European Coordination for Accelerator Research and Development

PUBLICATION

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06 June 2013

The research leading to these results has received funding from the European Commission under the FP7 Research Infrastructures project EuCARD, grant agreement no. 227579.

This work is part of EuCARD Work Package 8: **Collimators & materials for higher beam power beam.**

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Parameters identification in strain-rate and thermal sensitive visco-plastic material model for an alumina dispersion strengthened copper

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ARTICLE INFO

Article history:

Received 10 June 2011

Received in revised form

4 October 2011

Accepted 6 October 2011

Available online 24 October 2011

Keywords:

Inverse method

Optimization

Strain-rate

Temperature

Johnson–Cook model

ABSTRACT

The main objective of this paper is getting strain-hardening, thermal and strain-rate parameters for a material model in order to correctly reproduce the deformation process that occurs in high strain-rate scenario, in which the material reaches also high levels of plastic deformation and temperature. In particular, in this work the numerical inverse method is applied to extract material strength parameters from experimental data obtained via mechanical tests at different strain-rates (from quasi-static loading to high strain-rate) and temperatures (between 20 °C and 1000 °C) for an alumina dispersion strengthened copper material, which commercial name is GLIDCOP®. Thanks to its properties GLIDCOP® finds several applications in particle accelerator technologies, where problems of thermal management, combined with structural requirements, play a key role. Currently, it is used for the construction of structural and functional parts of the particle beam collimation system. Since the extreme condition in which the material could operate, it is fundamental to characterize it in a wide range both in strain-rate and temperature.

The numerical inverse method used in this work is particularly useful to reproduce experimental results when the stress–strain fields in the specimen cannot be correctly described via analytical models. Furthermore this procedure is useful to take into account thermal phenomena generally affecting high strain-rate tests in which the heat conversion of plastic work produces an adiabatic overheating. So, the applicability of this method is particularly indicated in special fields, such as aerospace engineering, ballistic, crashworthiness studies or particle accelerator technologies.

The attention is focused on evaluating the most suitable strategy of material model parameters optimization to obtain the best fit between experimental data and numerical results. In this regards, it is important to determine which material model coefficients can be considered as optimization variables and for each of them the most suitable range of variation.

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1. Introduction

The description of the relationship between stress and strain for a material implies the identification of work hardening, strain-rate sensitivity and thermal softening parameters in order to correctly reproduce the experimental material response with a specific material model.

If the material model is completely physically-based the model parameters are correlated to the physics and chemical material properties. Otherwise, if the material model is empirical

(phenomenological), it is necessary to obtain the model coefficients and, usually, the approach is fitting the experimental data analytically. With this standard approach, the quality of the results could be affected by geometrical effects, that lead to non uniform stress–strain field within the specimen, and thermo-mechanical coupling in case of high strain-rates, when the thermal softening effects become more relevant. On the other hand, a numerical inverse method is useful to extract material strength parameters from experimental results in all the cases in which the stress and strain fields are not correctly described or predictable with an analytical model. Usually, this happens in specimens with no regular shape, in specimens in which some instability phenomena occur (e.g. the necking phenomena in tensile tests) or in dynamic tests, in which the strain-rate field is not uniform due to the stress wave propagation. Besides, the inverse method is also useful in case of high strain-rate tests, in which the adiabatic heating due to

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plastic work conversion into heat leads to thermal softening phenomena.

The use of a numerical inverse method for the material model parameters identification is now widespread thanks to the larger computing power available at lower cost. In any case, the difficulty is often the understanding of which are the best strategies to choose to take advantage of the capabilities of the optimization methods applied to problem like system parameters identification.

In recent years, different authors applied a combined numerical and experimental technique with the aim to extract the material model parameters via an inverse method. One of the most important works on this topic is [1]. A specific treatment was developed according to the choice of the material model [2,3], the type of the experimental tests [4,5], the FEM code for the numerical solution and the algorithm on which the optimization was based on [6,7]. A lot of these works are related with the solution of problems in which the dynamic component could be relevant. For these reasons, the material behavior description was based on the definition of a visco-plastic material model and the experimental tests covered a wide range of strain-rates, from quasi-static up to high strain-rates. Since the loading conditions to simulate, often an explicit FEM code was used, like LS-DYNA, RADIOSS, AUTODYN and ABAQUS. Finally, the choice of the optimization algorithms was linked to the algorithms that are implemented in the commercial optimization codes, like LS-OPT and HYPERSTUDY.

