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FINITE TEMPERATURE CORRECTIONS IN $SU(N,1)$ SUPERGRAVITY

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A B S T R A C T

We compute finite temperature corrections to the scalar potential in maximally symmetric $SU(N,1)$ supergravity. In the large N limit, the zero temperature potential is only scaled by the thermal corrections. As an example, we apply the general result to a class of phenomenologically interesting no-scale supergravity models.

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Locally supersymmetric theories are nowadays considered as a promising solution¹⁾ to the gauge hierarchy problem, as well as an appealing possibility for superunification of gravity with strong and electroweak interactions. In particular, $N = 1$ supergravity is regarded as an effective (non-renormalizable) theory valid at energies below the Planck scale and where all gravitational radiative corrections are already included.

The breaking of gauge symmetries (grand unified and electroweak) in $N = 1$ supergravity has been extensively studied in the literature and models describing the breaking pattern $SU(5) \rightarrow SU(3) \times SU(2) \times U(1) \rightarrow SU(3) \times U(1)^{em}$ have been proposed¹⁾. However, the existence of the (zero temperature) potential with the correct vacuum structure is not enough to ensure the occurrence of the phase transition; in other words, the question arises why the field is not sitting in the global (broken) minimum from the very beginning. The answer is that finite temperature effects can trigger the phase transition if the field is driven to the local (symmetric) minimum at high temperatures.

Finite temperature corrections in minimal supergravity have been recently computed by Olive and Srednicki²⁾ and Binétruy and Gaillard³⁾. However, their results, and in particular the large N limit, where N hereafter will refer to the number of chiral superfields, cannot be generalized to non-minimal supergravity.

There is a particularly interesting class of non-minimal supergravity models⁴⁾⁻⁶⁾, where all light mass scales ($M_W, m_{3/2}$) are dynamically generated, $O(e^{-O(1)/\alpha_{M_P}})$, by non-gravitational radiative corrections and where the cosmological constant problem finds a natural solution, at least at the tree level. They have the structure of non-linear σ models where the scalar fields parametrize the coset space $SU(N,1)/SU(N) \times U(1)$. Thermal corrections in an inflationary supergravity model with non-minimal kinetic terms have recently been considered in Ref. 7).

In this letter we will compute the finite temperature corrections in maximally symmetric, $SU(N,1)/SU(N) \times U(1)$, supergravity. As an example, we will apply the general result to the simplest no-scale supersymmetric grand unified model.

The scalar potential of $N = 1$ supergravity is⁸⁾,

$$V = e^{\mathcal{G}} [q^i (q^{-1})^j_i q_j - 3] + \frac{1}{2} \text{Re} f_{\alpha\beta}^{-1} D^\alpha D^\beta \quad (1)$$

in $M = M_P / \sqrt{8\pi} = 2.4 \times 10^{18}$ GeV units, where $\mathcal{G}$ is the Kähler potential, $\mathcal{G}_i$ are the derivatives

$$q_i = \frac{\partial}{\partial y^i} \mathcal{G}, \quad q^i = \frac{\partial}{\partial y^+_i} \mathcal{G}, \quad (2)$$

the D term is given by

$$D^\alpha = g q_i (\tau^\alpha)^i_j \phi^j \quad (3)$$

and $f_{\alpha\beta}$ is an analytic function of the complex scalar fields (y^i) contained in the chiral multiplets. The superfield content is y^i ($i=0, \dots, N-1$), with $y^0 \equiv z$ and $y^j \equiv \phi^j$ ($j=1, \dots, N-1$): the z field is a gauge singlet - responsible for the spontaneous breaking of supergravity - and ϕ^i correspond to squarks, sleptons, $\underline{5}$ and $\bar{\underline{5}}$ of Higgses H and $\bar{H}$, $\underline{24}$ of Higgses Σ - responsible for $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$ breaking - and, possibly, other complex scalar fields.

