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DEMONIC CONSTRAINTS ON SUPERGRAVITY MODELS

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ABSTRACT

We consider renormalization group scaling of the CP-violating phases of softly broken supersymmetry. We show that unless $A(M_X)$ is real or close to zero, these phases are naturally large in conflict with the experimental bound on the dipole electric moment of the neutron (DEMON) d_N . We also show that for a large class of supergravity models $d_N \approx 10^{-28}$ e.cm (and could even be as large as 10^{-27} e.cm), a prediction that is two orders of magnitude higher than the standard model Kobayashi-Maskawa prediction.

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The dipole electric moment of the neutron (DEMON), d_N , provides an important testing ground for models of CP-violation. The standard three-generation electroweak model (with CP-violation through the Kobayaski-Maskawa phase δ) predicts a DEMON $\lesssim 10^{-30}$ e.cm, which is smaller than the present experimental bounds by five orders of magnitude. Future experimental measurements of d_N with higher precision will therefore be of great importance in discriminating between different models with predictions lying in between. In particular, if the measured value of d_N happens to be significantly larger than the standard model prediction, then one would be compelled to take a closer look at alternative models of CP-violation. In any case, any new proposal for a CP-violating model must respect the current experimental bound ($d_N < 4 \times 10^{-25}$ e.cm) and conversely the DEMON can be used to set restrictions on the parameters of the new model.

The current popularity of models of the fundamental interactions incorporating supersymmetry (SUSY) has naturally led to several investigations of their CP-violation predictions.¹⁾⁻⁴⁾ In addition to δ there are a number of new phases in these models, all of which provide potential sources of CP non-invariance. It turns out, however, that these new phases are strongly bound by the DEMON constraint and always lead to a very small value of the ϵ -parameter of the $K^0 - \bar{K}^0$ system. The phase δ is distinguished in this respect. Through intrinsically supersymmetric graphs (with internal gluino and squark lines) it can explain the ϵ -parameter with a light top quark and predicts a value for ϵ'/ϵ which is consistent with the new experimental results.^{2),5)-6)} For the DEMON, on the other hand, the gluino mediated contribution turns out to be exactly vanishing.²⁾ This is a consequence of the way in which the phases appear in the usual squark mass matrices realized in $N = 1$ supergravity (SUGRA) type models. Thus if δ is the only phase available, then these models would predict a DEMON of the same size as the standard model. If d_N turns out to be larger than this value, then in the context of SUSY models it would imply a non-zero (but small) value of a SUSY phase.

Another potential source of big d_N is the possibility of a non-vanishing value of Θ_{QCD} , the strong CP-violation parameter. Indeed, a non-zero but tiny ($\sim 10^{-9}$ - 10^{-13}) value of Θ_{QCD} will contribute to d_N as much as 10^{-25} - 10^{-29} e.cm. We hereafter disregard such a possibility with the belief that mechanisms naturally disposing of the strong CP problem either by dynamical relaxation,⁷⁾ or by use of the invisible axion⁸⁾ give a vanishing or less than $O(10^{-15})$ value for Θ_{QCD} respectively, i.e., they are harmless for d_N .

In this note we concentrate on the SUSY phases that can be present in the trilinear scalar coupling A and the Higgs (H_1-H_2) coupling B which arise naturally in the observable sector of $N = 1$ SUGRA models. It is known that these are the only two non-trivial ϕ SUSY phases in these theories, i.e., other phases (e.g., a phase in the gluino majorana mass) can be shifted into these by suitable phase rotations. We ask ourselves the following question: what is the range of the parameters of the theory that results in sufficiently small values of these phases at M_W ? In particular, if the phases of the theory are arbitrary, i.e., $O(1)$, at M_X , what must we require for the other parameters of the theory in order that the values of the phases at M_W should be compatible with the DEMON bound? Moreover, even if these phases were strictly zero at M_X , CP-violation would still leak to A and B from the Yukawa sector through renormalization scaling. In general, this implies that there is a lower bound on the magnitude of DEMON.

