of Fig.13.

The overall agreement is fair, but calculated peak intensities for positrons are about 50% higher than the measured ones, and calculated ω_m values are about 5% too high (the disagreement for energies smaller than 4 MeV is due to an experimental cut-off in the sodium iodide crystal). This is a general trend for all positron experiments to date including energies from MeV to GeV and diamond, silicon, and germanium crystals. The disagreement between calculated and experimental CR spectra is worse above 10 GeV/c. In Fig.5, theoretical curves are shown for 10, 20, and 55 GeV/c positrons penetrating the 0.100 mm thick silicon crystal along the {110} planes. No calculation for 5 GeV/c is presented since here the experimental resolution is of the order of $2\psi_p$. The calculations are carried out as discussed for Figs.12 and 13, but more harmonics are taken into account. For the 20 GeV/c case, the first and second harmonic peaks are still visible in both theoretical and experimental spectra and fall at the same photon energy; however, the theoretical intensity is higher (about 50%). For 55 GeV/c, the division into harmonics does not give any structure due to the many harmonics to be included: Experimentally the harmonics are also not visible, but there is a steady increase in intensity up to 2 GeV, which is the maximum energy that can be

detected by the NaI detector. However, the tagging system in the WA-64 setup (our previous experiment) is an excellent magnetic detector, and incident and exit momenta of the projectiles are easily obtained. A spectrum determined in this way is inserted in the upper corner. It is clear that the 55 GeV/c spectrum has an intensity maximum near 2.8 GeV, above which it drops off very fast.

The probabilities (experimental) for the incident positrons to radiate in the first harmonic as a function of incident angle to the $\{110\}$ plane are shown above the energy spectra. These show the same behaviour as in Fig.12, namely that the channeling radiation sets in abruptly at ψ_p , and that high transverse energies radiate the most. The more pronounced dips for 2 GeV/c as compared to 10 GeV/c is a matter of angular resolution in the experimental setup.

3. Possibility of subharmonic peaks

A Soviet collaboration¹³⁾ reported a structure near 25 MeV in the CR spectrum from 10 GeV/c positrons channeled along the $\{110\}$ planes of silicon. They claimed that it was only visible in a small angular range around ψ_p ; however, their experimental uncertainty appears to be quite large. More recently, a Khar'kov group¹⁴⁾ reported confirmation of a structure in this region. A very recent SLAC experiment¹⁵⁾ on diamond also seems to confirm such a structure. Figure 15 shows detailed spectra from 10 GeV/c positrons channeled along the $\{110\}$ planes of a 0.105 mm thick silicon crystal. The spectra are shown for the incident angle regions, $0 - \frac{1}{3}\psi_p$, $\frac{1}{3} - \frac{2}{3}\psi_p$, and $\frac{2}{3} - \psi_p$, respectively. The angular resolution in PS188 was around $\frac{1}{3}\psi_p$. Although the present experiment suffers from some long-term instability, the data do show

a faint structure near half the energy of the first harmonic. However, our statistical uncertainty does not allow confirmation of a subharmonic structure.

Such an effect could be caused by positrons oscillating between planes with an interplanar spacing of $2 d_p$, as illustrated in Fig.16a. Here an incident positron with transverse energy close to the potential barrier is backscattered at the top plane, transmitted through the middle plane, and finally backscattered from the bottom plane. Although this type of motion might seem very unlikely, binary collision calculations indicate that this may not be the case. Also such motion will give a structure in the emitted photon spectrum around half the energy of the first harmonic if there are oscillations enough to secure coherence. Figure 16b shows the results of calculations, using the binary collision channeling program¹⁶⁾ that has predicted most of the high-energy channeling results. For incident angles close to ψ_p , 1 - 2% of the particles seem to experience that type of motion. The effect is, of course, very sensitive to small variations in incident angle or instabilities in the directions of planes. The picture only applies for incident angles close to ψ_p , where the transverse energy is large enough to give a reasonable probability for barrier penetration. This effect needs to be investigated in more detail.

