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CHIRAL ANOMALY-FREE SYMMETRY GROUPS IN
EXTENDED SUPERGRAVITIES

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ABSTRACT

Anomaly-free subgroups of the local chiral symmetry groups of extended supergravities are classified. Their chiral part is at most $SU(2) \times U(1)$. Possible implications for superunification are discussed, in particular identification of the electroweak gauge group.

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Extended supergravity theories, and in particular $N=8$ supergravity ¹⁾, are often considered as possible candidates for unified theories of all fundamental interactions. One of the main reason for this belief is the appearance in these theories of chiral local symmetries. All (ungauged) N -extended supergravities ($N \geq 4$) possess essentially the same symmetry structure ^{1,2)}. They are invariant under the global transformations of a non compact group G_D ($G_D = SU(4) \times SU(1,1)$, $SU(5,1)$, $SO^*(12)$, $E_{7,7}$ for $N = 4, 5, 6, 8$ respectively). G_D is the duality group of vector field strengths and scalar fields are coordinates of the manifold G_D/G , G being the maximal compact subgroup of G_D ($G = U(N)$ for $N = 4, 5, 6$ and $G = SU(8)$ for $N = 8$). Supergravities can be formulated in such a way that G_D is a linear symmetry of the Lagrangian. This is achieved by adding scalar degrees of freedom, compensated by gauge invariance under G . The bosonic sector is then invariant under $G_D(\text{global}) \times G(\text{local})$. Fermions are G_D -inert. They however transform according to complex representations of G which is then a chiral local symmetry of the theory. Further gauging of supergravity ³⁾ will break G_D but preserve G . Choosing the non physical scalars to be zero corresponds to a gauge choice for G , the residual symmetry being $G(\text{global})$. In that gauge, G_D is non linearly realized. The gauge fields of G are simply auxiliary and can be expressed in terms of physical fields and their derivatives.

Attempts to embed a unified model of strong and electroweak interactions into the local symmetry G have been made ⁴⁾, based on the conjecture ¹⁾ that its gauge fields become dynamically propagating. Fermions and scalars were also bound states produced by dynamical effects. The choice of the gauge group and/or the choice of the spin $1/2$ representations were dictated by the necessary absence of anomalies in the unified gauge theory extracted from supergravity.

It has been however observed recently ⁵⁾ that σ -models formulated with local "hidden" symmetry G are anomalous when coupled to fermions in non anomaly-free representations. This is precisely the situation which occurs in extended supergravities, where all local chiral symmetries are anomalous ⁶⁾. This observation strongly suggests that the dynamical generation of propagating gauge fields may be in trouble ⁶⁾. A recent analysis ⁷⁾ of anomalies appearing in non linear models confirms in fact that dynamical generation of gauge fields should not occur in the non compact cases of interest in supergravity, at least in formulations with complete local symmetry G . Fermions of N -extended supergravities are however anomaly-free when G is restricted to suitable subgroups, and one can argue ⁷⁾ that dynamical generation could happen for gauge fields corresponding to anomaly-free symmetries. Since chiral fermions are a fundamental ingredient of superunified models, the possibility of finding anomaly-free subgroups of G which preserve chirality could be relevant for the superunification scheme.

The aim of this letter is to show that non-abelian, anomaly-free, chiral symmetries G_0 exist in N-extended supergravities with $N = 5, 6$ or 8 , and that G_0 is either $SU(2) \times U(1)$, or $SU(2) \times SU(2) \times U(1)$, one of the $SU(2)$ being non chiral.

Before proceeding to this classification, let us recall the transformation properties of fermions in supergravity theories and some simple group-theoretical results on the related anomalies.

The $U(N)$ (or $SU(8)$ for $N = 8$) representations of the states belonging to the supergravity multiplet are easily obtained. Supersymmetric charges are assigned to representation $\bar{N}_{-q}$ of $U(N)$, q denoting the $U(1)$ eigenvalue ($q = 0$ for $N = 8$). Then, left-handed gravitinos (with helicity $\lambda = -3/2$) transform according to

$$\bar{N}_{-q}, \quad (1)$$

while the $\lambda = -1/2$ states are in representations

$$\begin{aligned} N=4 &: \bar{4}_{-3q} \\ N=5 &: \bar{10}_{-3q} + \bar{1}_{5q} \\ N=6 &: \bar{20}_{-3q} + \bar{6}_{5q} \\ N=8 &: \bar{56} \end{aligned} \quad (2)$$

All these representations are complex, apart from the pseudoreal multiplet $\underline{20}$ of $SU(6)$. Taking $U(1)$ charges into account, all fermions are then chiral under the local symmetry G of supergravity.

