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NEW ANALYSIS OF THE TWO COMPETING PROCESSES :
CHANNELING - AND COHERENT PAIR PRODUCTION
AND A COMPARISON WITH EXPERIMENTAL DATA

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A B S T R A C T

The characteristics of channeling pair production are analysed and compared with those of the coherent analogue. Newly reported experimental findings are shown to fit perfectly theoretical predictions as opposed to the conclusion of the original paper. Separation of the two pair production mechanisms by energy and temperature variations is discussed.

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G E N E V A

Recently, the process of channeling pair production (ChPP) has received extensive interest. The prediction of the effect, being inverse to normal channeling radiation, dates back to 1978 (ref.1), and since then a series of theoretical papers has been published²⁻⁸⁾. However, so far only one ChPP experiment has been performed⁹⁾. In this work the authors claim to find a discrepancy in yields of a factor of around 10 as compared with theoretical predictions. It is one of the aims of this letter to show that the latter conclusion is erroneous and that the experimental findings in ref.9 indeed follow theoretical predictions when proper account is taken for the effect of coherent pair production (CPP). Further we have found it pressing to demonstrate how some of the features used in the literature to characterize ChPP and distinguish it from other electron-positron pair production mechanisms have a simple physical interpretation, while others are misleading.

Pair Production (PP) is inverse to the process of electron bremsstrahlung (BS) as is clear when PP is viewed as the excitation of an electron from a state of negative to one with positive energy by an incoming photon in an external field from, e.g. a single atomic nucleus, which is needed to ensure energy and momentum balance. In order to study PP in single crystals it is therefore instructive to recall the corresponding BS processes.

When a parallel beam of electrons traverses a crystal-line target far from low index axial and planar directions the well-known one-atom incoherent BS is emitted. As planar directions are approached coherence in successive interactions occurs, resulting in the interference structure of coherent BS. When the angle of incidence with respect to axial or planar directions becomes very small, the projectile motion becomes governed by the lattice continuum potential obtained by smearing the atomic charges along the considered axis or plane. The resulting strong steering effect is known as channeling and the corresponding bremsstrahlung as channeling radiation (for the GeV region, see ref.10).

For incoherent and coherent BS emitted by GeV electrons a first-order Born approximation is applicable for not too high values of the atomic charge Z . This means that the projectile scattering in the atomic field as well as its interaction with the radiation field are treated as perturbations, leading to a two-vertex process and, in turn, to the well-known Bethe-Heitler (BH) cross section. However, in the channeling case, the perturbative treatment of the interaction with the crystal field breaks down. In this case a description similar to the dynamical theory of electron diffraction is necessary, and therefore the origin of channeling radiation is radiative transitions between electronical Bloch-wave states. As opposed to incoherent and coherent BS, channeling radiation is then a one-vertex process since only the radiation field is treated as a perturbation.

By inverting the BS processes, all the above remarks apply to the corresponding PP mechanisms. In this letter we shall only consider photon incidence near the strongest axis in a given crystal, the transcription to other axes and planes being trivial.

ChPP is characterized by a threshold ω_{th} in photon frequency ω . For a photon incident at an angle θ to a major axis, conservation of energy and transverse momentum during pair production leads to the relation

$$\epsilon_+ + \epsilon_- = \frac{\hbar\omega}{2} \left(\theta^2 - \frac{(mc^2)^2}{E_+ E_-} \right) , \quad (1)$$

$$\epsilon_{\pm} = \frac{p_{\perp}^2}{2m\gamma_{\pm}} + V ,$$

where $\gamma_{\pm}$ is defined from the total energies $E_{\pm}$ of the produced pair as $\gamma = (E - V)/mc^2$, V being the potential energy corresponding to the crystal continuum potential. For $\theta = 0$, ChPP may then proceed provided

$$\hbar\omega > 2mc^2 \frac{mc^2}{-(\epsilon_+ + \epsilon_-)} \geq 2mc^2 \frac{mc^2}{V_{max}} \equiv \hbar\omega_{th} \quad (2)$$

where V_{max} corresponds to the depth of the continuum potential. A typical value for, e.g., silicon is $\hbar\omega_{th} \sim 5$ GeV; the approximate scaling to other crystals being $\propto d/Z$, where d denotes the atomic spacing along the considered axis.