4. {100} Si planes

In Fig.17 the photon energy spectra for positrons channeled along the {100} planes of silicon at 5, 6, and 8 GeV/c are presented. The calculated curves are determined as previously discussed and again the thermally averaged Doyle-Turner potential is used. The agreement is somewhat better than in the {110} case, especially in terms of the intensity level of the first harmonic. The

channeling radiation is harder because of the smaller oscillation period for a fixed momentum. There does not appear to be a γ -magic in this energy range as was predicted using the Moliere potential⁷⁾. This is clear evidence for the choice of the Doyle-Turner potential as can be seen from Fig.14b. In this figure $\hbar\omega_m$ vs. $E_\perp$ is plotted for both potentials, and in contrast to the $\{110\}$ case of Fig.14a, there is a large difference. The Doyle-Turner potential does not show a magic γ -value in the given energy range, consistent with the experimental spectra, whereas the Moliere gives $\hbar\omega_m$ relatively independent of $E_\perp$ for $m\gamma c^2 \approx 8$ GeV.

The experimental angular scans are shown above the energy spectra. Again the channeling radiation sets in abruptly at ψ_p ; however, the dips are not as deep as in the $\{110\}$ case. The latter is due to the narrowness of the planes and the fact that the planes are vertical, in the beam line, which degrades the angular resolution by dispersion in the small 10 mrad bending magnet on the incident side.

5. $\{111\}$ Si planes

Figure 18 shows the data for 5 GeV/c positrons on the $\{111\}$ planes of Silicon for three incident angle regions. These planes are made up of wide and narrow ones, with critical angles ψ_p^W and ψ_p^N , respectively. For ψ_p here is used $\psi_p^2 = [(\psi_p^N)^2 + (\psi_p^W)^2]/4$ and the three angle regions cover $(0 - \frac{1}{3}\psi_p)$, $(\frac{1}{3} - \frac{2}{3}\psi_p)$, $(\frac{2}{3} - 1\psi_p)$. Because of this complication, spectral calculations have not been done. However, the Moliere potential has been used to calculate the first harmonic peaks and these are at 27 MeV

for the wide planes and 59 MeV for the narrow ones. The first harmonic from the wide planes is clearly evident at a value slightly less than predicted. The second harmonic from the wide planes is also evident, especially for incident angles in the range $\frac{2}{3} - 1\psi_p$. Because less than $\frac{1}{4}$ of the particles are channeled between the narrow planes, the first harmonic from the narrow planes will be hidden in the second harmonic from the wide planes, due to the almost coincident energies.

6. {110} Ge planes

The spectrum from germanium for 5 GeV/c positrons on the {110} plane is shown in Fig.19 for a 0.205 mm thick crystal. The enhancement is not nearly as large as for silicon although the experimental angular resolution is relatively better due to a larger channeling angle. The enhancement should go down as can be easily estimated. The Bethe-Heitler yield from Eq.(7) scales with Z_2^2 . The channeling radiation intensity I from Eq.(8), however, scales as $Z_2^{4/3}$. This is easily obtained by remembering that I is proportional to the (acceleration)², which for the planar case gives $I \propto (Z_2^{2/3})^2$. Thus the ratio scales with $Z_2^{-2/3}$ or a reduction from silicon to germanium of a factor 0.57. Comparing the data for the 0.2-mm germanium target and the 0.5-mm Silicon target, which are approximately equal with respect to multiple scattering, a factor of 0.40 is observed, in good agreement with the $Z_2^{4/3}$ -scaling. The calculated intensity of the first harmonic is nearly a factor of three too high and the calculated $\omega_{\max}$ is 10% too high. The angular scan in the upper corner shows a nice dip indicating the expected good angular resolution. In general the picture here is the same as in the Si-case, namely that calculated intensities and energies of the first harmonic is higher than the experimental ones.

