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A PERTURBATIVE APPROACH TO THE CHIRAL SYMMETRY BREAKING

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ABSTRACT

In the framework of the 2-d Gross-Neveu model we show that the perturbative fermionic vacuum, in a space time with topology of a cylinder, $S^1 \times R$, exhibits spontaneous breaking of chiral symmetry. This is quite a different phenomenon from the finite temperature behaviour of the model. Indeed it is shown that at zero temperature the large N approximation reproduces the effect for small volumes.

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An important problem in AF gauge theories with massless fermions remains the mechanism of chiral symmetry breaking. It is usually assumed that the Nambu-Goldstone realization of chiral symmetry takes place, that is while the symmetry is explicit in the Lagrangian, it is spontaneously broken by the formation of a fermion-antifermion condensate in the vacuum^{1),2)}. Rigorous results³⁾, Monte Carlo techniques^{4a)} and strong coupling expansions^{4b)} of lattice gauge theories suggest the existence of a chiral symmetry breaking phase below some critical temperature or above some critical value of the coupling constant.

One attractive idea to attack the problem is the criterion of Banks and Casher⁵⁾ that one should have a physical environment for the fermions in which the density of eigenvalues $\rho(\lambda)$ of the Dirac operator is of the one-dimensional free fermion type near $\lambda = 0$ ⁵⁾. There is a continuous effort to characterize gauge field configurations which create this environment^{5),6),7)}.

In this work I would like to propose the idea of creating such an environment for the fermions by enclosing the AF theories with massless fermions in a d -dimensional space-torus, $T^d = (S^1)^d$, of side length L , and in the time direction fermions satisfy antiperiodic b.c.

$$\begin{aligned}\psi(x,0) &= -\psi(x,T) \\ \bar{\psi}(x,0) &= -\bar{\psi}(x,T)\end{aligned}\tag{1}$$

Although we shall work with the two-dimensional Gross-Neveu model⁸⁾, the method can be extended to four-dimensional for gauge theories^{6d)}.

To study chiral symmetry breaking we shall give a small mass to the fermions which we shall put equal to zero after the large time, T , limit is taken. Due to zero three-momentum modes, a condensate is formed, even for the free fermionic theory^{6a)}. The condensate is $\sim 1/L^3 \times$ logarithms of μL in every order of perturbative expansion in the renormalized coupling constant $a(\mu)$ and so when $L \rightarrow \infty$ it vanishes.

The philosophy to be taken here is analogous to that for the case of mass gap⁹⁾, that is if we could calculate exactly $\langle \bar{\psi}\psi \rangle(L)$ in a finite space box of side length L , this would differ from the exact $\langle \bar{\psi}\psi \rangle$ at $L = \infty$ by exponentially small terms $\sim e^{-a \cdot L}$ ¹⁰⁾.

We shall show that this happens indeed in the G-N model in the large N-approximation. Now, since we cannot calculate exactly $\langle \bar{\psi}\psi \rangle(L)$, it will be consistent if we estimate this quantity at values where it is flat in AF perturbation theory, as a function of L. That there is such a region is guaranteed by asymptotic freedom as we shall see. The question is if this region in L is near small values of the coupling constant.

Another point which is of importance in this attempt is to examine whether finite L mimics temperature effects. Although the p.b.c. in space give quite different spectrum of modes, it is not obvious that this does not happen. As we shall see in the large N expansion, temperature effects restore chiral symmetry¹¹⁾ at a critical temperature, while periodic space boundary effects disappear exponentially fast with L and we end up with a non-zero value of the condensate.

After this long introduction, let us now discuss the Gross-Neveu model. The Lagrangian of the model in Euclidean two-dimensional space-time is

$$\mathcal{L}_{GN} = \sum_{a=1}^N \bar{\psi}^a (i\not{\partial} - m) \psi^a - \frac{f}{2N} \left(\sum_{a=1}^N \bar{\psi}^a \psi^a \right)^2. \quad (2)$$

This model has been extensively studied in the literature^{8),11),12),13)}. It is AF and in the large N approximation exhibits dynamical breaking of the discrete chiral symmetry for $m = 0$:

$$\begin{aligned} \psi &\rightarrow \gamma^5 \psi \\ \bar{\psi} &\rightarrow \bar{\psi} \gamma^5. \end{aligned} \quad (3)$$

