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PROSPECTS AND PROBLEMS OF LOCALLY
SUPERSYMMETRIC KALUZA-KLEIN THEORIES

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ABSTRACT

In this talk, I will briefly review the role of supersymmetric Kaluza-Klein theories in the general quest for a unified field theory.

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In this talk, I will briefly review the role of supersymmetric Kaluza-Klein theories in the general quest for a unified field theory. I begin from a distance recalling that Grand Unified Theories¹⁾ (GUTs) provide a beautiful scheme for the understanding of three of the fundamental forces of Nature from a single standpoint and have many interesting implications: some of their cosmological ones have been discussed in other talks at this conference. They, however, suffer from some severe drawbacks:

- a) they fail to explain the observed hierarchy of gauge symmetry breaking scales, namely the huge difference between the grand unification scale M_X and the electroweak scale M_W

$$M_W / M_X \simeq 10^{-12} \quad (1)$$

- b) they do not incorporate the fourth force of Nature, namely gravitation.
- c) In general they do not provide any explanation of the families, namely of the apparently superfluous repetition of particles with the same $SU(3) \times SU(2) \times U(1)$ quantum numbers.

The efforts²⁾ to solve problem a) (the hierarchy problem) have led theorists to consider supersymmetric GUTs^{2),3)} which are globally supersymmetric gauge theories⁴⁾ where supersymmetry is the mechanism which protects the ratio of Eq. (1) from the devastating effect of radiative corrections.

Later it became clear that consistent supersymmetric GUTs could be formulated only if supersymmetry was made local and spontaneously broken⁵⁾ through super Higgs mechanism⁶⁾. This brought the theory of supergravity⁷⁾ onto the grand unification stage and therefore the quest for the solution of problem a) leads also to a solution of point b). Indeed supergravity is the supersymmetric extension of general relativity and any locally supersymmetric theory

necessarily includes Einstein gravity. Therefore, the framework which is presently used, with quite promising success for a unification of all the forces of nature within a theory explaining the features of low energy phenomenology, is provided by $N = 1$ supergravity coupled to $N = 1$ gauge theory and a suitable number of $N = 1$ chiral multiplets. The phenomenological aspects of this type of theories were presented at this conference in the talk by D.V. Nanopoulos⁸⁾ while the general form of the coupled Lagrangian was instead derived in Ref. 9). It appears that the explicit structure of the possible models depends on the following choices:

- i) choice of a gauge group G
- ii) choice of a Kähler manifold M_K with Kähler potential $G(z, z^*)$, whose coordinates are the scalar fields of the chiral multiplets introduced. (The chiral multiplets will eventually contain the observed quarks and leptons in the fermion sector.)
- iii) choice of a coupling function $f^{\alpha\beta}(z, z^*)$ which depends on the scalar fields z and belongs to the $\text{Adj} \otimes \text{Adj}$ representation of the gauge group G .

The function $f^{\alpha\beta}(z, z^*)$ determines the kinetic term of the gauge fields:

$$f^{\alpha\beta}(z, z^*) F_{\mu\nu}^\alpha F_{\mu\nu}^\beta \quad (2)$$

while the Kähler potential $G(z, z^*)$ is responsible for most of the features relating to supersymmetry and symmetry breaking and to the consequent mass-relations.

From the above discussion it appears that because of their high degree of arbitrariness and since they are certainly not renormalizable and most likely not finite, the $N = 1$ locally supersymmetric grand unified theories must be considered, in spite of their success, as effective theories, namely low energy approximations of some more fundamental theory which has as characteristic scale the Planck mass $M_p = 10^{19}$ GeV. It is at this point that Kaluza-Klein theories with local supersymmetry enter the stage. Let us discuss how. First of all we recall that the only way to decrease the degree of arbitrariness of supersymmetric models is to increase the number of supersymmetries. This can be done in two seemingly different ways:

First route: by increasing the number N , remaining in four space-time dimensions.