The main objective of this work is getting strain-hardening, thermal and strain-rate parameters for a material model in order to correctly reproduce the deformation process in a wide range of temperature and strain-rate via a numerical inverse method of which the main steps of the procedure are as follows.

- Performing the experimental tests at different speeds and temperatures.
- Optimizing the material parameters via numerical FEM simulation of the experimental tests using the commercial code LS-OPT [8] for the optimization phase and LS-DYNA [9] for the numerical simulations.

It is important to remark that a material characterization must count on a specified analytical model from which the number of strength parameters and the types of experimental tests to be performed depend. For this reason, it is very important that experimental tests and numerical modeling go hand in hand in order to avoid both an inadequate and an overflowing number of data. So, first of all it is necessary to choose the material model on which it is consequently possible to plan the experimental tests. Then, the numerical model of each experimental test is realized. The next aspect is the evaluation of the most suitable strategy of the parameters optimization estimating the influence of each model parameter on both the stress–strain relationship and the optimization error. Finally, once the best optimization strategy has been identified, it is possible to apply the numerical inverse method to extract the numerical model parameters for the investigated material.

2. Material model

In the past decades a lot of material models for the description of the elasto-visco-plastic behavior are proposed. The classification model makes a distinction between empirical, semi-empirical and physically-based models. The empirical models have no physical basis, but are obtained by interpolation of the experimental data. On the other hand the physically-based models are obtained starting from transformations in the material occurring during a deformation process.

Models such those proposed by Johnson–Cook (J–C) [10,11] and Cowper–Symonds (C–S) [12] are purely empirical models and they are the most widely used. An example of semi-empirical model is the Steinberg–Cochran–Guinan–Lund (S–C–G–L) model [13,14], which was first developed for the description of high strain-rates behavior [13], and after extended to low strain-rates [14]. Another semi-empirical model is the Zerilli–Armstrong (Z–A) model [15] that is obtained on the basis of the dislocation mechanics theory and presents different formulations for BCC and FCC materials. A more complex dislocation-based model is the Mechanical Threshold Stress (MTS) model [16].

The chosen material model for the numerical simulation is the J–C model because, since it is very simple, it is able to predict the mechanical behavior of the materials under different loading conditions. Besides, as mentioned before, it is one of the most used material models, so it is implemented in many FEM codes.

Several authors used the J–C model, or its modified formulations, in order to investigate and describe problems such as ballistic impacts or, more in general, problems in which the strain-rates component was relevant. Different methods for the material model calibration starting from experimental data were also suggested. A lot of different types of materials have been described using the J–C model, such as steels [17,18], aluminum alloys [19,20], titanium alloys [21–23], OFHC copper [24,25], tungsten alloy [26] and super alloys [27], with mainly application in automotive, aerospace, nuclear and military fields. In some cases, the experimental data were fitted on the basis of the analytical formulation of the material model, while other works performed the calibration of FEM models starting from experimental results.

Recently, a multi-objective procedure for the material model identification has been proposed in [26]. In the present paper, a similar approach is presented, but differently from [26], the method is based on the use of FEM models, in order to take into account also the development inside the specimen of non-homogeneous distribution in mechanical quantities (stress, strain, temperature and strain-rate).

2.1. Johnson–Cook model

The standard J–C model [10] expresses the flow stress as

$$\sigma_y = \left(A + B \varepsilon_{pl}^n \right) \left(1 + C \ln \frac{\dot{\varepsilon}_{pl}}{\dot{\varepsilon}_0} \right) \left(1 - \left(\frac{T - T_r}{T_m - T_r} \right)^m \right) \quad (1)$$

in which A is the elastic limit strength and fixes the stress value at which the plastic behavior starts, B and n are the work hardening parameters and influence the slope of the flow stress in the plastic domain. The parameter n usually assumes values between 0 (for perfectly plastic model) and 1 (for a bilinear model). C is the strain-rate sensitivity coefficient and m describes the thermal softening. In more detail, m determines the concavity of the temperature function: if $m < 1$ the function is convex, if $m > 1$ it is concave and if $m = 1$ the temperature influence is linear. The thermal effects are also described in function of T_r that is the reference temperature at which there are not any thermal effects and T_m that is the melting temperature at which the material mechanical strength goes to zero. In this condition the material loses its shear strength and starts to behave like a fluid.

In the LS-DYNA formulation [9], $\dot{\varepsilon}_0$ represents the quasi-static strain-rate threshold that represents the highest strain-rate for which the strain-rate effects on the flow stress are negligible.

