Testing the single-quark radiation hypothesis

M. G. Olsson and C. J. Suchyta III
Physics Department, University of Wisconsin, Madison, Wisconsin 53706

A. D. Martin
Physics Department, University of Durham, Durham, England

W. J. Stirling
Theory Division, CERN, CH-1211 Genève 23, Switzerland
(Received 12 October 1984)

In the quark model, radiative transitions between hadronic states are assumed to involve a single quark and not the hadron as a whole. We describe a possible test of this hypothesis using \( \bar{p}p \to \chi_{2} \to \psi \gamma \) with \( \psi \to e^{+}e^{-} \).

In the quark model, hadrons are composed of quarks with electric and color charges, one-half unit spin, and an effective mass. When a hadron emits or absorbs a photon we assume that one of the constituent quarks has played an active role while the others are spectators. Numerous consequences follow from this simple hypothesis concerning magnetic moments, photoproductions, and radiative hadronic decays.

A direct test of the single-quark radiation (SQR) hypothesis would follow from the observation of transition multipoles which could not be present through SQR. Such tests have been proposed within the Melosh-transformation formalism for \( A_{2} \to \rho \gamma \) and \( f \to \omega \gamma \) meson decays. Unfortunately, these transitions are difficult to observe.

Heavy-meson transitions offer certain advantages. First, these states are quite narrow below heavy-flavor threshold, so that the radiative processes compete successfully. Second, the dynamics of heavy-quark radiative transitions are easier to deal with since the nonrelativistic limit is a good first approximation.

We consider a meson of total angular momentum \( j' \) which decays electromagnetically to a state of angular momentum \( j \). The angular momentum \( j_{r} \) carried off by the photon must then be bounded by

\[
|j' - j| \leq j_{r} \leq j' + j
\]

from angular momentum conservation. In the SQR picture only one quark is assumed to radiate the photon as it makes a transition to a lower energy level in "orbit" around the other quark analogous to the way an electron makes a transition in an atom. The range of photon angular momenta with the SQR assumption can be less than the general range of Eq. (1).

\[ \chi_{2} \to \psi \gamma \]

MULTIPOLES

An example of a transition in which the SQR hypothesis can be tested is the decay of the \( 2^{++} \) charmonium state \( \chi_{2} \to \psi \gamma \). Here the \( \chi_{2} \) state has total angular momentum \( j' = 2 \), and, in the nonrelativistic limit, spin \( s' = 1 \) and orbital angular momentum \( l' = 1 \). The corresponding angular momentum quantum numbers of the \( \psi \) are \( j = 1 \), \( s = 1 \), and \( l = 0 \).

From Eq. (1) the allowed photon angular momenta are \( j_{r} = 1, 2, 3 \). Since the initial and final meson states have opposite parity, these photon states correspond to \( E1 \) (electric dipole), \( M2 \) (magnetic quadrupole), and \( E3 \) (electric octupole) radiation. In the SQR approximation we only consider the total angular momentum of the active quark in orbit around the other to determine the allowed \( j_{r} \)'s. In this case \( j' = \frac{3}{2} \) and \( j = \frac{1}{2} \) (\( j' = \frac{1}{2} \) is excluded since \( j' + s_{\text{spectator}} = 2 \)) and from Eq. (1) we have \( j_{r} = 1 \) and 2, which excludes the \( E3 \) multipole amplitude.

Measurement of the \( E3 \) transition amplitude in this decay provides a test of the SQR picture since \( E3 \) decay is forbidden independent of the type of quark involved. This test has not been possible in light-quark systems such as \( A_{2} \to \rho \gamma \) and \( f \to \omega \gamma \) because the branching fractions are quite small. The \( \chi_{2} \to \psi \gamma \) process is also difficult to investigate in \( e^{+}e^{-} \) collisions since an added cascade process \( \psi \to \chi_{2} \gamma \) is necessarily present.

\[ \chi_{2} \to \psi \gamma \text{ FROM } \bar{p}p \text{ COLLISIONS} \]

The \( \chi_{2} \) state can be formed exclusively in a \( \bar{p}p \) collision experiment in the chain

\[
\bar{p}p \to \chi_{2} \to \psi \gamma \quad ,
\]

The multipole structure of the \( \chi_{2} \) decay can then be determined from the angular distributions of the final products.