The contribution of the SUSY phases through gluino mediated graphs to the dipole moment of a quark can be written as^{5),2)}

$$d(q) = \frac{2e}{3\pi} |A(M_W)| \left(\frac{M_W}{\tilde{m}}\right)^2 \frac{\tilde{D}(x^2)}{x^3} |\sin 2\phi| \quad (1)$$

with

$$x = \tilde{M}/\tilde{m} \quad (2)$$

and

$$\tilde{D}(x) = \frac{x^2}{(x-1)^3} \left\{ \frac{x}{2} + \frac{5}{2} - (2x+1) \frac{\ln x}{x-1} \right\} \xrightarrow{x \rightarrow 1} \frac{1}{12} \quad (3)$$

Here $\tilde{m}$ and $\tilde{M}$ are the gluino and average squark masses. For a rough estimate we use the current expectations of $\tilde{m} \approx \tilde{M} \approx M_W/2$, $A(M_W) \sim O(1) - O(10)$, and the experimental bound on the DEMON to get

$$\phi \lesssim 10^{-2} \quad (4)$$

The two non-trivial phases of our interest must respect the bound of Eq. (4) at M_W . The evolution of these phases as a function of the mass scale can be extracted from the standard renormalization group equations⁹⁾ (RGE). For the moment, we will neglect all the contributions from the Yukawa couplings except from that of the top quark. The equations for the parameters A and B can then be integrated to give¹⁰⁾

$$A(M_W)(1+6r^2) = A(M_X) + C_A \xi \quad (5)$$

and

$$\begin{aligned} B(M_W) &= A(M_X) - 1 + C_B \xi - 3r^2 A(M_W) \\ &= A(M_X) \left(\frac{1+3r^2}{1+6r^2} \right) - 1 + \left(C_B - \frac{3r^2}{1+6r^2} C_A \right) \xi \end{aligned} \quad (6)$$

where C_A and C_B are integrals from M_X to M_W of functions of the running gauge coupling constants and in the model with the minimal particle content take the values 4.06 and 0.60 respectively. The constant r^2 is an integral that involves, in addition, the top-quark mass, m_t , and its value in the minimal model is 0.01 when $m_t = 40$ GeV. Finally

$$\xi = m_0 / M_0 \quad (7)$$

where M_0 and m_0 are respectively the flavour-blind scalar mass introduced by soft SUSY breaking through SUGRA and the gaugino mass, both at M_X . In deriving Eq. (6), the usual boundary condition $B = A-1$ at M_X has been used. The approximations involved in integrating the RGE to get Eqs. (5) and (6) have been checked numerically to be very good and reliable.

In order to answer the question that we have posed we must look at the real and imaginary parts of Eqs. (5) and (6). Defining $A = A_0 e^{i\phi_A}$ and $B = B_0 e^{i\phi_B}$ where A_0 and B_0 are real, we get from Eq. (5)

$$\sin \phi_A(M_W) = A_0(M_X) \sin \phi_A(M_X) / A_0(M_W)(1+6r^2) \quad (8)$$

and

$$A_0^2(M_W)(1+6r^2)^2 = A_0^2(M_X) + 2A_0(M_X)C_A \xi \cos \phi_A(M_X) + C_A^2 \xi^2 \quad (9)$$

From Eq. (6) we obtain, using Eq. (8)

$$\begin{aligned} \sin \phi_B(M_W) &= \frac{A_0(M_X)}{B_0(M_W)} \sin \phi_A(M_X) \frac{1+3r^2}{1+6r^2} \\ &= \frac{A_0(M_W)}{B_0(M_W)} (1+3r^2) \sin \phi_A(M_W) \end{aligned} \quad (10)$$

From Eq. (9) it is apparent that if $A_0(M_X)$ is chosen sufficiently small, then $A_0(M_W) \approx C_A \xi / (1+6r^2)$. From Eq. (8) it is then evident that for small $A_0(M_X)$ the value of ϕ_A at M_W will always be very small independently of the boundary condition on ϕ_A at M_X . Indeed, from Eqs. (8) and (9) one can determine an upper bound on $A_0(M_X)$ for a fixed value of ξ , such that $\phi_A(M_W)$ will be below the experimental bound $\phi_A^{\text{exp}} \approx 10^{-2}$. The exact calculation gives

$$A_0(M_X) \leq \frac{C_A \sin \phi_A^{\text{exp}} \sin [\phi_A^{\text{exp}} + \phi_A(M_X)]}{\sin^2 \phi_A(M_X) - \sin^2 \phi_A^{\text{exp}}} \xi \quad (11)$$