Second route: by increasing the number D of space-time dimensions while leaving N fixed to $N = 1$.

This statement is illustrated by the following recollection of quite well-known facts which will be useful for our further discussion:

First route: increasing N

i) For $N = 2$ we can still couple to supergravity both vector multiplets (containing the gauge fields) and hypermultiplets (containing the spin- $\frac{1}{2}$ and spin-0 matter), but the restrictions on the manifold of the scalar fields (now contained also in the vector multiplet) are much more severe^{10),11)}.

ii) For $N = 3$ and $N = 4$, the only multiplet which can be coupled is the vector one and although the explicit coupling has not yet been constructed, it is presumable that the allowed scalar manifold geometries will be further restricted¹²⁾.

iii) Beyond $N = 4$ up to the maximum number $N = 8$, no external multiplet can be coupled to supergravity whose spectrum already extends from spin-2 to spin-0 and the only available elementary gauge fields are those of the $SO(N)$ gauging the model. The scalar manifold geometry seems to be restricted according to a well-defined pattern¹²⁾ being completely determined in the maximal $N = 8$ case where the 70 scalar fields parametrize the

$$M_{\text{scalar}} = E_7/SU(8) \quad (3)$$

coset manifold¹³⁾. In this case there is a unique $N = 8$ supergravity model with gauged $SO(8)$ symmetry¹⁴⁾ which has a further $SU(8)$ invariance associated to composite gauge fields.

Second route: increasing D

i) In $D = 5$, the minimal $N = 1$ supergravity model has the same field content as $N = 2$ supergravity in four-dimensions $[v_\mu^a, \phi_\mu, B_\mu]$ and admits a beautiful geometric formulation¹⁵⁾ where the spin-1 field strength:

$$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu = \varepsilon_{\mu\nu\rho\sigma\tau} T_{\rho\sigma\tau} \quad (4)$$

is reinterpreted as the space-time torsion. Vector multiplets and hypermultiplets can be coupled to this theory as in four-dimensions, but an analysis of the vector couplings, which is the only one done so far¹⁶⁾, has revealed that only a subset, with special geometrical significance, of the permissible couplings to $D = 4$, $N = 2$ supergravity is available in $D = 5$.

ii) In $D = 6$, minimal supergravity¹⁷⁾ has the following spectrum

$$V_\mu^a, \psi_\mu, B_{\mu\nu} \quad (5)$$

where V_μ^a is the vierbein, ψ_μ is a left handed chiral gravitino and $B_{\mu\nu}$ is an antisymmetric two-index gauge field, whose field strength $F_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]}$ is self-dual on shell. As for the $D = 5$ theory, both vector and scalar multiplets can be in principle coupled but, due to the self-duality and chirality constraints of the gauge fields, for which the matter fields act as sources, we expect strong restrictions on the available scalar geometries. This is a problem which is still under investigation¹⁸⁾.

iii) From $D = 5$ to $D = 10$, only the vector multiplet can be coupled. In $D < 10$ this latter contains scalar fields whose geometry is so far unknown but for $D = 10$ a remarkable fact takes place: the vector multiplet contains only spin-1 and spin- $\frac{1}{2}$ fields so that there is no scalar manifold to be chosen. The coupling to supergravity is therefore unique up to the choice of the gauge group G and it was constructed in Refs. 19)-21). Presently this coupled theory is being rewritten in the formalism of the so-called "group manifold approach"²²⁾ and reveals a beautiful geometrical structure where all the supergravity bosonic fields are interpreted as vielbein and torsion tensors²³⁾.

iv) $D = 11$ is the maximal dimension in which supergravity can be formulated and no multiplet can be coupled: the theory is therefore unique²⁴⁾. It moreover admits a very aesthetically satisfying geometric formulation²⁵⁾ in terms of a free differential algebra completely determined by the super-Poincaré algebra.