The J–C model is a multiplicative model, in which the effects of plastic strain, strain-rate and temperature are assumed to act independently. It is clear from the Eq. (1) that a strain-rate or temperature variation implies only a scaling and not a modification

in the shape of the strain hardening curve. Obviously, this is a simplifying assumption.

The J–C model could take into account the thermal softening that is essentially due to heat conversion of plastic work occurring at high strain-rate deformations. For $\dot{\epsilon} \geq 10^2 \text{ s}^{-1}$ both thermal conduction and convection can be neglected and thermal softening can be evaluated under adiabatic assumption. Given this last hypothesis and the further assumption of uniform stress, strain and temperature fields, the temperature can be analytically computed as a function of plastic work [27,28]. In LS-DYNA the evaluation of the change in temperature due to plastic work conversion is performed by the material routine in case of structural analysis only. In case of coupled thermo-structural analysis, the temperature calculation is managed by the thermal solver.

3. Objective of the study

This study is performed on an alumina dispersion strengthened copper, known by the trade name GLIDCOP®. It is a composite material with metal matrix in copper strengthened with aluminum oxide ceramic particles. GLIDCOP® is available in several grades depending on the weight percentage of aluminum oxide content: in this work the GLIDCOP® AL-15 (0.3 wt. %) is used. Since the particle content is quite small the material keeps the OFE copper properties,

such as thermal and electrical conductivity, but with a higher yield strength, like a mild-carbon steel. Besides, with the addition of aluminum oxide, the good mechanical properties are retained also at high temperatures and the resistance to thermal softening is increased. In fact, the presence of the aluminum oxide particles in the copper matrix blocks the dislocation movement preventing the grain growth. Finally, the GLIDCOP® has a greater resistance to radiation damage in comparison with the OFE copper.

Thanks to these properties GLIDCOP finds several applications in particle accelerator technologies [29], where problems of thermal management, combined with structural requirements, play a key role. Currently, it is used for the construction of structural and functional parts of the particle beam collimation system of the Large Hadron Collider (LHC) at CERN (Geneva) [30]. Since the extreme condition in which the material could operate, it is fundamental to characterize it in a wide range both in strain-rate and temperature.

As a matter of fact, due to the interaction between the material and the high-energy particle beam, the operating conditions could be characterized by high temperature, high strain-rate or both of them (accidental case [31]).

According to the fact that the J–C material model is uncoupled in plastic strain, strain-rate and temperature effects, the

Fig. 1. Experimental results at different nominal temperatures. (a) True stress–effective plastic strain curves: $\times$: 20 °C, $\triangleright$: 100 °C, ∇ : 200 °C, $*$: 300 °C, $\diamond$: 400 °C, $\triangleleft$: 500 °C, $\square$: 600 °C, $\triangle$: 700 °C, $+$: 850 °C and $\bullet$: 1000 °C; (b) ratio between each experimental curve (σ) and the result obtained for the test at the lowest temperature (σ_{Tr}).

Fig. 2. Experimental results at different nominal strain-rates. (a) True stress–effective plastic strain curves: $\square$: 10 s^{-1} , $\diamond$: 10^{-1} s^{-1} , $*$: 10^{-2} s^{-1} , $\triangleright$: 10^{-3} s^{-1} (the experimental curve at 10^3 s^{-1} , performed on the SHPB, is interrupted at about 0.25 of effective plastic strain in accordance with the input wave length); (b) ratio between each experimental curve (σ) and the result obtained for the test at the lowest strain-rate (σ_{SR0}).

experimental tests are managed exchanging one parameter at a time. So, experimental tests are performed at different speeds (10^{-3} , 10^{-1} , 10 , 10^3 s^{-1} at room temperature) and different temperatures (room temperature, 100, 200, 300, 400, 500, 600, 700, 850 and 1000°C at 10^{-3} s^{-1}). For each loading condition three repetitions are performed, but for clarity the next figures show a single curve for each experimental test.

In this work problems concerned to the damage and the failure of the specimen are neglected. In this way, under the hypothesis that the mechanical behavior of the material is the same both in compression and in tension, it is chosen to carry out only the compressive tests as easier to perform than the tensile tests, especially for what concerns the tests in temperature.

Quasi-static loading condition is obtained via general purpose electro-mechanical testing machine while medium strain-rates tests are performed with servo-hydraulic equipments. High strain-rates compressive tests are carried out with a standard Hopkinson pressure bar setup [32].