The finite temperature corrections to the potential (1) arise through loops of scalar fields and their fermionic partners, of vector bosons and their partners (gauginos) and of gravitinos, that we assume to be in thermal equilibrium. They are given at one loop by⁹⁾

$$\Delta V_T = -\frac{\pi^2}{24} (N+1) T^4 + \frac{1}{24} T^2 [T_R M_0^2 + T_R M_{1/2}^2 + T_R M_{3/2}^2 + 3 T_R M_1^2] \quad (4)$$

where M_J^2 is the square mass matrix of properly normalized, i.e., with canonical kinetic terms, spin J fields. In particular the scalar fields y^i (and their supersymmetric partners) are normalized with the metric $\mathcal{G}_j^i$ as

$$y^i = (q^{-\frac{1}{2}})^i_j y^j_N \quad (5)$$

and gauginos, and gauge bosons, with the metric $f_{\alpha\beta}$

$$\lambda^\alpha = \text{Re } f_{\alpha\beta}^{-\frac{1}{2}} \lambda_N^\beta \quad (6)$$

In fact, the theory is defined completely once the functions $\mathcal{G}(y^i, y_1^+)$ and $f^{\alpha\beta}(y^i)$ have been specified. For instance, in minimal supergravity

$$g_j^i = \delta_j^i, \quad f^{\alpha\beta} = \delta^{\alpha\beta} \quad (7)$$

redefinitions (5) and (6) are not necessary and the calculation of finite temperature corrections (4) simplifies considerably^{2),3)}. Maximally symmetric supergravity is characterized by the Kähler potential⁴⁾

$$g = G + F(\phi^i) + F^+(\phi_i^+) \quad (8)$$

where

$$G = -3 \log \left(f(z) + f^+(z^+) - \frac{1}{3} \phi^i \phi_i^+ \right) \quad (9)$$

and $F(\phi^i)$ is the superpotential function.

The scalar potential is given by⁴⁾

$$V = e^{\frac{2}{3}G} e^{F+F^+} F_i F^i + \frac{1}{2} \text{Re } f_{\alpha\beta}^{-1} D^\alpha D^\beta \quad (10)$$

and the metric for chiral superfields

$$g_j^i = \frac{1}{3} G^i G_j + e^{G/3} \delta_j^i \quad (11)$$

Using (11) one obtains for the normalized mass matrix M_0^2

$$\begin{aligned} (M_0^2)_j^i &= 2 \left[e^{-G/3} (V_j^i + \text{Re } f_{\alpha\beta}^{-1} D^{\alpha i} D_j^\beta) \right. \\ &\quad \left. + \frac{1}{\sqrt{3}} e^{-G/2} \frac{f^i V_j^0 + f_j V_0^i}{|f'|^2} + \frac{1}{3} e^{-\frac{2}{3}G} \frac{f^i f_j}{|f'|^4} V_0^0 \right] \end{aligned} \quad (12)$$

so that a direct calculation shows

$$\begin{aligned}
 \text{Tr } M_0^2 = & 2e^{\frac{1}{3}G} e^{F+F^+} \left[F_{jk} F^{jk} + \frac{2}{3} g_k g^k F_j F^j + 2e^{\frac{1}{3}G} F_j F^j + \right. \\
 & + \frac{1}{3} F^{jk} F_j (2g_k + F_k) + \frac{1}{3} F_{jk} F^j (2g^k + F^k) + \\
 & \left. + \frac{2}{3} e^{G/3} (N-1) F_i F^i \right] + 2g_N^2 e^{G/3} C_2(R) |\phi|^2
 \end{aligned} \tag{13}$$

where the term $C_2(R) |\phi|^2$ has implicitly a sum over representations, $g_N = g \text{Re} h^{-\frac{1}{2}}$ is the normalized gauge coupling constant, and we have taken the simplest (non-minimal) choice for $f_{\alpha\beta}$ which leads to a Majorana mass for gauginos

$$\begin{aligned}
 f_{\alpha\beta} &= \delta_{\alpha\beta} h(z) \\
 m_{\tilde{V}} &= \left| e^{\frac{G}{2} + (F+F^+)/2} \frac{G_0}{G_0'} (\log \text{Re} h)_z \right|
 \end{aligned} \tag{14}$$