We see that both routes imply more and more restrictions on the permissible couplings leading finally to a unique theory, either gauged $N = 8$ supergravity in four dimensions or $N = 1$ supergravity in 11 dimensions. The attempts to derive $N = 1$ locally supersymmetric theories from a more fundamental one must

therefore rely on either one of the following two mechanisms, respectively related to the first and second routes:

First mechanism: spontaneous supersymmetry breaking

Second mechanism: spontaneous compactification

Let us discuss them.

Spontaneous supersymmetry breaking: is the well-known extension to supersymmetry of the Higgs mechanism. Some of the scalar fields parametrizing the scalar manifold $M_{(\text{scalar})}$ appropriate to the chosen initial N , develop a vacuum expectation value $\langle \phi \rangle = \phi_0$ and supersymmetry gets spontaneously broken from N down to $N_0 < N$. The main problems with this programme are the following two:

- i) It seems difficult to find a breaking pattern where $N_0 = 1$.
- ii) If N is made sufficiently large in order to kill all the arbitrariness of the couplings, then the freedom in choosing the gauge group G is also removed and one is left with $SO(N)$ which is always too small to accommodate the standard $SU(3) \otimes SU(2) \otimes U(1)$ model.

Spontaneous compactification: is a different mechanism which enables us to interpret a multidimensional gravitational theory as a four-dimensional field theory and brings back to the landscape of modern theoretical physics the old, time-honoured Kaluza-Klein idea²⁵⁾. The basic ingredient is, as in the Higgs mechanism, the appearance of a non-vanishing vacuum expectation value of a convenient field.

Any multidimensional supergravity is general relativity in dimension D coupled to a set of matter fields whose type and interactions are fixed by supersymmetry. Now imagine that the D -dimensional field equations admit an exact solution with the following properties:

- i) The D -dimensional manifold M_D is taken to be the following direct product:

$$M_D = M_4 \otimes M_{D-4} \quad (6)$$

where M_4 is Minkowski, de Sitter or anti-de Sitter space described by a constant Riemann curvature:

$$R^{ab}_{\cdot cd} = \Lambda_{(4)} \delta^{ab}_{cd} \quad (7)$$

$[\Lambda_{(4)}$ is the cosmological constant and $\Lambda_{(4)} = 0$ corresponds to the Minkowski case, $\Lambda_{(4)} > 0$ to the de Sitter one and $\Lambda_{(4)} < 0$ to the anti-de Sitter case].

ii) The M_{D-4} manifold is equipped with a metric which admits a group of isometries S . Typically M_{D-4} is a homogeneous space of the group S , namely the following coset manifold

$$M_{D-4} = S/R \quad (8)$$

where R is a subgroup of S . In general the Ricci tensor of these spaces, but not their Riemann tensor, can be chosen to be a Kronecker delta (in other words they are Einstein spaces²⁶⁾):

$$R^{\alpha\cdot}_{\beta\cdot} = \Lambda_{(D-4)} \delta^{\alpha}_{\beta} \quad (9)$$

where $\Lambda_{(D-4)}$ is some constant.

iii) The constants $\Lambda_{(4)}$ and $\Lambda_{(D-4)}$ are linked to the vacuum expectation value of a convenient matter field (a typically antisymmetric field strength $\langle F_{\mu_1 \dots \mu_r} \rangle$) whose energy-momentum tensor appears on the right-hand side of the Einstein equations and is the driving mechanism of the spontaneous compactification.

iv) All the fermion fields are taken to be zero.

