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THE CASIMIR EFFECT AS A SOURCE OF CHIRAL SYMMETRY BREAKING IN QCD

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A B S T R A C T

The vacuum of QCD, defined on a space-time topology $T^3 \times R$, breaks chiral symmetry. The responsible physical mechanism is the formation of fermionic condensates due to Casimir forces. Representations of coloured fermions, which possess asymptotic freedom, stabilize through their gauge interactions the formation of these condensates. Estimates of ratios of the order parameters are given for various representations.

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An outstanding problem of strongly interacting gauge theories with massless fermions is to determine whether chiral symmetry, possessed by the Lagrangians of these theories, is realized in the Nambu-Goldstone mode that is broken by the vacuum [1]. Rigorous results [2] and Monte Carlo techniques of lattice gauge theories [3] suggest the existence of chiral symmetry breaking for sufficiently strong couplings.

In this letter we concentrate on the relation of the scales for condensates formed by different fermions of different colour representations. This question is vital in the present day phenomenology of electroweak interactions [4] and results are already known for the SU(2) and SU(3) gauge groups [3].

The dynamical mechanism responsible for chiral symmetry breaking must create in the vacuum massless fermionic zero energy modes with a spectral density of the type of one dimensional fermionic free gas [5, 6, 7, 8].

To generate the necessary vacuum environment, we propose, following ref. [9], to put Euclidean QCD with massless fermions in a three dimensional torus T^3 , of size $L \times L \times L$, keeping infinite the time direction. The philosophy of this proposal is not to use the box as a physical bag, but rather extract in a systematic way information about the infinite volume situation [10]. The free-fermionic vacuum on the space-time $T^3 \times R$ is expected to have lower energy than the $R^3 \times R$ vacuum* due to Casimir attractive forces [11]. The renormalizability of field theories on space-time with boundary has been discussed in [12].

Before we discuss QCD we show that in the 2-d Gross-Neveu (G-N) model, with discrete chiral symmetry [13] which is exactly soluble in the $1/N$ expansion, the suggested chiral symmetry breaking mechanism works not only qualitatively but also quantitatively [9a]. The Euclidean version of the G-N model is described by the Lagrangian of N -flavours of fermions

$$\mathcal{L}_{G-N} = \bar{\psi} i \partial \psi - \frac{f}{2N} \cdot (\bar{\psi} \psi)^2 \quad (1)$$

* The time direction R is first compactified to $[0, T]$ with anti-periodic boundary conditions and then the $T \rightarrow \infty$ limit is taken.

where the fermionic fields ψ and $\bar{\psi}$ satisfy periodic boundary conditions in space

$$\begin{aligned}\psi^i(0, t) &= \psi^i(L, t) & \forall i=1, 2, \dots, N \\ \bar{\psi}^i(0, t) &= \bar{\psi}^i(L, t) & \forall t \in [0, T]\end{aligned}\quad (2)$$

and antiperiodic in time

$$\psi(x, 0) = -\psi(x, T) \quad \forall x \in [0, L] \quad (3)$$

The $1/N$ expansion, for the quantity $\langle \bar{\psi}\psi \rangle$, is studied either through the diagrammatical calculation of the effective potential or equivalently through the expansion around the $N=\infty$ saddle point of the functional integral [14].

Following the second method one obtains the gap equation

$$\frac{1}{2f_B} = \int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2 + \sigma_0^2 + m^2} \quad (4)$$

where

$$N\sigma_0 = f \cdot \langle \bar{\psi}\psi \rangle \quad (5)$$

Using the boundary conditions (2) and (3) we construct the propagator and we take the $T \rightarrow \infty$ limit and then $m \rightarrow 0$.

Then we obtain [9a]

$$\ln \frac{\sigma_0}{\Lambda_{\overline{MS}}} = \int_0^\infty \frac{dt}{t} \cdot e^{-tz^2} \cdot \sum_{k=1}^\infty e^{-\frac{k^2}{4t}} \quad (6)$$

where

$$Z \equiv \sigma_0 \cdot L = \frac{f}{N} \langle \bar{\psi}\psi \rangle \cdot L \quad (7)$$

In fig.1 the behaviour of the "condensate" σ_0 (which is also equal to the mass of the fermions), as a function of Z , is shown, together with the leading small Z behaviour. We see that $\sigma_0(L)$ develops the asymptotic value $\sigma_0(\infty) = \Lambda_{\overline{MS}}$, very fast at $Z \sim 3$.