The fermion mass matrix is given by

$$M_{\frac{1}{2}} = \begin{pmatrix} e^{(F+F^+)/2} m^{ij} & i\sqrt{2} D^{\alpha i} \text{Re} h^{-\frac{1}{2}} e^{-\frac{1}{6}G} \\ i\sqrt{2} D^{\alpha i} \text{Re} h^{-\frac{1}{2}} e^{-\frac{1}{6}G} & m_{\tilde{V}}^{\alpha\beta} \delta^{\alpha\beta} \end{pmatrix} \tag{15}$$

and the chiral fermion mass matrix is

$$\begin{aligned}
 m^{ij} &= \left(\frac{2}{3} g^i g^j + F^{ij} + \frac{1}{3} F^i F^j \right) e^{\frac{1}{6}G} \\
 m^{i0} &= -\frac{2}{\sqrt{3}} e^{G/3} g^i ; \quad m^{00} = 2e^{G/2} \quad (i, j = 1, \dots, N-1)
 \end{aligned} \tag{16}$$

Now

$$\begin{aligned}
 T_n M_{1/2}^2 &= e^{\frac{1}{3}G+F+F^+} \left\{ \frac{2}{3} g^i g^j + F^{ij} + \frac{1}{3} F^i F^j \right\} \times \\
 &\times \left\{ \frac{2}{3} g_i g_j + F_{ij} + \frac{1}{3} F_i F_j \right\} + \frac{8}{3} e^{F+F^+ + \frac{2}{3}G} g^i g_i \\
 &+ 4e^G + 4e^{\frac{1}{3}G} g_N^2 C_2(R) |\phi^i|^2 + \dim(\text{Adj}) m_V^2
 \end{aligned} \tag{17}$$

In the same way the contribution from gauge vector bosons is

$$T_n M_1^2 = 2e^{\frac{1}{3}G} g_N^2 C_2(R) |\phi^i|^2 \tag{18}$$

To compute the contribution of the gravitino, we proceed as Binétruy and Gaillard³⁾. In the gauge $\gamma^\mu \psi_\mu = 0$, the gravitino field decouples from the Goldstino and the effective action can be easily computed. This gives a contribution to the one-loop finite temperature corrections as

$$T_n M_{3/2}^2 = -2e^G \tag{19}$$

where the minus sign in (19) is a reflection that we have separated the contribution of the gravitino and Goldstino fields. In fact, in the fermion mass matrix (16) there is a term, $\frac{2}{3} g^i g^j$, which corresponds to the Goldstino field and which is eaten by the gravitino (super-Higgs effect) at the minimum of the potential. However, for the calculation of finite temperature corrections, we need the mass matrices as functions of the fields, and not only at the global minimum of the potential, so that we have to separate in the mass matrices the contributions from the (two-component) Goldstino, (17), and the (two-component) gravitino, (19).

The total finite temperature potential

$$V(y, T) = V(y) + \Delta V_T \tag{20}$$

can be read off from (1), (4), (13), (17), (18) and (19). In particular, the large N limit of (20) can be easily extracted and gives

$$V(y, T) \simeq \left(1 + \frac{T^2}{18} N\right) e^{\frac{2}{3} G + F + F^+} F_i F^i \quad (21)$$

plus a term which is field independent.