3.1. Experimental results

In a standard mechanical test the measured quantities are concerned with global properties of the specimen. Generally, the measured macroscopic quantities are forces and displacements at the specimen ends. The force measurement does not involve particular problems and it is obtained by means of a load cell. On the other hand, the displacement measurement requires more attention: the acquisition of the displacement of the clamping specimen system is not sufficient, but an extensometer is needed. In general, the measure of the displacement field of the extensometer gage length is substantially different from the stroke testing machine. This is due to the fact that also the clamping device can be deformed by applied loads [33]. If it is not possible to use an extensometer it is necessary to apply specific corrections on the displacement signal.

The force and displacement signals must be elaborated to achieve the true stress – effective plastic strain curve that is the classical input of FEM codes material models. First of all, starting from the force–displacement curve, the engineering stress–strain curve is obtained assuming the hypothesis of uniformity of the stress–

strain fields in the specimen. Then, under the assumption of no volume changes in plastic regime it is possible to obtain the material true stress–effective plastic strain curve.

Following these assumptions, results in terms of true stress–effective plastic strain curves are shown in Figs. 1a and 2a for the experimental tests at different temperatures and strain-rates, respectively. The points in Figs. 1b and 2b represent the ratio between each experimental curve and the lowest temperature and lowest strain-rate curves, respectively. Following a standard approach, in reference to the Eq. (1), the curve of the test performed at room temperature (20°C , Fig. 1a) could be representative of the first term (strain-hardening effects), the curve of Fig. 1b represents the last term (thermal effects) and the curve of Fig. 2b represents the second term (strain-rate effects).

4. Multi-objective optimization

As mentioned before, in this work, the experimental data are processed via a numerical inverse method based on FEM numerical simulations. The main objective of an inverse optimization method is the determination of a selected set of unknown parameters in a numerical model: starting from a trial point, the unknown parameters are estimated iteratively by comparing experimentally measured with numerically computed quantities for the same material test conditions. The material characterization is reached varying the material strength parameters of the FEM model, that reproduces the experimental loading and constraint conditions, and comparing the model results with the experimental data trying to obtain the best correlation. The great advantage of this procedure is that no hypothesis about the internal specimen stress–strain field is made: in fact, the comparison is made in terms of macroscopic quantities that, in general, are force and displacement. The main disadvantage of the inverse methods is the high computational times that these algorithms need: the optimization algorithms must perform iteratively a certain number of FEM simulations. Besides, the number of iterations dramatically increases when the degrees of freedom of the problem grow up or the trial parameters are far from the optimum ones.

Fig. 3. Results of the numerical identification process in which the optimization algorithm tries to reproduce, optimizing B and n , the reference J–C hardening curve ($A_0 = 90 \text{ MPa}$, $B_0 = 292 \text{ MPa}$, $n_0 = 0.31$) with an imposed error on the estimation of A (A is fixed in each optimization and varies between 0 and two times A_0) in order to understand the influence of this parameter.

Fig. 4. Results of the numerical identification process in which the optimization algorithm tries to reproduce, optimizing A and n , the reference J–C hardening curve ($A_0 = 90 \text{ MPa}$, $B_0 = 292 \text{ MPa}$, $n_0 = 0.31$) with an imposed error on the estimation of B (B is fixed in each optimization and varies between 0 and two times B_0) in order to understand the influence of this parameter.

The optimization of the parameters is performed with a dedicated algorithm included in the software LS-OPT [8], that manages the parameters variation strategy, runs the numerical simulation, analyzes the results and extracts the optimum set of parameters for each iteration.

In particular, if a FEM-based numerical inverse method is applied to a model like the J–C model, it requires necessarily a Multi-Objective Optimization (MOO, see Appendix), in which different objectives have to be satisfied simultaneously. As a matter of fact, both the thermal softening and the strain-rate parameters have to be estimated considering the variation of the corresponding properties (temperature and strain-rate) on the basis of multiple curves. From these considerations, one could intend to perform a single optimization step to achieve the complete material model parameters identification. Actually, the problem is that a MOO could have different solutions depending on the relative importance of the parameters and their influences on the global fit result.

4.1. Steps of the optimization

Since J–C model is a multiplicative model it would be possible to optimize separately each set of parameters. So, a first optimization would be performed in order to extract the strain dependence. Then the thermal parameters would be obtained on the basis of the static tests at different temperatures. Finally, the last optimization would be done to extract the strain-rate coefficients from the dynamic tests.