Following the notation of Martin, Olsson, and Stirling, the joint angular distribution is

\[
W(\theta, \theta', \phi') = \sum_{J} B_{J}^{2} \sum_{\ell', \ell} \frac{\sum d_{J}^{\ell} (\theta) d_{J}^{\ell'} (\theta') A_{J}^{\ell} A_{J}^{\ell'} \rho^{\sigma} (\theta', \phi')}{}
\]

where the \( \psi \) helicity \( \sigma = \nu - \mu \) and \( \sigma' = \nu' - \mu' \), and the density matrix for the \( \psi \) decay into an unpolarized \( e^{+} \) and \( e^{-} \) is

\[ W_{0} \]

©1985 The American Physical Society
\[ \rho^{\sigma}\delta^{\sigma}(\theta', \phi') = \sum_{n = \pm 1} \phi_n^{\sigma}(\phi', \theta', -\phi') \phi_n^{\sigma}(\phi, \theta', -\phi') . \] 

The angles and helicities are indicated in Fig. 1; \( \theta', \phi' \) specify the \( \psi \to e^+e^- \) decay in the \( \psi \) rest frame with the \( z \) axis aligned with the \( \psi \) direction in the \( x \) rest frame. Direct expansion of Eq. (3) yields

\[ W(\theta, \theta', \phi') = \frac{64\pi^2}{15} (B_0^2 + 2B_1^2) \tilde{W}(\theta, \theta', \phi') , \] 

with

\[ \frac{64\pi^2}{15} \tilde{W}(\theta, \theta', \phi') = K_1 + K_3 \cos^2 \theta + K_5 \cos^4 \theta + (K_4 + K_6 \cos^2 \theta)(K_0 + K_1 \cos^2 \theta) \sin^2 \theta \cos^2 \phi' + (K_9 + K_{11} \cos^2 \theta) \sin 2\theta \sin 2\phi \cos \phi' . \] 

The initial \( \bar{p}p \) state can have either helicity zero or one. At sufficiently high energies the helicity-one state should dominate according to the principle of hadronic helicity conservation. This result has been claimed to apply generally when quark-mass effects are negligible since vector-gluon radiation leaves the helicity of a massless quark unchanged. It remains an open question whether this limit is relevant for charmonium production by \( \bar{p}p \).

The \( \chi_c \to \psi \gamma \) transition involves three helicity amplitudes \( A_0, A_1, \) and \( A_2 \), which are normalized to

\[ A_0^2 + A_1^2 + A_2^2 = 1 . \] 

The observables \( K_i \) of Eq. (6) can be expressed in terms of these helicity amplitudes as

\[ 8K_1 = 2A_0^2 + 3A_0^2 - R (2A_0^2 - 4A_1^2 + A_2^2) , \]
\[ \frac{1}{2} K_2 = -2A_0^2 + 4A_1^2 - A_2^2 + R (4A_0^2 - 6A_1^2 + A_2^2) , \]
\[ 8K_3 = (6A_0^2 - 8A_1^2 + A_2^2)(3 - 5R) , \]
\[ 8K_4 = 2A_0^2 + 3A_0^2 - R (2A_0^2 + 4A_1^2 + A_2^2) , \]
\[ \frac{1}{2} K_5 = -2A_0^2 - 4A_1^2 - A_2^2 + R (4A_0^2 + 6A_1^2 + A_2^2) , \]
\[ 8K_6 = (6A_0^2 + 8A_1^2 + A_2^2)(3 - 5R) , \]
\[ 4K_7 = \sqrt{6}(R - 1)A_0A_1 , \]
\[ 4K_8 = \sqrt{6}(4 - 6R)A_0A_2 , \]
\[ 4K_9 = \sqrt{6}(5R - 3)A_0A_2 , \]
\[ (4/\sqrt{3})K_{10} = A_0A_1 + \sqrt{2}A_1A_2 - R (2A_0A_1 + \sqrt{2}A_1A_2) , \]
\[ 4\sqrt{3}K_{11} = (5R - 3)(3A_0A_1 + \sqrt{2}A_1A_2) . \]

FIG. 1. Helicities and angles for the process \( \bar{p}p \to \chi_c \to \psi \gamma \to e^+e^- \gamma \).
The constant $R$ measures the fractional contribution of the helicity-one initial production amplitude

$$R = \frac{2B_1^2}{B_0^2 + 2B_1^2} .$$  \hspace{1cm} (9)