The anomaly for a set of left-handed fermions with helicity λ , transforming according to the representation R of G , is proportional to ⁸⁾

$$n_\lambda \text{Tr}(\{T_R^a, T_R^b\} T_R^c) = n_\lambda d^{abc} A(R)$$

where T_R^a are generators of G for the representation R . The number n_λ is 1 for helicity $\lambda = \pm 1/2$, and 3 for $\lambda = \pm 3/2$ states ^{6,9)}. For semi-simple groups, all the dependence on R lies in $A(R)$. For simple groups, d^{abc} is non zero only if $G = SU(N)$, $N \geq 3$ ^{*}, since only these groups possess a third order Casimir invariant ⁸⁾. If G is not semi-simple, we will have three kinds of contributions to the anomaly. Apart from the case where all three generators are non-abelian (NNN-anomaly), non trivial traces will be obtained with one (NNA-anomaly) or three (AAA-anomaly) abelian generators. It will be convenient to normalize $SU(N)$ -generators in such a way that

^{*}) And $SO(6) \sim SU(4)$.

$$A(\underline{N}) = 1, \quad (N \geq 3), \quad (3)$$

which means that

$$\text{Tr} \left(T_{\underline{N}}^a T_{\underline{N}}^b \right) = \frac{1}{2} k \delta^{ab} \quad (4)$$

with

$$k = \left(\frac{2N(N-1)}{(N-2)^2} \right)^{1/3}. \quad (5)$$

The group theoretical factors $A(R)$ are then easily calculated ⁸⁾, using the transformations properties (1) and (2). The $SU(N)$ anomalies (NNN-anomalies) are

$$\begin{aligned} N=4: \lambda=-3/2: 3A(\underline{4}) &= -3, \quad \lambda=-1/2: A(\underline{4}) = 1, \\ N=5: \lambda=-3/2: 3A(\underline{5}) &= -3, \quad \lambda=-1/2: A(\underline{10}+\underline{1}) = 1, \\ N=6: \lambda=-3/2: 3A(\underline{6}) &= -3, \quad \lambda=-1/2: A(\underline{20}+\underline{6}) = -1, \\ N=8: \lambda=-3/2: 3A(\underline{8}) &= -3, \quad \lambda=-1/2: A(\underline{56}) = -5. \end{aligned} \quad (6)$$

There is no anomaly cancellation between the $\lambda = -3/2$ and $\lambda = -1/2$ sectors.

Let us remark that $N = 4$ Yang-Mills multiplets, with scalars living on the coset $SO(m,6)/SO(m) \times SU(4)$ ¹⁰⁾ (m is the number of Yang-Mills multiplets) cannot ensure the cancellation: their anomaly is $A = -m$. Truncation of $N = 8$ into $N = 4$ corresponds to $m = 6$, as can be checked from eq. (6). Mixed anomalies (NNA) and $U(1)$ anomalies (AAA) for the left-handed gravitinos of $N=4,5,6$ theories are the following:

$$\begin{aligned} \text{NNA: } A_{3/2}^{\text{mixed}} &= -3kq, \\ \text{AAA: } A_{3/2}^{U(1)} &= -3Nq^3, \end{aligned} \quad (7)$$

where k is given by eq. (5). The same quantities for the left-handed spin $1/2$ fields read:

$$\begin{aligned} N=4: A_{1/2}^{\text{mixed}} &= -3kq \\ A_{1/2}^{U(1)} &= -216 q^3 \\ N=5: A_{1/2}^{\text{mixed}} &= -9kq \\ A_{1/2}^{U(1)} &= -290 q^3 \\ N=6: A_{1/2}^{\text{mixed}} &= -13kq \\ A_{1/2}^{U(1)} &= 420 q^3 \end{aligned} \quad (8)$$

For both NNA and AAA anomalies, no cancellation occurs between gravitinos and spin 1/2 states.