Before starting with the optimization, since J–C model is a purely empirical model it is important to decide which of the parameters should be considered as optimization variables without any physical interpretation.

For what concerns the strain-hardening effect, three parameters (A , B and n) are available to perform the data fit and different procedures can be used: all the strain-hardening coefficients can be optimized or one of them can be fixed a priori and the other two optimized. In order to understand the best procedure, in this work, a numerical optimization (performed using MATLAB) is done. For this purpose, a reference curve with $A_0 = 90$ MPa, $B_0 = 292$ MPa and $n_0 = 0.31$ (parameter for copper [10]) is considered. In Figs. 3–5, the results obtained fixing one parameter and optimizing the other two

Fig. 5. Results of the numerical identification process in which the optimization algorithm tries to reproduce, optimizing A and B , the reference J–C hardening curve ($A_0 = 90$ MPa, $B_0 = 29$ MPa, $n_0 = 0.31$) with an imposed error on the estimation of n (n is fixed in each optimization and varies between 0 and 1) in order to understand the influence of this parameter.

Fig. 6. Temperature softening parameters influence.

are shown for a certain range of variation of the fixed parameter. In these figures, on the abscissa there are the values of the fixed parameter (normalized with respect to the reference value in case of A and B) and in ordinates there are the percentage Mean Squared Error (MSE) and the optimized values for the other two parameters (normalized with respect to the reference value in case of A and B).

In order to compare the results, it was considered that two repetitions of an experimental test performed under the same conditions could be different with a percentage error up to 1%. In reference to the Figs. 3–5, the threshold for the comparison is set to 0.05%: only the case in which the fixed parameter is A satisfies this condition in a wide range of variation of A respect to A_0 . This implies

Fig. 7. Numerical FEM model (a); velocity profiles of the compression plates for the FEM models of SHPB tests (b).

Fig. 8. Von Mises stress (Pa) distribution in the specimen (a); strain-rate (s^{-1}) distribution in a test nominally at 10^3 s^{-1} at 0.36 ms (b).

that the J–C model is able, after optimization of the other two parameters, to recover the estimate of the parameter A of about $\pm 50\%$ of the correct value. Starting from this consideration and with the aim to achieve a higher level of uniqueness in the results (it is evident as different sets of parameter could produce similar results, since they produce similar or equal MSE), the parameter A is fixed a priori. The evaluation of A can be performed on the basis of the experimental data in case of quasi-static tests and A can be set equal to the yield strength or the yield strength [offset = 0.2%] or as the end of the linear behavior.

In the J–C model, the temperature influence is expressed by the last term of the Eq. (1). The temperature parameters influence on the optimization process is investigated starting from the series of experimental data represented in Fig. 6 with dot markers. Each marker represents the ratio between the true stress–effective

plastic strain curve for the experimental test at temperature T and the curve corresponding to the experimental test performed at the lowest temperature.

Usually, the only J–C temperature parameter that is considered as an optimization variable is m . This approach leads to a bad fit with respect to Eq. (1) (Fit 1 in Fig. 6). An improvement could be achieved considering also T_m as an optimization variable: the addition of a variable adds a degree of freedom that allows to achieve a better fit to the experimental data. A further improvement could be obtained if the temperature range considered for the optimization is limited (Fit 2 in Fig. 6). This range reduction is possible if the material strength is considered negligible under a fixed level. In case of Fig. 6, for example, considering that the experimental data for which the temperature coefficient is less than 0.2 could be excluded from the optimization, the range of optimized temperature is reduced between 20°C and 600°C .

Another important consideration is about the possibility of optimizing B , n , m and T_m (for a fixed A) at the same time. In fact, if the strain dependence optimization is done on the experimental result of the test performed at the lowest temperature, it implies this shape will be assigned to all the other curves at different temperatures. The temperature dependence optimization thus only generates the best scale function. On the other hand, if all the parameters are optimized at the same time the reference shape is not constrained a priori but it is left free to be optimized. This last optimization strategy is however quite critical considering that some variables have similar influences on the results. In this case, as mentioned before, the optimized solution could not be unequivocal.

Finally, the second term of the J–C model (Eq. (1)) expresses the strain-rate influence on the material flow stress. Usually, only the parameter C is considered as an optimization variable, while $\dot{\epsilon}_0$ is set equal to 1 s^{-1} . In LS-DYNA this choice would mean that the strain-rate influence is neglected for all the experimental set data with $\dot{\epsilon}_0$ less than unity. If also $\dot{\epsilon}_0$ is used as optimization variable, an improvement in the experimental data fit could be obtained. With reference to Fig. 2b, one can guess that the variation of the position of the threshold $\dot{\epsilon}_0$ could entail a better fit.