We can state that, in the large N limit, the global minima of the total finite temperature potential are the same as those of the zero temperature potential, and given by the supersymmetric condition

$$F_i = 0 \quad (22)$$

However, there are often degenerate supersymmetric minima and non-leading terms in N can break this degeneracy. In particular, once we impose the condition (22), the global supersymmetric minimum is required to minimize the ϕ dependent part of the total potential

$$\begin{aligned} V = \frac{T^2}{24} \Big\{ & e^{\frac{1}{3} G + F + F^+} \left[3 F_{ij} F^{ij} + \frac{4}{3} e^{\frac{4}{3} G} |\phi^i|^4 + \right. \\ & \left. + \frac{2}{3} e^{\frac{2}{3} G} (\phi^i \phi^j F_{ij} + F^{ij} \phi_i \phi_j) + \frac{8}{3} e^G |\phi^i|^2 \right] \\ & \left. + \dim(\text{Adj}) M_V^2 + 2 e^{G + F + F^+} + 12 e^{G/3} g_N^2 C_2(R) |\phi^i|^2 \right\} \end{aligned} \quad (23)$$

where we have included the condition (22) and $D^\alpha = 0$.

Since the potential (23) is not positive definite, it is not evident that finite temperature corrections will prefer the symmetric phase $\langle \phi^i \rangle = 0$. On the contrary, for each particular model (particle content and superpotential), this has to be checked from (22) and (23). As an example, we will consider a particularly interesting and simple model: the no-scale supersymmetric GUT proposed in Ref. 4). There the adjoint Higgs has no superpotential so that the minimum condition (22) is automatically satisfied. Moreover, as the D terms vanish identically along the $SU(3) \times SU(2) \times U(1)$ invariant direction, the tree level potential is simply

$$V(T, \Sigma) = \frac{T^2}{12} \left[\frac{2}{3} m_{3/2}^{10/3} |\Sigma|^4 + 30 g_N^2 m_{3/2}^{2/3} |\Sigma|^2 + \frac{4}{3} m_{3/2}^{8/3} |\Sigma|^2 \right] \quad (24)$$

where $m_{3/2} = e^{G/2}$ is the gravitino mass. However, in the presence of supersymmetry breaking gaugino masses, radiative corrections will induce a radiative potential, which in the leading log approximation is

$$V_{\text{rad}} \simeq - \frac{1}{64\pi^2} m_{\tilde{V}}^2 |\Sigma|^4 \log |\Sigma|^2 \quad (25)$$

The potential (25) has been discussed in detail in Ref. 4) and is expected to favour the largest possible expectation value $\Sigma \simeq M$. However, at very high temperatures (24) will dominate over (25) and the field will be confined to the symmetric phase $\Sigma = 0$.

If $m_{3/2} \simeq M$, the critical temperature of the phase transition is expected to be $T_C \simeq O(m_{\tilde{V}})$. However, the dynamical determination of the electroweak scale fixes $m_{\tilde{V}} \simeq O(10^2)$ GeV, and therefore this possibility does not appear to be feasible, e.g., from the point of view of baryogenesis. On the other hand, in this case the height of the barrier separating the symmetric and broken phases is much higher than the thermal energy, so that the transition can only proceed through quantum tunnelling with a huge lifetime.

In the case where $m_{3/2} \ll M$, we find

$$T_C \simeq O(m_{\tilde{V}} m_{3/2}^{-1/3}) \quad (26)$$

Recently, such a class of $SU(N,1)$ models with $m_{3/2} \simeq m_{\tilde{V}}^P$, has been proposed^{6),10)}. They allow a dynamical relaxation of θ_{QCD} to zero while being compatible with all particle physics and cosmology constraints provided $1.2 < P < 1.4$. This would correspond to $T_C \lesssim 10^{10}$ GeV, automatically satisfying the Parker bound on the average monopole flux in the galaxy. We will return to these questions in more detail in a forthcoming publication¹¹⁾.

In conclusion, we have calculated thermal corrections in no-scale supergravity models. In the large N limit the minima of the total finite temperature potential are the same as those of the zero temperature potential. In this limit, the zero temperature potential is only scaled by the finite temperature corrections. The large N limit may, however, not be applicable as such to realistic models, and each case should be examined separately.

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R E F E R E N C E S

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