We now want to look for subgroups G_0 of the chiral symmetry groups of N-extended supergravities for which there is no anomaly induced by the fermionic sector. As long as G_0 is a subgroup of the semi-simple part $SU(N)$ of G , one can simply prove that the absence of anomaly cannot result from cancellation of the gravitino contribution by spin 1/2 fields. To see this, consider a generator T_R of G_0 . T_R is a linear combination of generators of $SU(N)$,

$$T_R = \sum_a \omega_a T_R^a, \quad (9)$$

the coefficients ω_a being the same for all representations R of G . The anomaly associated to T_R is

$$2 \text{Tr}(T_R^3) = \left(\sum_{a,b,c} d^{abc} \omega_a \omega_b \omega_c \right) A(R), \quad (10)$$

where $A(R)$ is the $SU(N)$ -anomaly. Since d^{abc} and ω_a do not depend on R , the ratio of the anomalies of $\lambda = -3/2$ and $\lambda = -1/2$ states will not be modified, except if

$$\sum_{a,b,c} d^{abc} \omega_a \omega_b \omega_c = 0. \quad (11)$$

In this case, T_R does not generate any anomaly on all representations of G ("anomaly-free generator"). If however G is not semi-simple, the tensor d^{abc} is not defined independently of R for abelian generators and we find that anomaly cancellation between gravitinos and spin 1/2 fermions is in general possible.

This discussion implies that the non-abelian part of G_0 is an anomaly-free subgroup of $SU(N)$. Since complex representations with dimension up to 8 give always anomalies, the non abelian part of G_0 will not be chiral. It will be a subgroup of $SO(N)$ or $Sp(2[N/2])$. Chirality can be obtained only if G_0 is not semi-simple, with suitable choices of $U(1)$ assignments.

Let us first consider the anomaly-free chiral subgroups G_0 , contained in $N = 8$ supergravity, for which the local symmetry $SU(8)$ is simple. Since anomalies should vanish for spin 3/2 and 1/2 separately, we need only to consider their cancellation on the fundamental representation $\underline{8}$, which classifies right-handed gravitinos. Under the semi-simple part H of G_0 , $\underline{8}$ will transform according to

$$\underline{8} = \sum_{i=1}^n r_i + k(\underline{1}), \quad (r_i \neq \underline{1}) \quad (12)$$

If the non trivial part is irreducible ($n=1$), absence of NNA anomalies implies that the $U(1)$ factors of G act only on the k singlet directions. $U(1)$ is chiral if its action on $\underline{8}$ and $\overline{\underline{8}}$ are inequivalent (i.e. if the charges $q_1, q_2, \dots, q_k$ are not a permutation of $-q_1, -q_2, \dots, -q_k$). The condition of zero AAA anomaly is

$$\sum_{i=1}^k q_i^3 = 0, \quad (13)$$

and $SU(8)$ generators are traceless:

$$\sum_{i=1}^k q_i = 0. \quad (14)$$

It is then simple to show that conditions (13) and (14) require at least five singlets to allow for a chiral $U(1)$. With five singlets, eqs. (13) and (14) have an infinite number of solutions, like for instance

$$q_1 = 1, q_2 = -1+\delta, q_3 = -1-\delta, q_4 = \alpha, q_5 = 1-\alpha \quad (15)$$

with

$$\alpha = \frac{1}{2} \left(1 + \sqrt{1 + 8\delta^2} \right) \quad (16)$$

and δ arbitrary. Defined by these charges, $U(1)$ is chiral, except for $\delta=0, \pm 1$. For instance, if $\delta = 6$, we get

$$q_1 = 1, q_2 = 5, q_3 = -7, q_4 = 9, q_5 = -8.$$

Classifying all possible chiral charges is not particularly instructive. Since we need at least five singlets, the non abelian part of G_0 acts on at most three directions of $\underline{8}$, which means that $G_0 = SU(2) \times U(1)$. We have then two anomaly-free chiral subgroups of the $SU(8)$ symmetry group of $N=8$ supergravity. They are defined by the embeddings

$$I: \quad \underline{8} = \underline{2}_0 + 6 \quad \text{charged singlets,} \quad (17)$$

$$II: \quad \underline{8} = \underline{3}_0 + 5 \quad \text{charged singlets,} \quad (18)$$

the $U(1)$ charges satisfying eqs. (13) and (14) in a chiral way.

