Testing the single-quark radiation hypothesis

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In the quark model, radiative transitions between hadronic states are assumed to involve a single quark and not the hadron as a whole. We describe a possible test of this hypothesis using $p\bar{p} \rightarrow \chi_2 \rightarrow \psi\gamma$ with $\psi \rightarrow e^+e^-$.

In the quark model, hadrons are composed of quarks with electric and color charges, one-half unit spin, and an effective mass. When a hadron emits or absorbs a photon we assume that one of the constituent quarks has played an active role while the others are spectators.¹ Numerous consequences² follow from this simple hypothesis concerning magnetic moments, photoproductions, and radiative hadronic decays.

A direct test of the single-quark radiation (SQR) hypothesis would follow from the observation of transition multipoles which could not be present through SQR. Such tests have been proposed³ within the Melosh-transformation formalism for $A_2 \rightarrow \rho\gamma$ and $f \rightarrow \omega\gamma$ meson decays. Unfortunately, these transitions are difficult to observe.

Heavy-meson transitions offer certain advantages. First, these states are quite narrow below heavy-flavor threshold, so that the radiative processes compete successfully. Second, the dynamics of heavy-quark radiative transitions are easier to deal with since the nonrelativistic limit is a good first approximation.⁴

We consider a meson of total angular momentum j' which decays electromagnetically to a state of angular momentum j . The angular momentum j_γ carried off by the photon must then be bounded by

$$|j' - j| \leq j_\gamma \leq j' + j \quad (1)$$

from angular momentum conservation. In the SQR picture only one quark is assumed to radiate the photon as it makes a transition to a lower energy level in "orbit" around the other quark analogous to the way an electron makes a transition in an atom. The range of photon angular momenta with the SQR assumption can be less than the general range of Eq. (1).

$\chi_2 \rightarrow \psi\gamma$ MULTIPOLES

An example of a transition in which the SQR hypothesis can be tested is the decay of the 2^{++} charmonium state

$\chi_2 \rightarrow \psi\gamma$. Here the χ_2 state has total angular momentum $j' = 2$, and, in the nonrelativistic limit, spin $s' = 1$ and orbital angular momentum $l' = 1$. The corresponding angular momentum quantum numbers of the ψ are $j = 1$, $s = 1$, and $l = 0$.

From Eq. (1) the allowed photon angular momenta are $j_\gamma = 1, 2, 3$. Since the initial and final meson states have opposite parity, these photon states correspond to $E1$ (electric dipole), $M2$ (magnetic quadrupole), and $E3$ (electric octupole) radiation. In the SQR approximation we only consider the total angular momentum of the active quark in orbit around the other to determine the allowed j_γ 's. In this case $j' = \frac{3}{2}$ and $j = \frac{1}{2}$ ($j' = \frac{1}{2}$ is excluded since $j' + s_{\text{spectator}} = 2$) and from Eq. (1) we have $j_\gamma = 1$ and 2, which excludes the $E3$ multipole amplitude.

Measurement of the $E3$ transition amplitude in this decay provides a test of the SQR picture since $E3$ decay is forbidden independent of the type of quark involved. This test has not been possible in light-quark systems such as $A_2 \rightarrow \rho\gamma$ and $f \rightarrow \omega\gamma$ because the branching fractions are quite small. The $\chi_2 \rightarrow \psi\gamma$ process is also difficult to investigate in e^+e^- collisions since an added cascade process⁴ $\psi' \rightarrow \chi_2\gamma$ is necessarily present.

$\chi_2 \rightarrow \psi\gamma$ FROM $p\bar{p}$ COLLISIONS

The χ_2 state can be formed exclusively in a $p\bar{p}$ collision experiment^{5,6} in the chain

$$p\bar{p} \rightarrow \chi_2 \rightarrow \psi\gamma \quad (2)$$

$$\quad \quad \quad \downarrow$$

$$\quad \quad \quad e^+e^-$$

The multipole structure of the χ_2 decay can then be determined from the angular distributions of the final products.