The result (2) has indeed a very simple interpretation. In incoherent PP the pair is emitted inside angles $\psi \sim mc^2/\hbar\omega$ with respect to the direction of the photon momentum. Consequently the emitted $e^\pm$ will be channeled provided

$$\psi \sim mc^2/\hbar\omega \lesssim \psi_1 \sim (V_{\max}/p^\pm c)^{\frac{1}{2}} \quad (3)$$

where ψ_1 denotes the critical angle for axial channeling corresponding to the $e^\pm$ -momenta $p_\pm$. With $p_\pm c \sim \hbar\omega$ eq.(2) is retrieved.

In a similar way we may argue that for $\omega > \omega_{th}$ the critical angle θ_c for photon incidence to lead to ChPP will be of order ψ_1 since the produced pair will be emitted within a cone around the photon momentum of opening angle small compared with ψ_1 . Indeed, in eq.(1) we may neglect the second term inside brackets for $\theta \gtrsim mc^2/\hbar\omega$ and since ChPP is confined to cases with $|\epsilon_+ + \epsilon_-| \lesssim V_{\max}$, higher values of the transverse energies correspond to the perturbation limit, i.e., to the case of CPP, we find a critical angle of

$$\theta_{cr} \approx \left(\frac{2V_{\max}}{\hbar\omega} \right)^{\frac{1}{2}} \approx \psi_1(p = \hbar\omega/2), \quad (4)$$

that is, θ_{cr} equals the usual critical angle for axial channeling ψ_1 corresponding to an $e^\pm$ -momentum of half that of the incident photon. The value (4) is in good agreement with that given numerically in ref.7 for diamond. Note that $\theta_{cr} \propto (\hbar\omega)^{-\frac{1}{2}}$.

In the so-called constant field approximation (CFA)^{11,3-4,6,8)}, the differential ChPP yield is for $\theta = 0$ given as

$$\frac{dW}{d\eta} = \frac{\alpha}{\sqrt{3}\pi\lambda} \frac{mc^2}{\hbar\omega} \left[\frac{1}{\eta(1-\eta)} K_{2/3}(\xi) - \int_{\xi}^{\infty} dt K_{5/3}(\xi) \right], \quad (5)$$

$$\xi = \frac{2}{3\eta(1-\eta)\kappa}, \quad \kappa \equiv |v| \frac{\lambda}{mc^2} \frac{\hbar\omega}{mc^2}, \quad \eta \equiv E_+/\hbar\omega,$$

cf., refs. 8 and 11, where $\alpha \approx 1/137$, $K_{n/3}$ denotes a Bessel function of order $n/3$, and $\lambda = \hbar/mc$ the Compton wavelength of the electron. As a consequence of application of the CFA the differential yield, eq.(5), is symmetric around $\eta = \frac{1}{2}$. Since the CFA is expected to be more reliable with increasing photon energy, it is difficult to understand the numerical results presented in fig.4 of ref.7, which show that $dW/d\eta$ should have a higher degree of asymmetry at high than at low energies.

Also the authors of ref.7 claim in various of their papers that a peaking around $\eta \sim \frac{1}{2}$ as opposed to the nearly flat BH spectra is a characteristic feature of ChPP. However, as is clear from fig.1, which shows the differential yield, eq.(5), as computed for tungsten this statement is not in general correct. Already at an energy of ~ 100 GeV the spectrum is rather flat, and in shape not very different from BH. At extremely high energies, where the rate decreases $\propto (\hbar\omega)^{-1/3}$, cf. also ref.8, the yield even peaks at $\eta = 0, 1$.

It may be noted that for lighter target materials as, e.g., germanium, the ChPP spectrum would still be peaked around $\eta = \frac{1}{2}$ at, e.g., 100 GeV since the excess over ω_{th} is not so high for this case as for tungsten.

For correct interpretation of experimental data it is essential to know the yield of CPP for which process the characteristic angles scale as

$$\theta^{\text{CPP}} \propto (\chi\omega)^{-1} \quad (6)$$

as opposed to the slower scaling for ChPP, eq.(4).