7. Temperature dependence in silicon

The channeling radiation spectra were measured on the 0.5 mm silicon crystal at 100 K and 300 K and four spectra are shown in Fig.20. In each case 5 GeV/c positrons are channeled along the {110} planes. In Fig.20a the incident angles are between 0 and $\frac{1}{3}\psi_p$ and only a small difference is seen between the 100 K and 300 K cases. In Fig.20b, the incident angles are between $\frac{2}{3}\psi_p$ and ψ_p and a reduction in the intensity of $\sim 10\%$ is seen for the crystal at room temperature. The reduction is largest at the second harmonic and higher, which is the region dominated by particles with high transverse energy. As expected, this difference shows that well channeled particles are kept far away from the nuclei in the planes, whereas particles with incident angles near ψ_p will have an increased probability for a close encounter with a nuclei. The cooling of the crystal from 300 K to 100 K corresponds to a reduction of the thermal vibration amplitude in silicon from 0.08 Å to 0.05 Å, giving an increase in the intensity in agreement with the results.

8. Thickness dependence in silicon and germanium

The theory presented in this paper predicts no thickness dependence on the photon energy spectra when normalized by the Bethe-Heitler formula for the same thickness amorphous foil. Experimental energy spectra for two thicknesses in silicon and two thicknesses in germanium are shown in Figs.21a and 21b, respectively. In each case, 5 GeV/c positrons are channeled along {110} planes and in each case the intensity has decreased for the

thicker crystal. In the thick germanium crystal, the radiation is near the Bethe-Heitler value, so apparently most particles are dechanneled, whereas in silicon crystals, the most significant change is in the peak intensity of the first harmonic.

The theory ignored both multiple scattering and line width effects and the narrowing of the first harmonic in the thick silicon crystal might be viewed as a line width effect whereas the decrease in intensity around the first harmonic might be viewed as a multiple scattering effect. However, Fig.13 shows that the width of the first harmonic is smaller experimentally than theoretically, hence line width effects must not be important.

Another important thickness effect not included in the theory is a treatment of the multiphoton process. The NaI photon detector gives the total radiated energy per incoming positron, hence, two photons from the same positron appears as a single signal at the sum of the photon energies. The probability of such an event is significant because the radiation length for CR in this case is approximately 2 mm.

The multiphoton effect can be estimated by self-convoluting the thin crystal data to obtain data for a thick crystal under the assumption of small multiple scattering. This calculation shows that approximately half the decrease of the first harmonic is due to emission of more than one photon. The remaining difference is apparently due to multiple scattering. Consequently, the peak height in the first harmonic is sensitive to the multiple scattering. This sensitivity may explain the observed difference between calculated and measured intensities as shown, for example, in Fig.12a.

9. Discrepancy between calculated and measured spectra

The difference between calculated and measured photon energy spectra from planar channeled positrons, as seen, for example, in Figs.12 and 13, can be divided into a difference in intensity of the first harmonic and a difference in energy, as measured, at the sharp upper edge of the first (and second) harmonic. Multiple scattering is neglected in the calculations and the difference in intensity can probably be explained from the observed multiple scattering, as mentioned above. The multiple scattering leads to a broadening and a downwards shift of the line spectrum seen at fixed observation angle, but when an average over incident and emission angle is made, the energy shift of the first harmonic is less than 1%¹⁷⁾, compared to the observed difference of ~5%. In the Mev region, where distinct photon peaks emerged for both electrons and positrons, discrepancies are also found for positrons. Here calculated photon energies are 5 - 10% higher than the measured ones, whereas very good agreement is found for electrons.

This fact led to a careful investigation of possible experimental errors and it was realized that the transmitted momentum of the high-energy secondary beam line might be uncertain up to ~5%, due to stray-fields, alignment errors, etc. Since the ω_m values around 5 GeV/c scale as γ , this could explain the discrepancy. So a measurement of the actual beam momentum was started by measuring the deflection of the beam in the 100 mrad bending magnet (BM2 on Fig.7). Preliminary results indicate that the actual beam momentum is ~4% lower than specified for 5 GeV/c. This value has to be confirmed, together with accurate measurements of the particle path for all momenta

in the bending magnet. Figure 22 shows the measured spectra from Fig.13 with new momenta. The agreement, ignoring the intensity level, is now almost perfect. Such large uncertainties are unusual and detailed inspection of the whole beam line is in progress.