To pursue the analysis, consider cases for which eq. (12) reads

$$\underline{8} = \underline{r} + \underline{r} + k(\underline{1}) \quad (19)$$

Absence of NNA anomalies requires that the $U(1)$ charges q, q' of the two multiplets $\underline{r}$ satisfy $q + q' = 0$. G_0 is then non chiral in the H-non trivial sector. Since k is now at most four, there is no way to obtain a chiral G_0 with the embedding (19). As a consequence, the non-abelian part of G_0 must possess two- or three-dimensional representations, and G_0 cannot contain simple groups larger than $SU(2)$. It is then straightforward to study the remaining possible embeddings into $SU(8)$. We find the following chiral anomaly-free subgroups for $N=8$ supergravity, in addition to eqs. (17) and (18):

$$\text{III: } SU(8) \supset G_0 = SU(2)_1 \times SU(2)_2 \times U(1)_X$$

$$\underline{8} = (\underline{2}, \underline{1}, 0) + (\underline{1}, \underline{3}, 1) + (\underline{1}, \underline{2}, -4) + (\underline{1}, \underline{1}, 5) \quad (20)$$

$$\text{IV: } SU(8) \supset G_0 = SU(2) \times U(1)_X$$

$$\underline{8} = \underline{4}_2 + \underline{3}_{-5} + \underline{1}_7 \quad (21)$$

$$\text{V: } SU(8) \supset G_0 = SU(2) \times U(1)_X$$

$$\underline{8} = \underline{4}_2 + \underline{2}_{-20} + \underline{1}_7 + \underline{1}_{25} \quad (22)$$

and $SU(2) \times U(1)^2$ or $SU(2) \times U(1)$ subgroups of case III.

Notice that the first $SU(2)$ which appears in case III only transforms a $U(1)_X$ -neutral doublet which does not contribute to any anomaly condition. We can then remove this doublet to obtain a chiral anomaly-free $SU(2)_2 \times U(1)_X$ symmetry of $N=6$ supergravity:

$$SU(6) \supset G_0 = SU(2) \times U(1)_X$$

$$\underline{6} = \underline{3}_1 + \underline{2}_{-4} + \underline{1}_5 \quad (23)$$

G_0 is embedded in $SU(6)$ only, and then spin 3/2 and spin 1/2 states are separately anomaly-free. This solution is in fact the only case in which one can truncate the $N=8$ theory into a lower N supergravity without spoiling the anomaly cancellation.

Another remark which may be of interest for case III is that the $SU(2)_1$ group acts in the same way as the $SU(2)$ automorphism group of $N=2$ supersymmetry. All fields of the $N=8$ graviton multiplet can be classified in $N=2$ multiplets transforming in some representation of $SU(2)_2 \times U(1)_X$. We will come back shortly to this point later.

We now turn to $N=4,5$ and 6 supergravities for which the local chiral symmetry group is $U(N)$. We have already seen from the analysis of $N=8$ supergravity that the $N=6$ theory possesses an anomaly-free chiral subgroup of $SU(6)$ only, $G_0 = SU(2) \times U(1)$ (see eq. 23). We still have to look for anomaly-free symmetries involving also the $U(1)_q$ factor of $U(N) = SU(N) \times U(1)_q$. In this situation, anomalies due to gravitinos can be compensated by contributions of spin 1/2 states.

It is a straightforward task to compute the overall anomalies of fermions transforming according to eqs. (1) and (2), for all chiral subgroups G_0 of $U(N)$, and to obtain the conditions for anomaly cancellation. Let us outline how solutions come out. We clearly need subgroups of $SU(N)$ of the form $H \times U(1)^n$, $n > 0$ (H is semi-simple) to be able to construct an anomaly-free combination of the abelian generators and of $U(1)_q$. This combination will have n real coefficients. Cancellation of AAA anomaly will give one constraint (for each combination), and NNA anomalies will give one condition for each simple component of H . For instance, if $H = SU(2) \times SU(2)$, anomaly cancellation will in general give three conditions, suggesting that n is at least three. The rank five group $SU(2) \times SU(2) \times U(1)^3$ can in fact be found in $SU(6)$, but not in smaller supergravities. It may happen however that some of the conditions are not independent, allowing to reduce n . But this simple counting argument is in general not sufficient, simply because in most cases the equations for zero anomalies in terms of the n real coefficients do not admit non-trivial solutions. A systematic search leads to the following results: $N=5$ and $N=6$ supergravity theories possess $SU(2) \times U(1)$ chiral subgroups, for which the anomalies due to spin 1/2 and to spin 3/2 states cancel each other. These groups are obtained in the following way: for $N=5$, consider the $SU(2) \times U(1)_X$ subgroup of $SU(5)$ defined by