In fig.2a we show the CPP yield as computed¹²⁾ for $\eta = \frac{1}{2}$ for 100 GeV photons incident near the $\langle 110 \rangle$ axis in germanium in the $\{100\}$ plane and at an azimuthal angle of $\varphi = 0.1$ to the $\{110\}$ plane since this situation corresponds closely to the experimental situation reported in ref.9. The crystal temperature is $T = 100$ K. For incidence in the $\{100\}$ plane the interference peaks are located at positions $\propto 1/n$; $n=1,2,\dots$. When a small azimuthal angle is chosen each peak splits up into a series of sidebands and at larger azimuthal angles as, e.g., $\varphi = 0.1$, the spectrum is smeared out to a great extent, however, the average level, which at moderate angles is far beyond BH, being maintained. The insert in fig.2a shows the variation of the total yield $W = \int_0^1 d\eta dW/d\eta$ obtained for $\theta = 4$ mrad as a function of φ . Clearly, only minor variations are observed, the overall level being $\approx 2.5 \times \text{BH}$.

In fig.2a we have further plotted the expected ChPP yield as a dashed line; the yield for $\eta = \frac{1}{2}$ is calculated for $\theta = 0$ according to eq.(5) and is assumed to be reduced to half of this value at $\theta = \theta_{\text{cr}}$, eq.(4). We note that the maximal yields in the

two processes are comparable, both exceeding the BH rate by a factor of $\sim 8 - 10$, but that CPP extends over a much wider angular region. As is clear from the figure there is a region where the curves overlap; in this grey area neither the ChPP nor the CPP perturbation approach can be assumed valid.

In fig. 2b we show the differential spectra. It is clear that the CPP yield at $\theta = 0.5$ mrad is close to that obtained for ChPP, both in shape and absolute numbers, whereas the CPP yield at large angles is close to BH in shape. For the present case, both of the first show a peaking at $\eta = 1/2$, which means that even when this feature shows up in ChPP it is not specific to this process. Consequently ChPP cannot be identified alone by the shape of the differential spectrum.

Fig.3a,b shows the experimental points presented in fig. s.1,3 of ref.9. The points are normalized to a "non-aligned (NA)" case which, as is clear from fig.3b, corresponds to $\theta = 3.91$ mrad, since this point is presented without error bars and positioned at $(W - W_{NA})/W_{NA} \equiv 0$. In comparing with theory, however, it is crucial to observe that the yield at this angle is not equal to BH as the authors assume but rather to a few times this yield due to CPP as is clear from fig.2a. Correspondingly, we show in fig.3a the computed yields $W^{\text{ChPP}}(\theta=0)/W^{\text{CPP}}(\theta= 3.91 \text{ mrad}) - 1$, where for W^{CPP} we have used an average over azimuthal angles (which is less important as the variation in W with ϕ is small, cf., fig.2a). Clearly the theore-

tical curve fits nearly perfectly the experimental points and no dubious factor (of $1/10$) is involved as in the original publication⁹⁾. The reason for the theoretical yields being slightly lower than the experimental ones is that background contributions to ChPP have been neglected. If we assume ChPP to be sitting on top of a yield corresponding to the amorphous part of the CPP yield¹²⁾, at $T = 100$ K corresponding to $.84 \cdot BH$, the dashed curve in fig.3a is obtained which, indeed, shows a fully perfect agreement with the experimental findings.

Similarly, in fig.3b we show our theoretical results corresponding to the angular scan presented in ref.9 for $\varphi = 0.1$. We have computed the results for $\hbar\omega = 80,100$ GeV and weighted these with $1/\hbar\omega$ corresponding to normal bremsstrahlung. The interference peaks of CPP have disappeared due to integration over η . Again it is clear that our computed values fit the experimental ones without any dubious reduction factor involved. As to the width of the ChPP, we have assumed eq.(4) to apply. The experimentally recorded value seems to be lower. If this is real, it might be connected to the well-known fact that critical angles for electrons usually are considerably smaller than ψ_1 , cf.ref.13. Further on, the estimate in eq. (4) could be off by a factor of 2. In any case, however, θ_{cr} characterizes the incident angle region where perturbation theory breaks down.