10. A New Strong Source for Polarized γ -Rays in the MeV Region

In the last few years, much attention has been given to the possibility of making strong monoenergetic γ sources for photo-absorption studies of a large variety of phenomena in nuclear and atomic physics. For such studies, synchrotron radiation, bremsstrahlung, and positron annihilation in flight are mostly considered, and much effort is currently made to construct machines for the production of these types of radiation. In many cases, electron/positron storage rings equipped with magnetic wigglers are used. Unfortunately, the energy of the radiation is normally below 1 MeV, so for most nuclear-physics experiments, this type of radiation is not suitable. Channeling radiation from GeV positrons, however, turns out to produce sharp photon peaks for positron energies close to the magic γ - value and emitted within an angular cone of $1/\gamma$. The radiation is so strong that a silicon crystal about 1 mm thick can yield nearly one photon per positron in the narrow peak, which is orders of magnitude stronger than, e.g. positron annihilation in flight.

If the emerging photon beam is collimated to around $\frac{1}{3}\gamma$, a highly monoenergetic supply of photons becomes available. However, this requires a highly parallel positron beam as is available from a Linac and which can supply more than 10^9 particles

per second. This enormous strong source in the MeV region is eminently suitable for nuclear physics, especially since the photon energy can be varied continuously. For 1 - 10 GeV/c positrons, the photon energy is in the 1 - 100 MeV region and like synchrotron radiation, plane polarized.

11. Conclusions

For planar channeled GeV/c electrons and positrons traversing single crystals, the low energy part of the photon spectra is enhanced by more than a factor of 30 above the incoherent bremsstrahlung. Positrons move in a nearly harmonic planar potential with equidistant energy levels, which gives rise to a peak structure in the spectrum, whereas the electrons move in a strongly anharmonic potential and the oscillation frequency becomes a function of transverse energy and a broad spectrum is obtained.

The present investigations confirm the predicted magic positron momentum, $\gamma_{\text{magic}} c$, between 1 and 10 GeV/c, where, for most crystal planes, the maximum photon energy in the first harmonic is independent of incident angle within the planar critical angle. The predictions were based on calculations using a Moliere potential which gave a narrow momentum range for this γ_{magic} . The present investigations using the more realistic thermally averaged Doyle-Turner potential results in a momentum range of a few GeV/c for which ω_m is relatively independent of transverse energy and a sharp peak in the spectrum is obtained. The confirmation of the predicted magic γ value has some very interesting possibilities for making extremely strong photon sources in the 1 - 100 MeV region, since every positron emits nearly one photon for crystal thicknesses around 1 mm.

The experimental results have been compared to calculations based on classical electrodynamics. The break-down of the dipole approximation, and the following increased influence of higher harmonics, seen in experimental spectra, is well described by the theory. The overall agreement is fair but calculated intensities are too high, which to some extent could be due to multiple scattering. The energy of the sharp peaks in the photon spectra from 1 - 10 GeV/c positrons can be compared to calculated ω_m -values, and it turns out that calculated ones are around 5% too high. This discrepancy is found in nearly all positron experiments covering an energy region from 20 MeV to around 50 GeV. For electrons, on the other hand, the agreement is excellent. In general, one would have expected the opposite situation, because classical channeling calculation for relativistic positrons usually are in good agreement with experiments. For negative channeled particles, such calculations are much more complicated because of difficulties in including the enhanced multiple scattering. This puzzling situation, on the other hand, could be explained for the present experiments by an error in beam momenta. If the error is confirmed for all present momenta, the positron data agree very well with calculations. Other positron experiments are urged to perform the same detailed checks of experimental parameters so that hopefully the problems around channeling radiation from positrons could be cleared up.

With regard to the theoretical description, in general it is found that the Lindhard model developed for positive, non relativistic particles can easily be extrapolated to the GeV region by replacing mass and energy terms by their relativistic expressions. The agreement between calculated and

measured critical angles is very good for both electrons and positrons up to the highest energies measured here. This means that the simple expressions for channeling angles are found to hold over a range of 5 - 6 orders of magnitude in momentum.