$$\frac{5}{\sim} = \frac{2}{\sim}(-6q) + \frac{1}{\sim}(4q + \beta + \gamma) + \frac{1}{\sim}(4q - \beta + \gamma) + \frac{1}{\sim}(4q - 2\gamma) \quad (24)$$

(The number q appearing in $U(1)_X$ -assignments is the same quantity as in eqs. (1) and (2). Denote by $U(1)_{X+q}$ the group generated by the sum of the generators of $U(1)_X$ and $U(1)_q$. Then, $SU(2) \times U(1)_{X+q}$, with $U(1)_q$ assignments as defined in eqs. (1) and (2) is anomaly-free, if the condition

$$1000q^3 + 90q\gamma^2 + 30q\beta^2 - 3\gamma(\gamma^2 + \beta^2) = 0 \quad (25)$$

is fulfilled. This subgroup is chiral as long as q and γ do not vanish. The $N=6$ theory possesses two solutions, obtained with the two $SU(2) \times U(1)_X$ subgroups of $SU(6)$ defined by

$$i) \underline{6} = \underline{3}(-4q) + \underline{1}(4q+\beta+\gamma) + \underline{1}(4q-\beta+\gamma) + \underline{1}(4q-2\gamma), \quad (26)$$

and

$$ii) \underline{6} = \underline{2}(-4q) + \underline{1}(2q+\beta+\gamma) + \underline{1}(2q-\beta+\gamma) + \underline{1}(2q+\delta-\gamma) + \underline{1}(2q-\delta-\gamma). \quad (27)$$

In both cases, $SU(2) \times U(1)_{X+q}$ is anomaly-free, provided the corresponding condition is satisfied. These conditions are, respectively:

$$i) 184q^3 + 8q(\beta^2 + 3\gamma^2) + \gamma(\gamma^2 - \beta^2) = 0 \quad (28)$$

(chiral for q and γ different from zero), and

$$ii) 72q^3 + 12q\gamma^2 + 6q(\beta^2 + \delta^2) + \gamma(\beta^2 - \delta^2) = 0 \quad (29)$$

(chiral, except if $q=\gamma=0$). These rather strange constraints are obtained from the vanishing of AAA anomalies.

It is tempting at this point to identify the chiral anomaly-free groups G_0 of N -extended supergravities with the $SU(2)_L \times U(1)_Y$ gauge group of electroweak interactions. To do this, we first have to investigate the structure of the electric charge operators Q obtained from G_0 . Electric charge is as usual given by

$$Q = T_{3L} + Y, \quad (30)$$

where T_{3L} is the diagonal generator of $SU(2)_L$ and Y is the weak hypercharge. To avoid parity violations in electromagnetic interactions and massless charged fermions, Q should be vector-like (real) on $\lambda = -3/2$ and $\lambda = -1/2$ states. Q is then traceless on both gravitinos and spin 1/2 left-handed fermions. This last requirement can be satisfied only if $SU(2)_L \times U(1)_Y$ is a subgroup of $SU(N)$. If Y is a combination of $SU(N)$ generators and of $U(1)_q$, for $N = 4, 5, 6$, then, on gravitinos,

$$\text{Tr } Q = -Nq. \quad (31)$$

Since chirality is lost for $q = 0$, these embeddings do not possess a convenient electric charge. Consider now the anomaly-free $SU(2) \times U(1)$ subgroups of the $N=8$ theory, defined by eqs. (17) and (18). For gravitinos, the weak hypercharge acts only on $SU(2)$ -singlet states. Imposing that $Q = T_{3L} + Y$ is vector-like is the same as requiring Y to be vector-like, and the embedding into $SU(8)$ is no more chiral.