There exist two means by which it is possible to separate experimentally the ChPP and CPP effects and to study systematically

their mutual interplay. One method to adjust the width of the grey zone, the overlap region of fig.2a, is to lower the photon energy. Since the characteristic angles for ChPP and CPP scale as $(\hbar\omega)^{-1/2}$ and $(\hbar\omega)^{-1}$, respectively, cf., eqs.(4) and (6), the width of the grey zone is reduced at low energies. This point is illustrated in fig.4a-c for germanium kept at $T=100$ K for $\eta = \frac{1}{2}$. A decrease in $\hbar\omega$ from 200 GeV to 50 GeV nearly separates the two effects which, if added, essentially are melted together at the higher energy.

Another method to separate the effects is to heat the crystal as illustrated in fig.4b,d. Due to a Debye-Waller factor on the interference terms in CPP¹²⁾, the effect of increasing the temperature will be to damp the yield at distant reciprocal lattice points, i.e., primarily at smaller incident angles; in fig.4b the region of, say, $0.1 < \theta < 1$ mrad. This gives an effective larger width of the central "hole" in the CPP spectrum which leads to a gradual separation of CPP from ChPP with increasing temperature. Further the diffuse background yield, which is the only one left for $\theta \sim 0$, where the CPP perturbation approach actually breaks down, approaches BH at high temperatures. Increasing the target temperature from 100 K to 500 K has dramatic effects on all yields as demonstrated in fig.4b,d.

To conclude: Critical angles and threshold energies for ChPP have simple physical interpretations. Experimentally ChPP is hard to discern from CPP because of similar characteristics for the pairs and angular overlap - only close to threshold and for different crystal temperatures the two effects can be separated. These new aspects need detailed knowledge of PP yields in a large angular region around a crystal axis¹⁴⁾. High Z-materials give a very abrupt and sensitive onset of ChPP effects with the overlap setting in at lower photon energies where the ratio of ChPP to CPP is more favourable. The first experimental data have been re-analyzed and are in perfect agreement with our predictions for ChPP and CPP - the present CPP calculations agree also with older data¹⁵⁾. Polarization effects^{3,4)} could be investigated using crystal radiators.

** Postscript :

Private communications indicate that the normalization point in Fig.3G has been redefined to a value of ~ 40 mrad. Unfortunately this spoils the perfect agreement shown in Fig.3 - both for ChPP and CPP.

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Figure Captions

Fig.1: Differential ChPP yield, eq.(5), for incidence along the $\langle 111 \rangle$ axis in tungsten at a target temperature of 100K as a function of the fractional positron energy for various values of the photon energy. The dashed curve on a) corresponds to the BH yield.

Fig.2: Differential CPP (full drawn), ChPP (dashed), and BH (dotted) yield for 100 GeV photons incident at an angle θ to the $\langle 110 \rangle$ axis in germanium at a target temperature of 100K as a function of a) θ and b) η . In a) symmetric PP and incidence at two different azimuthal angles are considered. The insert shows the variation with φ of the total CPP yield for $\theta = 4$ mrad together with the total BH yield. In b) incidence in the $\{100\}$ plane is assumed and the ChPP yield is computed for $\theta = 0$, eq.(5).

Fig.3: Experimental findings for the dependence of total ChPP yield on a) photon energy and (b) incidence angle as presented in ref.9 for 50-110 GeV photons incident on a $\langle 110 \rangle$ germanium crystal cooled to 100K together with the present theoretical results, all normalized to the yields encountered at $\theta = 3.91$ mrad. In a) the full drawn curve corresponds to the ChPP yield given in eq.(5) whereas in the dashed one also the amorphous CPP yield is included. In b), CPP for $\varphi = 0.1$ (full drawn), ChPP (dashed), and BH (dotted) is shown for the experimental energy window of $80 < \hbar\omega < 110$ GeV. The dashed curve is obtained by adding the amorphous yield to ChPP.

Fig.4: Differential CPP (full drawn), ChPP (dashed), and BH (dotted) yields as a function of incidence angle to the $\langle 110 \rangle$ axis in a germanium crystal for a photon energy and a target temperature of a) 50 GeV, 100 K, b) 100 GeV, 100 K, c) 200 GeV, 100 K, d) 100 GeV, 500 K. Symmetric PP and incidence in the $\{100\}$ plane are assumed.

Fig. 1

Fig. 2

Fig. 3

Fig. 4