What happens is that a condensate of fermionic pairs of opposite chirality is formed in the vacuum and $\langle \bar{\psi}\psi \rangle$ is different from zero. We study this model in a space time $[0, L] \times [0, T] \times \mathbb{R}^{-\epsilon}$, where ϵ is the dimensional regulator for UV divergences, $\epsilon = 2-d$. The fermions satisfy p.b.c. in space

$$\begin{aligned} \psi(0, t) &= \psi(L, t) \\ \bar{\psi}(0, t) &= \bar{\psi}(L, t) \end{aligned} \quad \forall t \in [0, T] \quad (4a)$$

and antiperiodic in time

$$\begin{aligned}\psi(x,0) &= -\psi(x,T) \\ \bar{\psi}(x,0) &= -\bar{\psi}(x,T)\end{aligned}\quad \forall x \in [0,L] \times \mathbb{R}^{-\varepsilon} \quad (4b)$$

These b.c. determine the massive free fermion propagator to be

$$S_F(x,y;m) = \frac{1}{TL} \int \frac{d^D k}{(2\pi)^D} \sum_{\ell, n=-\infty}^{+\infty} \frac{e^{ik(x-y)}}{k+m} \quad (5)$$

where $D = -\varepsilon$,

$$\begin{aligned}k_0 &= \frac{(2n+1)\pi}{T} \\ k_1 &= \frac{2\pi\ell}{L}\end{aligned}\quad n, \ell = 0, \pm 1, \pm 2, \dots \quad (6)$$

and the 2×2 γ -matrices are chosen to satisfy the $2-\varepsilon$ dimensional Euclidean Clifford algebra

$$\{\gamma_\mu, \gamma_\nu\} = -2\delta_{\mu\nu} \quad (7)$$

We define the condensate for $m \neq 0$ as

$$\int d^{2+D}x \langle \bar{\psi}\psi(x) \rangle_m \equiv \frac{\partial}{\partial m} \ln Z(m) \quad (8a)$$

where

$$Z(m) = \int d\psi d\bar{\psi} e^{-\int \mathcal{L} d^{2+D}x} \quad (8b)$$

and $\mathcal{L}$ is the massive G-N Lagrangian

$$\mathcal{L} = \mathcal{L}_{GN} = \bar{\psi} (i \not{\partial} - m) \psi - \frac{f}{2N} (\bar{\psi}\psi)^2. \quad (9)$$

Summation over the "flavour" $O(N)$ group indices is implied.

At the tree level it is easy to see that

$$\langle \bar{\psi}\psi \rangle = \frac{N}{L} \frac{\sinh mT/2}{\cosh mT/2} + O(m, e^{-T/L}) \quad (10)$$

So if the limit $T \rightarrow \infty$ is taken first and then $m \rightarrow 0$, we find ($f = 0$):

$$\langle \bar{\psi} \psi \rangle = \frac{N}{L} . \quad (11)$$

The result is not surprising: the zero momentum modes are responsible for this effect. In higher - say three - dimensions, this effect survives only if we take space to be a torus $T^3 = (S^1)^3$ and time T the real line.

In perturbative theory we know that the operator $\bar{\psi}\psi$ has an anomalous dimension $\gamma_{\bar{\psi}\psi}(f)$, defined through the renormalization group equation (RGE)¹⁴:

$$\left[\mu \frac{\partial}{\partial \mu} + \beta(f) \frac{\partial}{\partial f} + \gamma_{\bar{\psi}\psi}(f) \right] \langle \bar{\psi} \psi \rangle(L) = 0 . \quad (12)$$

Here $\beta(f)$ is the RG β -function

$$\beta(f) = -f^2 [b_0 + b_1 f + \dots] \quad (13)$$

and

$$\gamma_{\bar{\psi}\psi} = -f [\gamma_1 + f \gamma_2 + \dots] \quad (14)$$

The RGE (12) implies that $\langle \bar{\psi}\psi \rangle(L)$ (here $m = 0$) has the form

$$\langle \bar{\psi} \psi \rangle(L) = \frac{1}{L} \{ a_{00} + f (a_{11} \ln \mu L + a_{10}) + \dots \} , \quad (15)$$

where a_{11} and a_{00} are related through the RG Equation:

$$a_{11} = \gamma_1 \cdot a_{00} . \quad (16)$$

From relation (11) we have

$$a_{00} = N \quad (17)$$

We calculate now the first order term in f of $\langle \bar{\psi}\psi \rangle$: $\langle \bar{\psi}\psi \rangle(L) = \langle \bar{\psi}\psi \rangle^{(0)}(L) + f \cdot \langle \bar{\psi}\psi \rangle^{(1)}(L)$.