Also the leading small volume behaviour develops a minimum value, numerically very near the infinite volume one, with a tendency to become stable against changes in Z , (compare the case of the $O(N)$ mass gap [10a]).

Let us discuss now QCD. By a simple calculation one realizes that at the free field level the zero momentum modes produce a condensate

$$\langle \bar{\psi} \psi \rangle(L) = \frac{a_{00}}{L^3} \quad (8)$$

where

$$a_{00} = 2 \cdot n_{\text{colours}} = 2 \cdot d_R$$

d_R is the dimension of the fermionic representation. We see that $\langle \bar{\psi} \psi \rangle(L) \rightarrow 0$ as $L \rightarrow \infty$. This effect is not due to a wrong normal ordering, the normal ordering must be chosen so that the $L=\infty$ vacuum has zero energy [11].

To form a Renormalization Group (RG) invariant quantity we multiply $\langle \bar{\psi} \psi \rangle$ by powers of the running coupling constant with exponents the anomalous dimension of the $\bar{\psi} \psi$ operator [15,16]. For the calculation of $\langle \bar{\psi} \psi \rangle$ at the next order we find the Renormalization Group Equation (RGE) consistent form [9b],

$$Z \equiv L^3 \langle \bar{\psi} \psi \rangle(L) = \left[a_{00} + \frac{a}{\pi} (a_{11} \log L + a_{10}) + O\left(\frac{a^2}{\pi^2}\right) \right] \quad (9)$$

We use now the anomalous dimensions of $\bar{\psi} \psi$ (see for instance [16]), $\gamma(\alpha)$ and the $\beta(\alpha)$ function ($\alpha = g^2/4\pi$), and we define the RG invariant quantity

$$\langle \widehat{\bar{\psi} \psi} \rangle = \left(\frac{a}{\pi} \right)^{\frac{\gamma_1}{\beta_1}} \langle \bar{\psi} \psi \rangle \quad (10)$$

where

$$\gamma_{\bar{\psi} \psi} = \gamma_1 \frac{a}{\pi} + \gamma_2 \left(\frac{a}{\pi} \right)^2 + O\left(\frac{a^3}{\pi^3}\right) \quad (11)$$

and

$$\beta(\alpha) = -\beta_1 \frac{a}{\pi} - \beta_2 \left(\frac{a}{\pi} \right)^2 + O\left(\frac{a^3}{\pi^3}\right) \quad (12)$$

The coefficients $\beta_1, \beta_2, \gamma_1, \gamma_2$ are [16].

$$\gamma_1 = \frac{3}{2} C_F \quad (13a)$$

$$\gamma_2 = \frac{3}{16} C_F^2 + \frac{97}{48} C_F C_A - \frac{5}{24} C_F m_F \quad (13b)$$

$$\beta_1 = 11 - \frac{2}{3} m_F \quad (14a)$$

$$\beta_2 = 102 - \frac{38}{3} m_F \quad (14b)$$

Here C_F and C_A are the eigenvalues of the quadratic Casimir operators of the fermionic and the adjoint representation respectively. In the coefficients β_1 and β_2 we include only fermion triplets (n_F number of flavours). We substitute now the coupling constant α from rel.(10) into the expression for the RG invariant scale of QCD Λ_{MS}

$$\Lambda_{ms} = \mu \left(\frac{\beta_1 \alpha}{2\pi} \right)^{-\frac{\beta_2}{\beta_1^2}} \cdot e^{-\frac{\pi}{\beta_1 \alpha}} \quad (15)$$

to obtain a scheme independent or universal function,

$$\frac{\langle \bar{\psi} \psi \rangle(L)}{\Lambda_{ms}^3} = k \cdot Z^{1 + \frac{3\beta_2}{\beta_1 \gamma_1}} \cdot e^{-\frac{3}{\gamma_1}} \equiv X(Z) \quad (16)$$

where

$$k \equiv \alpha_{00} \cdot 2^{-\frac{3\beta_2}{\beta_1^2}} \cdot e^{\frac{3\alpha_{10}}{\gamma_1 \alpha_{00}}} \quad (17)$$

and

$$Z \equiv \frac{\langle \bar{\psi} \psi \rangle \cdot L^3}{\alpha_{00}} \quad (18)$$