The bounds in Fig. 1 have been obtained using Eq. (11). Once $\phi_A(M_X)$ is fixed, then the slope of the straight line is determined. We exhibit curves for $\phi_A(M_X) = \pi/4$ and $\pi/8$. It is clear from the figure that the bounds on $|A(M_X)|$, though dependent on $\phi_A(M_X)$ are not especially sensitive. We have checked that even if $A(M_X)$ is chosen to be purely imaginary, i.e., $\phi_A(M_X) = \pi/2$, then the bounds on $|A(M_X)|$ are not drastically different. Of course it is clear from Eq. (8) that if $A(M_X)$ is real [$\phi_A(M_X) = 0$], then, when neglecting off-diagonal Yukawa couplings, $A(M_W)$ is real as well, irrespective of $|A(M_X)|$.

The bounds that we obtain on $|A|$ in this manner can actually be translated into bounds on the mass splitting, $\Delta \tilde{M}$, between the left- and right-handed squarks at M_W , using

$$\Delta \tilde{M} / \tilde{M} = A_0(M_W) \quad (12)$$

At M_W the gluino and squark masses are given by

$$\tilde{m} = \frac{\alpha_3}{\alpha_G} m_0 \approx 3 m_0 \xi \quad (13)$$

and

$$\tilde{M}^2 = m_0^2 (1 + C_Q \xi^2) \quad (14)$$

where $C_Q = 7.26$ in the minimal model. Using Eqs. (12) to (14), we get the bounds on $\Delta \tilde{M} / \tilde{M}$ as a function of $\tilde{m}$ shown in Fig. 2. The nature of this curve can be understood by noting that fixing $\tilde{M}$ sets an upper bound on $\tilde{m}$ given by

$$\tilde{m} \leq 3 \tilde{M} / \sqrt{C_Q} \quad (15)$$

For the figure we have chosen $\tilde{M} = 50$ GeV. If the squark mass $\tilde{M}$ is scaled by a factor d , then $\tilde{m}$ is similarly scaled and $\Delta\tilde{M}/\tilde{M}$ is scaled by d^{-1} .

Similarly, we can use $|\phi_B^{\text{exp}}| \lesssim 10^{-2}$ to set a limit on $A_0(M_X)$. We find

$$A_0(M_X) \leq \frac{1+6r^2}{1+3r^2} \frac{\tan \phi_B^{\text{exp}} [C\xi - 1]}{\cos \phi_A(M_X) [\tan \phi_A(M_X) - \tan \phi_B^{\text{exp}}]} \quad (16)$$

where $C = C_B - 3r^2 C_A / (1+6r^2)$. The resulting stringent bound is also shown in Fig. 1. We conclude that if the SUSY phases ϕ_A and ϕ_B were $O(1)$ at M_X , the current upper bound on the DEMON implies a value of $A_0(M_X)$ very near zero, typically $O(10^{-2})$ at most. We would like to point out that there exists a class of interesting models where $A_0(M_X) = 0$ naturally, namely the no-scale supergravity models^{11),7)} based on the non-compact $SU(n,1)$ Kähler manifold. On the other hand, if we use popular values of $A_0(M_X)$, such as $A_0(M_X) = 3$, compatibility with the DEMON bound forces us to choose $\phi_A(M_X) \lesssim 10^{-3}$.