5. Numerical model

The FEM models are developed with the commercial code LS-DYNA [9], which includes an implicit and explicit solver with thermo-mechanical and highly non-linear capabilities.

The realized numerical model (see Fig. 7a) is 2D axisymmetric (in order to reduce the computational time) and the performed analysis is a coupled thermo-structural analysis, in which both conduction and convection contributions are taken into account. In particular the thermal analysis is performed with an implicit solver.

The models for the simulation of the compressive tests include the specimen and the plates with a contact algorithm in which the friction parameters are set in order to reproduce the real experimental barreling effect. Since the problem is mesh-dependent, a study of the mesh influence is performed and the mesh shown in Fig. 7a is a good compromise between solution accuracy and computational costs.

Fig. 9. A sequence of steps of experimental test performed with SHPB filmed with high velocity camera (20,000 fps): due to the friction between specimen and testing equipment the deformation is not uniform (barreling). The sequence is used to validate the numerical simulations friction model.

Fig. 10. Temperature distribution (K) at two different strain-rates: nominally (a) 10^{-1} s^{-1} , and (b) 10^3 s^{-1} .

For high strain-rate tests ($\dot{\epsilon} = 10^3 \text{ s}^{-1}$), the load that is applied to the specimen by input and output bars of the SHPB is reconstructed and implemented into the FEM model: the experimental velocity profile is applied to the rigid plates (see Fig. 7b).

The numerical FEM model-based optimization should be useful in order to avoid the following simplifying assumptions.

- *Uniaxial stress and strain inside the specimen.* Actually, three-axial stress and strain fields inside the specimen are caused by the friction between specimen and testing equipment (see Figs. 8a and 9).
- *Constant strain-rate inside the specimen.* Actually the strain-rate is not constant and uniform during dynamic tests (Fig. 8b), thus influencing strain-rate sensitivity parameters.

Table 1
Step 2, multi-objective identification results for the thermal softening parameters.

Case	T range	Variable	Fixed
1	20–1000 °C	$m = 0.5102$	$T_m = 1089 \text{ °C}$
2	20–1000 °C	$m = 0.5409$; $T_m = 1037 \text{ °C}$	–
3	20–850 °C	$m = 0.5111$	$T_m = 1089 \text{ °C}$
4	20–850 °C	$m = 0.6501$; $T_m = 877 \text{ °C}$	–
5	20–700 °C	$m = 0.5123$	$T_m = 1089 \text{ °C}$
6	20–700 °C	$m = 0.6920$; $T_m = 816 \text{ °C}$	–
7	20–200 °C	$m = 1.1089$; $T_m = 473 \text{ °C}$	–

Fig. 11. Step 2 – absolute mean squared error (MSE) in each optimization case on the basis of the load–stroke curves for the experimental tests at different temperatures.

- *Uniform temperature inside the specimen.* Actually the temperature has a certain distribution proportional to the distribution of plastic strain inside the specimen, and the effect is even higher for dynamic tests (see Fig. 10). This should be considered to identify material strength parameters.

6. GLIDCOP® data

The strategy of the optimization with the J–C model is now applied in case of GLIDCOP® based on the experimental compression tests of Figs. 1 and 2.

For what concerns the optimization of the experimental data at different temperatures, the multi-objective optimization is based on the minimization of the absolute squared error (W_p and s_p are set equal to 1, see Appendix). This implies more weight is given to the curves at lower temperatures: from a mechanical point of view it is more relevant to be able to predict the material behavior in a range in which its strength properties are still significant (there is more than one order of magnitude between the stress–strain

Fig. 12. Step 2 – comparison between experimental data (marker) and numerical results (line) for the tests at different temperatures (case 6): x: 20 °C, v: 100 °C, *: 200 °C, *: 300 °C, diamond: 400 °C, <.: 500 °C, square: 600 °C and triangle: 700 °C.

Fig. 13. Step 2 – comparison between experimental data and numerical results for the tests performed at different temperatures (case 7): $\times$: 20 °C, $\triangleright$: 100 °C, ∇ : 200 °C.

curves at 20 °C and 1000 °C). Also for the dynamic stress–strain curves, the weights are set to 1, since being the curves of the same order of magnitude to use the absolute MSE or the relative one does not affect the results.