We expand $Z(m)$ up to order f and we find [see relations (8),(9)]

$$\langle \bar{\psi} \psi \rangle^{(0)}(L) = \frac{N}{L} \quad (18a)$$

$$\langle \bar{\psi} \psi \rangle^{(1)}(L) = \frac{N}{L} \cdot \frac{2N-1}{2\pi N} \left[\ln \mu L - \frac{1}{2} (\ln 4\pi - \gamma_E) \right]. \quad (18b)$$

From (18b) and (16) we obtain

$$\gamma_1 = \frac{2N-1}{2\pi N}, \quad (19)$$

$\gamma_E = 0.5772$ is the Euler-Macheroni constant. Note also that for large N , $b_0 = 1/\pi$ and $b_1 \sim O(1/N)^8$. We are interested in a result which is RG invariant, so we form the quantity

$$\widehat{\langle \bar{\psi} \psi \rangle}(L) \equiv (b_0 f)^{\gamma_1/b_0} \cdot \langle \bar{\psi} \psi \rangle(L), \quad (20)$$

which satisfies the RGE (up to first order in f)

$$\left[\mu \frac{\partial}{\partial \mu} + \beta(f) \frac{\partial}{\partial f} \right] \widehat{\langle \bar{\psi} \psi \rangle}(L) = 0. \quad (21)$$

Our result for $\langle \bar{\psi} \psi \rangle(L)$ should be compared to a natural scale of the model which we take to be $\Lambda_{\overline{MS}} = e^{\frac{1}{2}(\ln 4\pi - \gamma_E)}$. $\Lambda_{\overline{MS}}$ and

$$\Lambda_{\overline{MS}} = \mu (b_0 f)^{-b_1/b_0^2} \cdot e^{-1/b_0 f} \quad (22)$$

At infinite L we should predict the number $c = \widehat{\langle \bar{\psi} \psi \rangle}(L = \infty) / \Lambda_{\overline{MS}}$, so the best we can do is to look for stable values of the function

$$F(z) \equiv \widehat{\langle \bar{\psi} \psi \rangle}(L) / \Lambda_{\overline{MS}} \quad (23)$$

with respect to z changes.

Here z is defined to be

$$z \equiv L \cdot \langle \hat{\bar{\psi}} \psi \rangle (L) \quad (24)$$

This variable appears naturally when we solve

$$z = L \cdot \langle \hat{\bar{\psi}} \psi \rangle (L) = (b_0 f)^{\gamma_1/b_0} \left[N + \frac{f}{2\pi} (2N-1) (\ell_m L + \frac{1}{2} (\gamma_E - \ell_m 4\pi)) \right] \quad (25)$$

with respect to f in perturbative expansion in the variable z and we substitute the result into the definition of Λ_{ms} . We find

$$F(z) = K \cdot z^{1 + b_1/b_0 \gamma_1} \cdot e^{(a_{00}/z)^{b_0/\gamma_1}} \quad (26)$$

where

$$K = a_{00}^{-b_1/b_0 \gamma_1} e^{a_{11}/a_{00} \gamma_1 - \frac{1}{2} (\ell_m 4\pi - \gamma_E)} \quad (27)$$

From (15) and (18) we have

$$\begin{aligned} a_{00} &= N \\ a_{10} &= -\frac{2N-1}{4\pi} (\ell_m 4\pi - \gamma_E) \end{aligned} \quad (28)$$

In this work we shall not give numerical estimates for $\langle \bar{\psi} \psi \rangle$ for finite N . We shall show that the perturbative result (26) is reproduced for finite volume in the large N approximation and for small values of the variable z .

In order to derive the exact leading large N expression for the function F , we proceed to write down the gap equation as the saddle point condition at $N \sim \infty$ of the action:

$$\begin{aligned} Z(m) &= \int d\psi d\bar{\psi} d\sigma e^{-\int d^D x [\bar{\psi} (i\cancel{\partial} - m) \psi + \frac{N}{2f} \sigma^2 \sigma \bar{\psi} \psi]} \\ &= \int d\sigma e^{-\frac{N}{2f} \sigma^2 + N \text{tr} \ln [i\cancel{\partial} - m - \sigma]} \end{aligned} \quad (29)$$

We expand $\sigma(x)$ around a constant σ_0 value.