Following the notation of Martin, Olsson, and Stirling^{6,7} the joint angular distribution is

$$W(\theta; \theta', \phi') = \sum_{\lambda} B_{|\lambda|}^2 \sum_{\nu, \nu' = -j, \mu = \pm 1}^j d_{\lambda\nu}^j(\theta) d_{\lambda\nu'}^j(\theta) A_{|\nu|}^j A_{|\nu'|}^j \rho^{\sigma'}(\theta', \phi') \quad (3)$$

where the ψ helicity $\sigma \equiv \nu - \mu$ and $\sigma' \equiv \nu' - \mu$, and the density matrix for the ψ decay into an unpolarized e^+ and e^- is

$$\rho^{\sigma'\sigma}(\theta', \phi') = \sum_{\kappa=\pm 1} \mathcal{D}_{\sigma'\kappa}^1(\phi', \theta', -\phi') \mathcal{D}_{\sigma\kappa}^{1*}(\phi', \theta', -\phi') . \quad (4)$$

The angles and helicities are indicated in Fig. 1; θ', ϕ' specify the $\psi \rightarrow e^+ e^-$ decay in the ψ rest frame with the z axis aligned with the ψ direction in the χ rest frame. Direct expansion of Eq. (3) yields

$$W(\theta, \theta', \phi') \equiv \frac{64\pi^2}{15} (B_0^2 + 2B_1^2) \hat{W}(\theta, \theta', \phi') , \quad (5)$$

with

$$\begin{aligned} \frac{64\pi^2}{15} \hat{W}(\theta, \theta', \phi') = & K_1 + K_2 \cos^2 \theta + K_3 \cos^4 \theta + (K_4 + K_5 \cos^2 \theta + K_6 \cos^4 \theta) \cos^2 \theta' \\ & + (K_7 + K_8 \cos^2 \theta + K_9 \cos^4 \theta) \sin^2 \theta' \cos^2 \phi' + (K_{10} + K_{11} \cos^2 \theta) \sin 2\theta \sin 2\theta' \cos \phi' . \end{aligned} \quad (6)$$

The initial $p\bar{p}$ state can have either helicity zero or one. At sufficiently high energies the helicity-one state should dominate according to the principle of hadronic helicity conservation.⁸ This result has been claimed⁸ to apply generally when quark-mass effects are negligible since vector-gluon radiation leaves the helicity of a massless quark unchanged. It remains an open question whether this limit is relevant for charmonium production by $p\bar{p}$.

The $\chi_2 \rightarrow \psi\gamma$ transition involves three helicity amplitudes A_0, A_1 , and A_2 , which are normalized to

$$A_0^2 + A_1^2 + A_2^2 = 1 . \quad (7)$$

The observables K_i of Eq. (6) can be expressed in terms of these helicity amplitudes as

$$\begin{aligned} 8K_1 &= 2A_0^2 + 3A_2^2 - R(2A_0^2 - 4A_1^2 + A_2^2) , \\ \frac{4}{3}K_2 &= -2A_0^2 + 4A_1^2 - A_2^2 + R(4A_0^2 - 6A_1^2 + A_2^2) , \\ 8K_3 &= (6A_0^2 - 8A_1^2 + A_2^2)(3 - 5R) , \\ 8K_4 &= 2A_0^2 + 3A_2^2 - R(2A_0^2 + 4A_1^2 + A_2^2) , \\ \frac{4}{3}K_5 &= -2A_0^2 - 4A_1^2 - A_2^2 + R(4A_0^2 + 6A_1^2 + A_2^2) , \\ 8K_6 &= (6A_0^2 + 8A_1^2 + A_2^2)(3 - 5R) , \\ 4K_7 &= \sqrt{6}(R - 1)A_0A_2 , \\ 4K_8 &= \sqrt{6}(4 - 6R)A_0A_2 , \\ 4K_9 &= \sqrt{6}(5R - 3)A_0A_2 , \\ (4/\sqrt{3})K_{10} &= A_0A_1 + \sqrt{\frac{3}{2}}A_1A_2 - R(2A_0A_1 + \sqrt{\frac{3}{2}}A_1A_2) , \\ 4\sqrt{3}K_{11} &= (5R - 3)(3A_0A_1 + \sqrt{\frac{3}{2}}A_1A_2) . \end{aligned} \quad (8)$$

FIG. 1. Helicities and angles for the process $p\bar{p} \rightarrow \chi_2 \rightarrow \psi\gamma \rightarrow e^+e^-\gamma$.

FIG. 2. Observables K_1 to K_5 as a function of the electric octupole amplitude a_3 near $a_3 = 0$ for different helicity admixtures of χ_2 production [with R specified in Eq. (9)]. For illustration we have arbitrarily taken $a_2 = 0.5$.