6.1. Step 1

The first set of optimization is done in order to extract the strain-hardening parameters B and n of the Eq. (1) from the quasi-static experimental results at the lowest temperature. The parameter A is set equal to 250 MPa, that corresponds to the end of the linear behavior. In Fig. 12 there is the comparison between the experimental and the numerical results for the tests at 20 °C in terms of load–stroke curves: the optimization leads to $B = 313.7$ MPa and $n = 0.1551$ with a MSE equal to 7.4×10^4 N².

6.2. Step 2

In order to obtain the best fit of the experimental data for the tests at different temperatures a series of different optimization cases (see Table 1) are evaluated varying the number of optimization degrees of freedom and the range of temperatures on which the optimization is performed.

The results in terms of absolute MSE are shown in Fig. 11 and the comparison between experimental and numerical load–stroke curves are in Fig. 12 for the case 6 and in Fig. 13 for the case 7 that are the two cases with the lowest MSE.

From the results it can be stated that there is an improvement in the fit (lower MSE) both adding the T_m as an optimization variable (obviously it no longer represents the melting temperature, but the temperature under which the softening effects are neglected) and reducing the range of temperature on which the optimization is performed. For the latter aspect more precise information is needed. If the J–C model is used to evaluate conditions of self-heating, it should be more suitable to greatly restrict the

Table 2
Step 3, multi-objective identification results for the strain-rate parameters.

Case	Variable	Fixed	Reference case for thermal softening
1	$C = 0.0548$	$\dot{\epsilon}_0 = 1 \text{ s}^{-1}$	6
2	$C = 0.0352$; $\dot{\epsilon}_0 = 0.0559 \text{ s}^{-1}$	–	6
3	$C = 0.0269$; $\dot{\epsilon}_0 = 0.0405 \text{ s}^{-1}$	–	7

Fig. 14. Step 3 – absolute mean squared error (MSE) in each optimization case on the basis of the load–stroke curves for the experimental tests at different strain-rates.

considered temperature range. Otherwise, if the J–C model is used to simulate the behavior of a component that has to work in high temperature conditions, one should be forced to extend the considered range.

6.3. Step 3

As for the temperature, also for the strain-rate effects, a series of different optimization cases are evaluated (see Table 2), varying the number of optimization variables and the reference parameters for the thermal softening effects evaluation.

In each optimization case, the absolute MSE, calculated on the load–stroke curves for each strain-rates, is shown in Fig. 14 and the comparison between experimental and numerical curves are in Fig. 15 for the case 2 and in Fig. 16 for the case 3, that are the two cases with the lowest MSE.

As told in advance, considering $\dot{\epsilon}_0$ as an optimization variable a better fit could be reached, since there is one more degree of freedom available for the optimization (comparison between case 1 and case 2 in Fig. 14). A further reduction of the absolute MSE for all

Fig. 15. Step 3 – comparison between experimental data and numerical results for the tests at different strain-rates (case 2): $\square$: 10^3 s^{-1} , $\diamond$: 10 s^{-1} , $*$: 10^{-1} s^{-1} , $\triangleright$: 10^{-3} s^{-1} .

Fig. 16. Step 3 – comparison between experimental data and numerical results for the tests at different strain-rates (case 3): $\square$: 10^3 s^{-1} , $\diamond$: 10 s^{-1} , $*$: 10^{-1} s^{-1} , $\blacktriangleright$: 10^{-3} s^{-1} .

the strain-rates (comparison between case 2 and case 3 in Fig. 14) is achievable starting with the thermal softening parameters obtained from the fit of the experimental data in the reduced temperature range (case 7 of the step 2).

7. Conclusions

In this work, a combined experimental and numerical technique, based on an inverse optimization approach, is fully developed and applied to characterize an alumina dispersion strengthened copper (GLIDCOP®). In particular the attention is focused on the critical review of the numerical optimization strategies to apply in the parameters identification process in case of strain-rate and thermal sensitive material models.

The first step is the choice of the analytical model for the description of the mechanical material behavior and in this work the Johnson–Cook model is used. It is important to underline that the J–C material model is extremely simple and probably quite inaccurate to describe the material behavior over a wide range of temperatures and strain-rates. Besides, it is well known that other material models (such as the physically-based ones) should be more suitable for the description of the behavior in case of FCC materials, like copper. Nevertheless, the J–C model is by far the most commonly used in commercial FEM codes, hence the choice to use it.