Acknowledgements

The authors are grateful to J.U.Andersen, E.Bonderup, and E.Lægsgaard for fruitful discussions and helpful criticism. We are especially indebted to members of the WA64 collaboration, who played an active part in the first channeling-radiation experiment at the SPS. The Danish participation in the collaboration was made possible by funds from the Danish Committee for Accelerator Physics. One of us (O.P.) gratefully acknowledges a grant from the Danish Natural Science Research Council.

Appendix

Continuum model of planar channeling for relativistic projectiles

When the angle of incidence, ψ , to a crystallographic plane is of the order of the critical channeling angle ψ_p (Eq.(1)), the equation of motion in the crystal is to a good approximation

$$\frac{d}{dt}(m\dot{x}) = -\frac{dU}{dx} \quad x(0) = 0 \quad \dot{x}(0) = v\sin\psi \quad (A1)$$

$$\frac{d}{dt}(m\dot{z}) = 0 \quad z(0) = 0 \quad \dot{z}(0) = v\cos\psi,$$

where x and z are the transverse and longitudinal coordinates shown in Fig.1, $m\gamma$ is the relativistic mass of the projectile, and v is the projectile velocity outside the crystal surface ($t < 0$). $U(x)$ is the planar continuum potential, obtained by averaging the crystal potential over the coordinates parallel to the plane, which is periodic and symmetric:

$$U(x/d_p) = U(x); \quad U(x) = U(-x); \quad U\left(\frac{d_p}{2}\right) - U(0) = U_0.$$

The depth of the channel, U_0 , is of the order of a few Rydbergs, e.g., for Si {110} at room temperature $U_0 \approx 22$ eV. The motion is two-dimensional due to the initial velocity and the transverse direction.

Eq.(A1) implies conservation of total energy, which means:

$$\frac{d}{dt}(\gamma mc^2) = -\frac{d}{dt}U(x) = -\dot{x}\frac{dU}{dx}. \quad (A2)$$

Now Eq.(A1) may be expanded to first order in $\epsilon \equiv \frac{U_0}{\gamma_0 mc^2}$, where

$\gamma_0 = (1 - (v/c)^2)^{-\frac{1}{2}}$, and we arrive at

$$m\gamma_0 \ddot{x} = - \frac{dU}{dx}(x) \quad , \quad x(0) = x_0 \quad , \quad \dot{x}(0) = v \sin \psi \quad (A3a)$$

$$\dot{z} = v \cos \psi \left(1 - \frac{U(x_0) - U(x)}{\gamma_0 mc^2}\right) \quad , \quad z(0) = 0 \quad , \quad (A3b)$$

where terms of the order ε^2 have been neglected. If. Eq.(A3a) is multiplied by $\dot{x}$, it can be seen that the transverse energy

$$E_{\perp} = \frac{1}{2} m \gamma_0 c^2 \left(\frac{\dot{x}}{c}\right)^2 + U(x) = \frac{1}{2} m \gamma_0 c^2 \psi^2 + U(x_0) \quad (A4)$$

is conserved, here we have neglected a term of relative size γ_0^{-2} and used the fact $\psi^2 \sim \psi_p^2 \sim \varepsilon$. For 5 GeV positrons in Si {110} $\gamma_0^{-2} \approx 10^{-8}$ and $\varepsilon \sim 4 \cdot 10^{-9}$.

The solutions of Eq.(A3a) are periodic when $E_{\perp} < U_0$, due to the symmetry of U , and the period can be derived from Eq.(a4):

$$T(E_{\perp}) = 4 \cdot \sqrt{\frac{m\gamma_0}{2}} \int_0^{x_m} \frac{dx}{\sqrt{E_{\perp} - U(x)}} \quad , \quad (A5)$$

where x_m is the amplitude of the oscillation, $U(x_m) = E_{\perp}$. It is seen that $T \sim \frac{d_p}{c} (m\gamma_0 c^2 / U_0)^{\frac{1}{2}}$, which means that a 5 GeV positron in Si{110} performs approximately 200 oscillations in 1 mm of crystal length. Equations (A3b) - (A4) give $\dot{z} = v(1 - \frac{1}{2}(\dot{x}/c)^2)$, which means that $\dot{z}$ is periodic with a period $\frac{T}{2}$.