The constant R measures the fractional contribution of the helicity-one initial production amplitude

$$R = \frac{2B_1^2}{B_0^2 + 2B_1^2} \quad (9)$$

The factor of 2 appears because helicity ± 1 contributes equally. Integration over two angles gives the angular distributions discussed earlier.⁶ The total rate is normalized to unity

$$\int d\Omega d\Omega' \tilde{W}(\theta, \theta', \phi') = 1 \quad (10)$$

As alluded to earlier, the transition multipole amplitudes a_j are linearly related to the decay helicity amplitudes. The general relation⁴ is

$$A_\nu = \sum_k a_k \left(\frac{2k+1}{2j'+1} \right)^{1/2} \langle k, 1; 1, \nu-1 | j', \nu \rangle \quad (11)$$

where the transition multipoles are normalized to

$$\sum_k a_k^2 = 1 \quad (12)$$

If we write the three-vector $\mathbf{A} = (A_0, A_1, A_2)$ and the three-vector $\mathbf{a} = (a_1, a_2, a_3)$, where a_1 , a_2 , and a_3 are the multipole amplitudes corresponding to $E1$, $M2$, and $E3$ transitions, then the 3×3 orthogonal matrix relating the vectors is from Eq. (11)

$$\mathbf{A} = \tilde{\mathbf{R}} \cdot \mathbf{a} \quad (13)$$

$$\tilde{\mathbf{R}} = \begin{pmatrix} \sqrt{\frac{1}{10}} & \sqrt{\frac{1}{2}} & \sqrt{\frac{2}{5}} \\ \sqrt{\frac{3}{10}} & \sqrt{\frac{1}{6}} & -\sqrt{\frac{8}{15}} \\ \sqrt{\frac{3}{5}} & -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{15}} \end{pmatrix} \quad (14)$$

The inverse matrix is the transpose of Eq. (14).

DETERMINING THE $E3$ MULTIPOLE

The observables $\{K_i; i=1, 11\}$ are all generally expressible in terms of the three real constants R , a_2 , and a_3 . The electric dipole a_1 is fixed by the normalization

$a_1^2 = 1 - a_2^2 - a_3^2$ and by convention is taken to be positive. An optimal analysis would search this three-parameter space for solutions which best account for the observed K_i 's.

Although the $E1$ transition is dominant, there is evidence⁹ for a large $M2$ multipole from the Crystal Ball experiment. In addition, preliminary results⁵ from the CERN $\bar{p}p$ experiment also indicate⁶ a large $M2$ multipole for $\chi_2 \rightarrow \psi\gamma$ decay. Measurement of some of the K_i observables may be enough to determine the multipole and helicity structure of the process of Eq. (2). As pointed out previously,⁶ the single-angle distributions in θ and θ' suffice to fix R and a_2 , assuming that $a_3=0$.

To get a feeling for the a_3 dependence of the observables we assume for example $a_2 = \frac{1}{2}$ and plot K_1 through K_5 near $a_3=0$, for $R=0, \frac{1}{2}$, and 1. In Fig. 2 we see that in the case of helicity-zero absorption the observables K_2 and K_3 vary rapidly with a_3 . For $R = \frac{1}{2}$ the dependence on a_3 is less dramatic, but for pure helicity one ($R=1$), again K_2 and K_3 are sensitive to the electric octupole moment. Referring again to Eq. (6), we see the importance of an accurate measurement of the photon angular distribution over a wide range of $\cos\theta$ to distinguish between $\cos^2\theta$ and $\cos^4\theta$ dependences. As we see from Fig. 2, the observable K_5 is quite sensitive to the initial-helicity parameter R . Finally, we note that after integration over θ the remaining angular distribution in θ' and ϕ' is independent of R ; it only depends on the decay multipole amplitudes.

We have seen that the SQR assumption of the quark model can be directly tested by observing the angular distributions of $\chi_2 \rightarrow \psi\gamma \rightarrow e^+e^-\gamma$ from direct production from $\bar{p}p$ collisions. The fractional angular distributions $\tilde{W}(\theta, \theta', \phi')$ are fixed in terms of three parameters, two of which describe the multipolarity of $\chi_2 \rightarrow \psi\gamma$ decay. The absence of one of these multipoles ($E3$) is required by the SQR hypothesis.

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