It appears that there is some unnaturalness in the above choices needed to satisfy the DEMON bound [although it has been argued that in some models the SUSY phases are naturally tiny⁴⁾]. An obvious possibility would be to insist that the correct model should have only real $A(M_X)$ and real $B(M_X)$. However, even then $A(M_W)$ and $B(M_W)$ will contain complex phases, the origin of which are the renormalization effects due to the KM phases present in the off-diagonal Yukawa coupling contribution to the RGEs of A and B . (Since the Yukawa couplings enter into the RGE in the form $hh^\dagger$, there are no phases in the diagonal terms.) However, we expect the resulting phases at M_W to be much below the limit 10^{-2} . Indeed, if $A(M_X)$ is real, we find from the complete RGE¹²⁾ that in the leading approximation

$$\sin \phi_A(M_W) \simeq \frac{3}{(4\pi)^2} h_t h_c \sin [\delta(M_W) + \phi_A(M_W)] \quad (17)$$

where h_i 's are the Yukawa couplings and the relative phase between the two Yukawa couplings is defined as δ . We expect to find a similar order of magnitude estimate for ϕ_B , too. From Eq. (17) we find that, assuming $\delta \approx O(1)$ and two Higgs doublets with equal vevs $v_1 = v_2 = v$,

$$\phi_A \lesssim 4 \times 10^{-5} \quad (18)$$

Indeed, as we have no reason to expect that the phase δ will be small, Eq. (18) probably gives the correct order of magnitude for ϕ_A in the case where $A(M_X)$ is real.

This means that the DEMON due to the supersymmetric CP-violating phases could be as high as 10^{-27} e.cm. Taking as a conservative estimate $\sin\delta \approx 0(10^{-1})$, we find that the natural order of magnitude of the DEMON is predicted to be

$$d_N \approx 10^{-28} \text{ e.cm} \quad (19)$$

two orders of magnitude larger than what one would expect from the standard model alone. Such a relatively large DEMON falls in the range that will be experimentally accessible in the very near future.

In the case where $A(M_X)$ is real, it is interesting to note that d_N can be related to the ϵ -parameter of the Kaon system, since both depend on the angle δ . In supersymmetry, the leading contribution to ϵ is proportional to $(m_t/m_c)^4$, and combining Eq. (17) with the results of Ref. 2) we find very roughly that

$$\phi_A \approx \frac{4 \times 10^6}{f(x)} \left(\frac{\tilde{M}}{M_W} \right)^6 \left(\frac{m_c}{m_t} \right)^3 \left(\frac{m_c}{v} \right)^2 \epsilon \quad (20)$$

where $f(x)$ is a function of $x = \tilde{M}/\tilde{m}$ as discussed in Ref. 2). For $x \approx 1$, $f(x) \approx 0.2$. If $\tilde{M} \approx 40-50$ GeV, we obtain $\phi_A \approx 0(10^{-2}-10^{-3})\epsilon$.

In conclusion, we have discussed the various supersymmetric contributions to the dipole electric moment of the neutron. If the soft supersymmetry breaking terms have complex coupling constants (masses), the smallness of the present experimental bound on the DEMON seems to imply a naturality problem with the relevant phases at M_X , unless the trilinear coupling $A(M_X) \approx 0$. If $A(M_X) \approx 0$, or if it is real, there will still appear SUSY CP-violating phases at M_W due to renormalization effects of off-diagonal Yukawa couplings. These, while naturally small enough, will ensure the existence of a lower bound of the supersymmetric DEMON that is two, perhaps even three orders of magnitude above the standard model Kobayashi-Maskawa prediction and can be measured experimentally in the near future. Thus the magnitude of the DEMON can act as a decisive test for or against phenomenological low-energy supergravity.

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FIGURE CAPTIONS

Fig. 1 The bounds imposed by the DEMON on $A_0(M_X)$ and ξ for

a) $\phi_A(M_X) = \pi/8$ and

b) $\phi_A(M_X) = \pi/4$.

The allowed region is below the lines shown in the figures.

Fig. 2 The DEMON bound in the $(\tilde{m}, \tilde{\Delta M}/\tilde{M})$ plane in the case $\tilde{M} = 50$ GeV,

$\phi_A(M_X) = \pi/4$. The cut at $\tilde{m} = 51.6$ GeV corresponds to the condition that the lightest slepton should weigh more than 20 GeV.

Fig. 1a

Fig. 1b

Fig. 2