A careful analysis of Eq.(A1) shows that the solutions of these equations are periodic, too, and that the periods differ relative to Eq.(A5) by a term of the order ε . The average value of the longitudinal velocity is

$$\langle \dot{z} \rangle = \bar{\beta}_z c = v \frac{1}{T} \int_0^T dt \left(1 - \frac{\frac{1}{2} m \gamma_0 c^2 \left(\frac{\dot{x}}{c} \right)^2}{m \gamma_0 c^2} \right) = v \left(1 - \frac{K_{\perp}}{m \gamma_0 c^2} \right), \quad (A6)$$

where the γ_0 -independent quantity $K_{\perp}$ is the average value of the transverse kinetic energy. $U(x)$ is nearly harmonic for positrons, and therefore $K_{\perp} \approx \frac{1}{2} E_{\perp}$. This is thus a correction of the order ϵ .

It is now clear that the trajectory of the particle is to a good approximation given by:

$$\vec{r}(t) = x(t) \vec{e}_x + (\bar{\beta}_z c t + z_p(t)) \vec{e}_z, \quad (A7)$$

where $x(t)$ and $z_p(t)$ are periodic with periods T and $\frac{1}{2}T$, respectively. x is of the order of $d_p \sim \text{\AA}$, while $z_p \sim c \cdot T \cdot \epsilon \sim d_p \cdot \sqrt{\epsilon} \ll x$. This means that z_p is $\sim 10^{-4} \cdot d_p$.

Such small changes may seem unimportant, but when we consider radiation wavelengths of the order $|z_p|$, it may become important, because wavelengths for MeV photons are of the same order.

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Figure Captions

- Fig.1 Typical planar channeling positron trajectories oscillating between crystal planes. The initial position is $x_0 = 0$ and the initial angle ψ and amplitude of oscillation x_m are shown for the shorter wavelength trajectory. There is a factor of $10^3 - 10^4$ difference in the x- and z scale.
- Fig.2 Number of photons per 2.275 MeV interval for random (solid line) and $\{110\}$ planar channeled (circles) (10 GeV/c) positrons in 0.1 mm silicon. Only positrons in $(-\psi_p, \psi_p)$ are shown in the planar spectra.
- Fig.3 Photon intensity as function of photon energy (horizontal) and angle to the $\{110\}$ plane (vertical), normalized to the Bethe-Heitler expression, for 10 GeV/c positrons traversing 0.1 mm silicon. The data are the same as in Fig.2.
- Fig.4 Photon energy spectra for 10 GeV/c (a) positrons and (b) electrons channeled $(0 - \psi_p)$ along the $\{110\}$ plane in 0.1 mm silicon, normalized to the Bethe-Heitler spectrum. The data for positrons is the same as in Figs.2 and 3.
- Fig.5 Photon energy spectra for 5, 10, 20, and 55 GeV/c positrons channeled along the $\{110\}$ plane in 0.1 mm silicon. The insert in the 55 GeV/c spectrum shows the measured intensity spectrum for 55 GeV/c positrons up to 10 GeV photons. The solid lines (except for the insert) are from theoretical calculations.
- Fig.6 Maximum photon energy of the first harmonic, $\omega_m(E_\perp, \gamma_0, 1)$ as a function of transverse energy, for 5, 10, 20, and 55 GeV/c positrons channeled along the $\{110\}$ planes of silicon