If such a solution exists, it can be regarded as a vacuum state of the full theory which can be re-expanded around the solution. In the case where M_{D-4} is a coset manifold S/R the essential tool to be used is the harmonic analysis on S/R ²⁷⁾. Every field $\phi_1(x, z)$ of the theory depends both on the co-ordinates x of $M_{(4)}$ and the co-ordinates z of $M_{(D-4)}$: moreover, since it has definite transformation properties under the D -dimensional Lorentz group $SO(1, D-1)$, it will belong to definite representations of the four-dimensional Lorentz group $S(1, 3)$ and of the tangent group $SO(D-4)$ to the M_{D-4} manifold. This allows the following expansion

$$\Phi_{(i)}^I(x, z) = \sum_I \Phi^I(x) Y_{(i)}^I(z) \quad (10)$$

where $Y_{(i)}^I(z)$, the spherical harmonics on S/R , belong to an infinite spectrum of representations of S whose structure is decided by the simultaneous embedding of R both in S and in $SO(D-4)$. In this way the original theory in D -dimensions becomes an ordinary four-dimensional theory with an infinite number of fields $\Phi^I(x)$, whose four-dimensional masses can be related via a formula, the explicit form of which depends on the model considered, to the eigenvalues of a certain invariant differential operator $\Box_{(1)}$ on $M_{(D-4)}$ which is specific to the $SO(D-4)$ representation in question. Therefore one obtains the following mass formulae

$$m_{(i)} = m(\Box_{(i)}) \quad (11)$$

and the traditional Kaluza-Klein philosophy consists of considering the truncation of the theory to the massless modes $\Phi^I(x)$ ($m^I = 0$) as a viable low-energy approximation on the ground that the discarded modes have masses of the order of the Planck mass. (This can be arranged considering the characteristic length scale of the compact manifold M_{D-4} as enormously small.) We should make some comments on the correctness of this picture when $M_{(4)}$ is de Sitter or anti-de Sitter space^{27b)} but let us accept it for the moment, since it is certainly correct in the Minkowski case where $\Lambda_{(4)} = 0$. What we find is that the observable features of the effective four-dimensional theory obtained in this way depend on the geometry and topology of the compactified manifold M_{D-4} . In particular we find

- i) Massless gauge fields of a group $G' = S \otimes H$ where S is the isometry of $M_{(D-4)}$ and $H \subset G$ is a suitable subgroup of the gauge group G in D dimensions (if we are in a situation where it existed).
- ii) N_0 remaining supersymmetries where the number N_0 is essentially linked to the number of Killing spinors admitted by the manifold M_{D-4} .
- iii) A set of spin- $\frac{1}{2}$ and spin-0 fields whose number and representation assignments under $S \otimes H$ depends on the structure of zeros of Dirac, Lichnerowicz and Laplace operators on $M_{(D-4)}$.

- iv) A number of Abelian gauge multiplets associated to central charges which is determined by some of the B_k Betti numbers of the manifold $M_{(D-4)}$.

As one can see, the situation is such that, in principle, we can expect to be able to derive the $N = 1$ phenomenological models from a supergravity theory in higher dimensions. Particularly satisfying would be to obtain them from either one of the following two theories

A) $D = 11$ supergravity

B) $D = 10$ supergravity coupled to the super Yang-Mills multiplet of a suitably chosen group G .

The first is the most ambitious programme since there is no arbitrariness in the choice of the higher dimensional theory while the second is the next most ambitious one since the whole arbitrariness allowed is the choice of a group G . The programme A) has been the focus of extensive work by many authors in the last two years²⁸⁾ due to the discovery, by Freund and Rubin²⁹⁾, that the presence in $D = 11$ supergravity of a three-index photon $A_{\mu\nu\rho}$ naturally induces compactification via the vacuum expectation value of the field strength $F_{\mu\nu\rho\sigma}$ which is taken to be of the form:

$$\langle F_{\mu\nu\rho\sigma} \rangle = e \varepsilon_{\mu\nu\rho\sigma} \quad (12)$$

The $\Lambda_{(4)}$ and $\Lambda_{(D-4)}$ constants are then related to the constant e by the formulae:

$$\Lambda_{(4)} = -16e^2 \quad ; \quad \Lambda_{(D-4)} = 12e^2 \quad (13)$$

which show that $M_{(4)}$ is always anti-de Sitter space.