$$\sigma(x) = \sigma_0 + \left(\frac{f}{2N}\right)^{1/2} \sigma'(x) \quad (30)$$

and we get the saddle point condition; in finite volume

$$\frac{1}{2f} = +r \frac{1}{-\partial^2 + \sigma_0^2 + m^2} \quad (31)$$

For $m \rightarrow 0$, this has the solution

$$\ell n \frac{\sigma_0}{\Lambda_{\overline{ms}}} = \int_0^\infty \frac{dt}{t} e^{-t\xi^2} \sum_{\ell=1}^\infty e^{-\frac{\ell^2}{4t}} \equiv 2 \sum_{\ell=1}^\infty K_0(\ell \cdot \xi), \quad (32)$$

where σ_0 represents the dynamically generated mass of the fermions, $K_0(x)$ is the modified Bessel function and $\xi = \sigma_0 L$.

Now, from (29) it is easy to obtain the relation

$$N \langle \sigma \rangle = f \langle \bar{\psi} \psi \rangle \quad (33)$$

which for large N becomes

$$N \sigma_0 = f \langle \bar{\psi} \psi \rangle \quad (34)$$

From (32) we get the leading small ξ behaviour,

$$\frac{f \langle \bar{\psi} \psi \rangle (L)}{N \cdot \Lambda_{\overline{ms}}} = \frac{1}{4\pi} e^{\gamma_E \xi} \xi e^{\pi/\xi}, \quad (35)$$

where

$$\xi \equiv \frac{f \langle \bar{\psi} \psi \rangle (L)}{N} \cdot L$$

Now we observe two interesting things. First, (32) goes exponentially fast to zero, so $f \langle \bar{\psi} \psi \rangle (L) / N \xrightarrow{L \rightarrow \infty} \Lambda_{\overline{ms}}$, which is the infinite volume value of the condensate⁸⁾: Second, the leading small ξ behaviour, relation (33), reproduces the large N form of the perturbative result (26). In Fig. 1 we plot (a) the exact large N function $F(z)$, and (b) its leading small ξ behaviour. We see that the best value from the perturbative result (26) for large N is the minimum or "flat" region of $F(\xi)$.

Finite volume corrections¹⁰⁾ will change the numerical values and we shall present them in another place.

Finally, we want to show that finite time situation with antiperiodic b.c. and infinite space $L \rightarrow \infty$ (finite temperature), has completely different physics for large N from the finite L with periodic b.c. and $T \rightarrow \infty$. We look again in the mass gap equation (31) but now with the new b.c. We get

$$\lim_{\Lambda_{\overline{m}s}} \frac{\sigma_0}{\Lambda_{\overline{m}s}} = 2 \sum_{\ell=1}^{\infty} (-1)^{\ell} K_0(\ell \cdot \xi), \quad (36)$$

in agreement with Ref. 11).

In Fig. 2 we plot $\sigma_0/\Lambda_{\overline{m}s}$ as a function of the temperature $t = 1/T$ in units of $\Lambda_{\overline{m}s}$, $t/\Lambda_{\overline{m}s} = 1/T\Lambda_{\overline{m}s}$. For small temperatures we get a non-zero condensate which gradually vanishes as we approach the critical temperature t_c . For higher than t_c temperatures there is restoration of the discrete chiral symmetry. We note the difference in the behaviour with the finite space infinite time situation. In Fig. 3 we plot for both cases, $L < \infty$, $T \rightarrow \infty$ and $T < \infty$, $L \rightarrow \infty$ the variation of the function $\sigma_0/\Lambda_{\overline{m}s}$ with $1/L\Lambda_{\overline{m}s}$ and $1/T\Lambda_{\overline{m}s}$ respectively.

For a theory of bosons, both cases would be identical.

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FIGURE CAPTIONS

Fig. 1 (a) The large N condensate. $N^{-1}\hat{\langle\bar{\psi}\psi\rangle}(L) = N^{-1}f\langle\bar{\psi}\psi\rangle(L)$ as a function of $\xi = N^{-1}\langle\bar{\psi}\psi\rangle \cdot L$.

(b) The leading small ξ behaviour of the large N condensate as a function of ξ .

Fig. 2 The temperature dependence of the large N condensate $N^{-1}\hat{\langle\bar{\psi}\psi\rangle}(T)$ as a function of $(T\Lambda_{\overline{ms}})^{-1}$.

Fig. 3 Comparative plot of the large N, finite L, $T = \infty$ or finite t, $L = \infty$ condensate as a function of $(L\Lambda_{\overline{ms}})^{-1}$ and $(T\Lambda_{\overline{ms}})^{-1}$ respectively.

Fig. 1

Fig. 2

Fig. 3