as calculated for the Doyle-Turner and the Molière potentials

- Fig.7 Experimental setup. Here SC designates scintillators, DC drift chambers, and BM bending magnets. The photons emitted in the target are detected with the NaI detector.
- Fig.8 Measured drift velocity as function of distance to anode wire x for the two drift fields 1.6 kV/cm and 1.4 kV/cm.
- Fig.9 Intensity of positive particles transmitted through a $\langle 110 \rangle$ germanium crystal in two-dimensional angle space, as measured with the drift chambers, a) for the correct drift velocity of Fig.8a, and b) for a constant drift velocity of 52.1 $\mu\text{m}/\text{nsec}$.
- Fig.10 Azimuthal scan of Fig.9a for polar angles between 3 and $4\psi_1$. The planes are identified with their Miller indices.
- Fig.11 a) Colored version of Fig.9a. The circle indicate the size of the axial critical angle ψ_1 . b) Intensity in two-dimensional incident angle space of positrons emitting a 10 - 40 MeV photon in a 0.2 mm thick $\langle 110 \rangle$ germanium crystal.
- Fig.12 Photon energy spectra for $\{110\}$ planar channeling radiation from 7 GeV/c positrons and electrons traversing a 0.105 mm thick silicon crystal. The circles are experimental points and the curves are theoretical. The incident angles are in $(-\psi_p, \psi_p)$. The inserts show the probability relative to random of emitting a photon in the energy range 28 - 35 MeV for positrons and 19 - 38 MeV for electrons, as a function of incident angle.

Fig.13 Photon energy spectra ($-\psi_p, \psi_p$) and angular scans of 2, 4, 5, 7, and 10 GeV/c positrons channeled along the {110} plane in a 0.1 mm silicon crystal. Circles are experimental and curves theoretical.

Fig.14 Maximum frequency of the first harmonic as function of transverse energy for positrons moving in the Doyle-Turner and the Moliere {110} and {100} planar potentials. Taken from Eq.(13) of the text with $\vartheta = 0$ and $n = 1$.

Fig.15 Intensity spectra from 10 GeV/c positrons incident in the intervals (a) $0 - \frac{1}{3}\psi_p$, (b) $\frac{1}{3} - \frac{2}{3}\psi_p$, and (c) $\frac{2}{3} - \psi_p$ to the {110} plane in a 0.1 mm silicon crystal.

Fig.16 a) Simulation in angle space of a positron with $\psi \approx \psi_p$. b) Schematic trajectory of the motion from a). First the positron is channeled between planes with spacing $2d_p$, then it is channeled between planes with spacing d_p , and finally it is dechanneled.

Fig.17 Intensity energy spectra ($0 - \psi_p$) and angular scans of 5, 6, and 8 GeV/c positrons planar channeled between {100} planes in a 0.1 mm silicon crystal. The circles are from the experimental data and the curves are the result of calculations using the Doyle-Turner potential.

Fig.18 Intensity energy spectra of 5 GeV/c positrons incident on {111} planes in a 0.1 mm silicon crystal in the intervals $0 - \frac{1}{3}\psi_p$, $\frac{1}{3} - \frac{2}{3}\psi_p$, and $\frac{2}{3} - \psi_p$, respectively.

- Fig.19 Intensity energy spectra from 5 GeV/c positrons incident within ψ_p to the {110} plane in 0.2 mm germanium. Inserted is an angular scan of the positrons emitting a photon in the range 14 - 24 MeV. The measurements are compared to calculations as described in the text.
- Fig.20 Temperature dependence from 5 GeV/c positrons incident on the {110} planes in 0.1 mm silicon in the angular intervals (a) $(0 - \frac{1}{3}\psi_p)$ and (b) $(\frac{2}{3} - \psi_p)$, respectively. Data are shown for crystal temperatures of 300-K (open circles) and 100 K (closed circles).
- Fig.21 Thickness dependence of radiation from 5 GeV/c positrons incident within ψ_p of the {110} planes in silicon (a) and germanium (b), respectively.
- Fig.22 As fig.13, but the beam energy used in the calculations has been decreased by 4%, as described in the text.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

127-324

Fig. 7

Fig. 8

Fig. 9

Fig. 10

Fig. 11

Fig. 12

Fig. 13

Fig. 14

Fig. 15

127 - 325

Fig. 16

Fig. 17

Fig. 18

Fig. 19

Fig. 20

Fig. 21

Fig. 22