The classification of coset manifolds S/R which, being seven-dimensional Einstein spaces, are solutions of the $D = 11$ field equations, has been completed by L. Castellani, L. Romans and N. Warner³¹⁾ after many separate cases had been worked out by various authors³¹⁾. The number of Killing spinors and the second Betti number of these spaces, which decides in this case the number of gauged central charges, is also known by now and the spectrum of representations has been calculated for the principal solutions³²⁾. The outcome of the classification is shown in Table I and from this battery of results, one can extract the following negative conclusions.

Cosmological problem:

There is always a relation between the gauge coupling constant and the cosmological term which forces one of the two to be unrealistic. This relation is typical of all gauged supergravity models and is due to the fact that the residual supersymmetry one obtains is N_0 -extended de Sitter supersymmetry instead of N_0 -extended Poincaré supersymmetry.

Chirality problem:

The fact that the fermions in the $D = 11$ theory are Majorana spinors forbids the appearance of chiral fermions after compactification. [This is a possibility which is open only if the starting point is a Majorana-Weyl spinor available in $2 \bmod 8$ dimensions³³⁾ (for instance in $D = 10$).]

 $N = 1$ problem:

There is only one class of coset manifolds yielding the standard model gauge group $SU(3) \otimes SU(2) \otimes U(1)$. These are the M^{pqr} spaces defined in Ref. 34) and explicitly constructed and studied in Ref. 31c). They lead to either $N_0 = 0$ or $N_0 = 2$ supersymmetry but never to $N_0 = 1$ as desired.

It is obvious that the $D = 11$ theory is still interesting as a mathematical laboratory to test the features of supersymmetric Kaluza-Klein models but fails in the goal of producing the standard $N = 1$ model. All the hopes have therefore, in my opinion, to be concentrated on the $D = 10$ case where at the price of introducing an arbitrary gauge group one gains a much more promising starting point with respect to the chirality and cosmological problems. Indeed in that theory one has Majorana-Weyl spinors and a torsion induced by a 7-index antisymmetric field strength, which might help in cancelling the cosmological term as it has been constantly advocated by Orzalesi and collaborators³⁴⁾. The study of $D = 10$ spontaneous compactifications has only just begun. As a final remark¹⁹⁾, we can report the following very recent developments relating to the $D = 10$ and $D = 11$ programmes:

$D = 10$ programme - A recent result by Green and Schwarz indicates that there is a unique choice of the gauge group G which yields an anomaly-free theory [$G = SO(32)$]. This removes the drawback of the arbitrariness of G ³⁵⁾.

$D = 11$ programme - Recent investigations reveal that if the internal manifold M_7 is chosen to be non-compact but of finite volume, both the chirality and cosmological problem might be overcome³⁵⁾.

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Table I - Coset manifolds with Killing spinors

N	HOL	$\mathcal{S}$	$\mathcal{R}$	$\mathcal{S}'$	Manifold name
8	1	$SO(8)$	$SO(7)$	1	S_7^{round}
2	SU_3	$SU_3 \times SU_2 \times U_1$	$SU_2 \times U_1 \times U_1$	$SU_3 \times SU_2$	M^{111}
2	SU_3	$SU_2 \times SU_2 \times SU_2 \times U_1$	$U_1 \times U_1 \times U_1$	$[SU_2]^3$	Q^{111}
3	SU_2	$SU_3 \times SU_2$	$SU_2 \times U_1$	SU_3	N^{011}
1	G_2	$SO_5 \times SO_3$	$SO_3 \times SO_3$	$SO_5 \times SO_3$	$S_7^{(\text{squashed})}$
1	G_2	$SU_3 \times U_1$	$U_1 \times U_1$	$SU_3 \times U_1$	N^{pqr}
2	SU_3	$SO_5 \times SO_2$	$SO_3 \times SO_2$	SO_5	?
1	G_2	SO_5	SO_3	SO_5	?