The only cases of interest are then given by eqs. (20) to (23). Consider first N=8 supergravity with anomaly-free subgroup $SU(2)_1 \times SU(2)_2 \times U(1)_X$ (see eq. 20). We have three possibilities to identify $SU(2)_L$: if $SU(2)_L = SU(2)_1$, the weak hypercharge acts only on $SU(2)$ -singlet gravitinos and imposing that Q is vector-like destroys chirality. If $SU(2)_L = SU(2)_2$, there are two possible electric charges $Q = T_{3,2} + Y$ (electric charge is always defined up to an overall sign). The first hypercharge is

$$Y_1 = \alpha T_{3,1} + \frac{1}{2} X, \quad (32)$$

with α arbitrary; the corresponding charges of gravitinos, classified in $\underline{8}$, are

$$Q_1 = \left(\frac{\alpha}{2}; -\frac{\alpha}{2}; \frac{3}{2}; \frac{1}{2}; -\frac{1}{2}; -\frac{3}{2}; -\frac{5}{2}; \frac{5}{2} \right). \quad (33)$$

The second possibility is:

$$Y_2 = \alpha T_{3,1} + \frac{1}{6} X, \quad (34)$$

with α arbitrary and

$$Q_2 = \left(\frac{\alpha}{2}; -\frac{\alpha}{2}; \frac{7}{6}; \frac{1}{6}; -\frac{5}{6}; -\frac{1}{6}; -\frac{7}{6}; \frac{5}{6} \right). \quad (35)$$

Notice that if $SU(8)$ with these embeddings of $SU(2)_L \times U(1)_Y$ classifies the fields of a unified model (at M_{Planck}), one can estimate the corresponding Weinberg angle; using

$$\sin^2 \theta_w = \frac{\text{Tr}(T_{3L}^2)}{\text{Tr}(Q^2)}, \quad (36)$$

where traces are taken on any $SU(8)$ representation. We then obtain for Q_1

$$\sin^2 \theta_w = \frac{5}{35 + \alpha^2}, \quad (37)$$

and for Q_2

$$\sin^2 \theta_w = \frac{45}{75 + 9\alpha^2}. \quad (38)$$

The third choice is to identify $SU(2)_L$ with the diagonal $SU(2)$ subgroup of $SU(2)_1 \times SU(2)_2$. We also have two possible electric charges, defined by the two following $SU(2)_L \times U(1)_Y$ assignments of gravitinos:

$$\underline{\bar{8}} = \underline{\bar{2}}_0 + \underline{\bar{2}}_{-1/2} + \underline{\bar{2}}_2 + \underline{\bar{2}}_{-5/2}, \quad (39)$$

and

$$\bar{\mathcal{Q}} = \mathcal{Q}_0 + 3_{-1/6} + 2_{2/3} + 1_{-5/6} \quad (40)$$

The electric charges of gravitinos are respectively $\pm 1/2, \pm 1/2, \pm 3/2, \pm 5/2$ and $\pm 1/2, \pm 1/6, \pm 5/6, \pm 7/6$, and the Weinberg angles are

$$\sin^2 \theta_w = 1/6, \quad (41)$$

and

$$\sin^2 \theta_w = 9/14, \quad (42)$$

respectively.

We still have to discuss the possible electric charge operators for $SU(2) \times U(1)_X$ anomaly-free subgroups as given by eqs. (21) and (22) for $N=8$ supergravity. We find the following $SU(2)_L \times U(1)_Y$ gravitino assignments:

$$\text{I: } \bar{\mathcal{Q}} = 4_{-1} + 3_{5/2} + 1_{-7/2} \quad (43)$$

with charges $\pm 1/2, \pm 3/2, \pm 5/2, \pm 7/2$ and

$$\sin^2 \theta_w = 1/6, \quad (44)$$

$$\text{II: } \bar{\mathcal{Q}} = 4_{-1/3} + 3_{5/6} + 1_{-7/6} \quad (45)$$

with charges $\pm 1/6, \pm 5/6, \pm 7/6, \pm 11/6$ and

$$\sin^2 \theta_w = 9/14, \quad (46)$$

$$\text{III: } \bar{\mathcal{Q}} = 4_{-1/9} + 2_{10/9} + 1_{-7/18} + 1_{-25/18} \quad (47)$$