From RGE we know that $\alpha_{11} = \gamma_1 \alpha_{00}$. The constant α_{10} is proportional to $\alpha_{00} C_F$ [9b]. Due to asymptotic freedom β_1 and β_2 are positive and

the function $X(Z)$ develops a minimum, becoming stable against changes in Z at

$$Z_{\min} = \left(\frac{3 \beta_1^2}{\beta_1 \gamma_1 + 3 \beta_2} \right)^{\frac{\gamma_1}{\beta_1}} \quad (19)$$

with the value

$$X_{\min} = K \cdot \left(\frac{3 e \beta_1^2}{\beta_1 \gamma_1 + 3 \beta_2} \right)^{\frac{\beta_1 \gamma_1 + 3 \beta_2}{\beta_1^2}} \quad (20)$$

To calculate the constant K we must perform a two loop calculation [9b]. We observe however that the combination $3\alpha_{10}/\gamma_1 \cdot \alpha_{00}$ does not depend on the fermionic representations, so by taking ratios of $X_{\min}$ for two different representations we eliminate the unknown constant α_{10} . Note that the variable Z is different for every representation (rel.(18)) and to get ratios of X 's at the same value of L we need to know α_{10} .

So in this work we can only give the ratios, for different representations, of the $X_{\min}$. In table 1, we give the ratios of the $X_{\min}$ values for the sextet, octet, decuplet to the triplet. In fig. 2 assuming $\frac{\alpha_{10}}{\alpha_{00} \cdot C_F} = .005$, we give the ratios $X_6/X_3, X_8/X_3$ and X_{10}/X_3 as functions of $\frac{\alpha}{\pi}$ or the corresponding Z 's. In fig. 3 we plot X_3, X_6, X_8, X_{10} for $\alpha_{10} = .005 \alpha_{00} C_F$.

Finally we observe that for small values of $Z_{\min}$ the coupling constant is given

$$\left. \frac{\bar{\alpha}(1/L)}{\pi} \right|_{\min} \cong \frac{1}{\beta_1} \cdot Z_{\min}^{\frac{\beta_1}{\gamma_1}} = \frac{3 \beta_1}{\beta_1 \gamma_1 + 3 \beta_2}$$

or

$$\alpha_{eff.} \equiv C_F \cdot \frac{\bar{a}(1/L)}{\pi} \cong 2 - \frac{4 \beta_2}{\beta_1 C_F + 2 \beta_2} \quad (21)$$

that is, weak dependence on the fermionic representations. In table 2 we give a few values of $\alpha_{eff.}$. This result is consistent with the Casimir scaling idea [17,18]. The above considerations for evaluating $\langle \bar{\psi}\psi \rangle$ do not take into account finite volume corrections. These may alter drastically the values of the ratios as one can observe in fig.2. It would be very important to prove sum rules which connect $\langle \bar{\psi}\psi \rangle(L)$ with $\langle \bar{\psi}\psi \rangle(\infty)$, similar to those, which hold in the case of the mass gap [19]. These sum rules would give us some information about the error due to finite size effects and show how reliable are our results.

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Table 1

X_{10}/X_3	X_8/X_3	X_6/X_3
4.83	3.35	2.59

Ratios of minimum values of X's for $\alpha_{10}=0.005 \alpha_{00} \cdot C_F$

Table 2

	<u>3</u>	<u>6</u>	<u>8</u>	<u>10</u>
α_{eff}	1.26	1.34	1.35	1.02

Values of $\alpha_{eff}=C_F \cdot \frac{\alpha}{\pi} \Big|_{min}$ for different representations

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Figure Captions

Fig.1 : Curve a,the exact leading $1/N$,G-N condensate as a function of Z .
Curve b,the truncated leading $1/N$,leading small Z behaviour of
the same quantity.

Fig.2 : Ratios of (a) X_{10}/X_3 , (b) X_8/X_3 , (c) X_6/X_3 for $\alpha_{10}=0.005 \alpha_{00} \cdot C_F$,
as functions of $Z=\langle \bar{\psi}\psi \rangle \cdot L^3$

Fig.3 : (a) X_{10} , (b) X_8 , (c) X_6 and (d) X_3 as functions of Z , for $\alpha_{10}=0.005 \alpha_{00} \cdot C_F$

Fig. 1

Fig. 2

Fig. 3