The factor of 2 appears because helicity $\pm 1$ contributes equally. Integration over two angles gives the angular distributions discussed earlier.\footnote{F. E. Close, \textit{An Introduction to Quarks and Partons} (Academic, London, 1979); D. H"{u}fflin, in \textit{Proceedings of the 1983 International Symposium on Lepton and Photon Interactions at High Energies}, Ithaca, New York, edited by D. G. Cassel and D. L. Kreinick (Newman Laboratory of Nuclear Studies, Cornell University, Ithaca, 1984).} The total rate is normalized to unity

$$\int d\Omega \, d\Omega' \, W(\theta, \theta', \phi') = 1 .$$  \hspace{1cm} (10)

As alluded to earlier, the transition multipole amplitudes $a_j$ are linearly related to the decay helicity amplitudes. The general relation\footnote{F. J. Gilman and I. Karliner, Phys. Lett. \textbf{46B}, 426 (1973); Phys. Rev. D \textbf{10}, 2194 (1974).} is

$$A_j = \sum_k a_k \left( \frac{2k + 1}{2j' + 1} \right)^{1/2} \langle k, 1, 1, \nu - 1 | j', \nu \rangle ,$$  \hspace{1cm} (11)

where the transition multipoles are normalized to

$$\sum_k a_k^2 = 1 .$$  \hspace{1cm} (12)

If we write the three-vector $A = (A_0, A_1, A_2)$ and the three-vector $a = (a_1, a_2, a_3)$, where $a_1$, $a_2$, and $a_3$ are the multipole amplitudes corresponding to $E1$, $M2$, and $E3$ transitions, then the $3 \times 3$ orthogonal matrix relating the vectors is from Eq. (11)

$$A = \mathbf{R} \cdot a ,$$  \hspace{1cm} (13)

$$\mathbf{R} = \begin{pmatrix} \sqrt{\frac{10}{15}} & \sqrt{\frac{1}{2}} & \sqrt{\frac{2}{15}} \\ \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{15}} \\ \sqrt{\frac{2}{15}} & -\sqrt{\frac{1}{2}} & \sqrt{\frac{10}{15}} \end{pmatrix} .$$  \hspace{1cm} (14)

The inverse matrix is the transpose of Eq. (14).

\section*{DETERMINING THE E3 MULTIPOLe}

The observables $\{ K_i; i = 1, 11 \}$ are all generally expressible in terms of the three real constants $R$, $a_2$, and $a_3$. The electric dipole $a_1$ is fixed by the normalization $a_1^2 = 1 - a_2^2 - a_3^2$ and by convention is taken to be positive.

An optimal analysis would search this three-parameter space for solutions which best account for the observed $K_i$'s.


To get a feeling for the $a_3$ dependence of the observables we assume for example $a_2 = \frac{1}{2}$ and plot $K_1$ through $K_5$ near $a_3 = 0$, for $R = 0$, $\frac{1}{2}$, and $1$. In Fig. 2 we see that in the case of helicity-zero absorption the observables $K_2$ and $K_3$ vary rapidly with $a_3$. For $R = \frac{1}{2}$ the dependence on $a_3$ is less dramatic, but for pure helicity one ($R = 1$), again $K_2$ and $K_3$ are sensitive to the electric octupole moment. Referring again to Eq. (6), we see the importance of an accurate measurement of the photon angular distribution over a wide range of $\cos \theta$ to distinguish between $\cos \theta$ and $\cos^3 \theta$ dependences. As we see from Fig. 2, the observable $K_5$ is quite sensitive to the initial-helicity parameter $R$. Finally, we note that after integration over $\theta$ the remaining angular distribution in $\theta'$ and $\phi'$ is independent of $R$; it only depends on the decay multipole amplitudes.

We have seen that the SQR assumption of the quark model can be directly tested by observing the angular distributions of $\chi_c \to \psi \gamma \to e^+ e^- \gamma$ from direct production from $\bar{p}p$ collisions. The fractional angular distributions $W(\theta, \theta', \phi')$ are fixed in terms of three parameters, two of which describe the multipolarity of $\chi_c \to \psi \gamma$ decay. The absence of one of these multipoles ($E3$) is required by the SQR hypothesis.

This research was supported in part by the University of Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation, and in part by the Department of Energy under Contract No. DE-AC02-76ER00881.