with charges $\pm 7/18, \pm 11/18, \pm 25/18, \pm 29/18$ and

$$\sin^2 \theta_w = \frac{891}{1636} \sim .54 \quad (48)$$

$N=6$ supergravity has an anomaly-free $SU(2) \times U(1)$ subgroup of $SU(6)$ (see eq. 23). To obtain a convenient electric charge, the embedding should be either

$$\bar{\mathcal{Q}} = 3_{-1/2} + 2_2 + 1_{-5/2} \quad (49)$$

with gravitino charges $\pm 1/2, \pm 3/2, \pm 5/2$, and

$$\sin^2 \theta_w = 1/7, \quad (50)$$

or

$$\bar{\mathcal{Q}} = 3_{-1/6} + 2_{2/3} + 1_{-5/6} \quad (51)$$

with gravitino charges $\pm 1/6$, $\pm 5/6$, $\pm 7/6$, and

$$\sin^2 \theta_w = 3/5. \quad (52)$$

All these tentative embeddings of $SU(2)_L \times U(1)_Y$ into N-extended supergravities are not expected to give realistic quark and lepton electric charges in the graviton multiplet. The fundamental spin 1/2 fermions always contain $SU(2)_L$ - and $U(1)_Y$ -exotic states. Such a scheme could possibly be made realistic only with composite states participating in the effective theory which should emerge from supergravity. The gauge fields themselves are composite and dynamically generated. This bound state hypothesis was in fact always present in attempts at superunification ⁴⁾. It is however interesting to try to obtain among the fundamental spin 1/2 fields some of the usual $SU(2)_L \times U(1)_Y$ multiplets classifying quarks and leptons. This may also be necessary in order to construct bound states with correct assignments. There is a unique case where some quark electric charges appear. This case corresponds to weak hypercharge (34) with $\alpha = 0$. The anomaly-free $SU(8)$ -subgroup is $SU(2) \times SU(2)_L \times U(1)_Y$. The embedding is defined by

$$\begin{aligned} \mathcal{G} = & (2, 1, 0) + (1, 3, -1/6) \\ & + (1, 2, +2/3) + (1, 1, -5/6). \end{aligned} \quad (53)$$

The electric charges of gravitinos are 0, 0, $\pm 1/6$, $\pm 5/6$, $\pm 7/6$. As already mentioned, the two neutral gravitinos correspond to a N=2 supersymmetry commuting with the gauge group $SU(2)_L \times U(1)_Y$. All fields of the N=8 supergravity multiplet can be classified into N=2 massless supermultiplets (λ, R) , with maximal helicity λ and transforming according to the representation R of the gauge group. This decomposition reads:

$$\begin{aligned} & (2, 1_0) \\ & + (3/2, 3_{-1/6} + 2_{2/3} + 1_{-5/6}) \\ & + (1, 4_{-1/2} + 3_{-1/3} + 3_{-1} + 1_{-4/3} + 2_{-1/6} + 2_{1/2}) \\ & + (1/2, 4_{-1/3} + 3_{-7/6} + 2_{-1/3} + 1_{-1/2} + h.c.) \end{aligned} \quad (54)$$

The vector multiplets $(1, 2_{-1/6} + 2_{1/2})$ contain two quark doublet and two lepton doublets; the right-handed partners in the graviton multiplet are however not $SU(2)_L$ -singlet states. This particular embedding of the electroweak gauge group gives

$$\sin^2 \theta_w = 3/5. \quad (55)$$

The obvious weakness of this approach is the absence of strong interactions. One may argue that they appear when supergravity is gauged, for instance as a subgroup of $SO(8)$ in gauged $N=8$ supergravity ³⁾. This idea leads however to a basic difficulty: $SU(2)_L$ -doublet and singlet states will transform in general in the same colour representations. Chirality will be preserved, but "mirror" fermions will appear (antiquarks with left-handed weak interactions, for instance). Quarks with integer charges and leptons with fractional charges will also be present in general. A proliferation of exotic states seems in any case quite difficult to avoid. One could also relax our assumption that electric charge has to be vector-like on the fields of the supergravity multiplet, and invoke this restriction on bound states only. This last approach seems rather artificial, and painful to realize in practice.

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