Once the analytical model is chosen, experimental tests are planned and performed at different strain-rates and temperatures. The experimental characterization is performed for wide ranges both in temperatures and strain-rates. The temperature range considered is wider with respect to the temperature increment resulting from the self-heating of the material in high dynamic tests. This is because the investigated material could operate both in high strain-rate and high temperature conditions.

Before starting with the numerical inverse optimization, the analysis of the most suitable optimization strategy is performed in order to understand the role of each material coefficient, which and how many of them could be considered as optimization variables. As a matter of fact, each material model parameters entails different effects on the flow stress description.

The performed analysis shows that the influence of some parameters could generally be different in relation to the range of variation of strain-rate and temperature of the experimental tests

to simulate. In general, it is found that there is little influence of the strain-hardening parameter A and the possibility exists of considering material constants, like the melting temperature, and model parameters, like the quasi-static strain-rate threshold, as optimization variables instead of constants.

A multi-objective optimization is strictly needed in the case of a material model like the J–C model, in which both thermal softening and strain-rate parameters have to be estimated considering the variation of the corresponding properties (temperature and strain-rate) on the basis of several curves. In particular, the effectiveness of different optimization strategies is analyzed on the basis of the sum of the absolute mean squared error that is calculated for each numerical simulation.

In conclusion, it is important to underline that any choice, about material model, experimental tests and optimization strategy have to be assessed as a function of the range of variation of the variables of interest for the specific problem. As a matter of fact, especially for very simple material models, there do not exist a unique optimum set of parameters, but, in general, the optimization has to be adapted on the basis of the application of interest.

Nevertheless the treatment of the optimization procedure could appear too complex and elaborate, but with a state-of-the-art computer resource, the procedure takes only few hours for the complete material characterization. Besides, the strategy used has to be understood only once for each material model and then it can be applied each time this particular model is used. Finally, when one uses a plasticity model for FEM analysis, one always has to perform benchmarking simulations for comparisons with experimental tests to check if everything is working properly. In this case the benchmarking phase is automatically performed during the optimization process.

Acknowledgments

The authors are grateful to the Mechanical and Materials Engineering Group (EN-MME) of CERN (Geneva), in particular to A. Dallochio for his support in this research.

Appendix

Multi-Objective Optimization (MOO) problems are significantly different to single objective optimization problems. MOO problems do not have a single optimal solution. Instead there is a set of solutions that reflects trade-offs among objectives. In general, there should not be a single solution that minimizes each objective to its fullest. The optimal solution is such that any further attempts to optimize on a single objective leads to worse results for the other(s).

Mathematically, the MOO unconstrained problem is defined as follow

$$\min F(\Phi_1, \Phi_2, \dots, \Phi_N) \quad (2)$$

where F represents the multi-objective function and $\Phi_i = \Phi_i(x_1, x_2, \dots, x_n)$ are the various objective functions with x_i the n design variables. In particular the MOO function is define as

$$F = \sum_{k=1}^N \omega_k \Phi_k \quad (3)$$

in which ω_k are the weights to assign to each single objective function. It is important to note that each objective function has a target. In this work all the weights ω_k are set to unity, so all the objectives are equally important.

For the present study, the single objective function is the Mean Squared Error (MSE) Φ_i

$$\Phi_i = \frac{1}{P} \sum_{p=1}^P W_p \left(\frac{f_p(x) - G_p}{s_p} \right)^2 \quad (4)$$

where P is the number of points in which the MSE is calculated, G_p , varying p , are the values on the target curve G and $f_p(x)$ the corresponding components of the computed curve f . W_p and s_p are scale functions for each point p and finally x is the design vector.

Several definitions of the MSE are possible varying the value of the scale function s_p . In this work two definitions were used: the absolute MSE if s_p is constant and equal to unity and the relative MSE if s_p is constant and equal to the maximum of G .

The chosen optimization algorithm is the Leap-Frog Algorithm for Constrained Optimization (LFOPC) [8,34], that is a gradient method that generates a dynamic trajectory path, from any given starting point, toward a local minimum. In particular the method seeks the minimum of a function of n variables by considering the associated dynamic problem of the motion of a particle of unit mass in a n -dimensional conservative force field, in which the potential energy of the particle is the function to be minimized.

In order to accelerate convergence a domain reduction strategy is adopted to reduce the size of the subregion of the feasible points. During a particular iteration, the subregion is used to bound the position of new points. This method is ideal for the system identification. The experimental design is the selection procedure for finding the feasible points in the design space that must be analyzed. In this work a D-optimal method is applied [8].

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