Testing $O(\alpha)$ corrections of the electroweak theory: A systematic overview

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Various possibilities of testing $O(\alpha)$ corrections in the electroweak theory are classified and in each case the experimental precision needed for an unambiguous test is given.

I. INTRODUCTION AND GENERAL REMARKS

We discuss the experimental precision which is needed in various experiments to test unambiguously $O(\alpha G_F)$ corrections in processes which are allowed in the zeroth-order approximation of the electroweak standard model. Precision tests of the electroweak theory have already been widely discussed¹ but we believe it is still worth systematizing certain points.

Apart from the fermion mass matrix and the Higgs-sector parameters, the Glashow-Salam-Weinberg (GSW) model is a three-parameter theory. Order by order in perturbation theory, physical quantities M_k are given in terms of the expansion parameter α_R and two other parameters, say, θ_R and ν_R :

$$M_k^{(n)}(R) = M_k^{(n)}(\alpha_R, \theta_R, \nu_R).$$

One can measure M_k , $k=1,2,3$, extract α_R , θ_R , ν_R , and substitute into M_4 . Thus, in principle, any four measurements test the theory. In practice, there is a well-known renormalization-scheme ambiguity² (R dependence in the above formula): in any finite order n of perturbation theory the $M^{(n)}(R)$'s depend on our choice of α_R , θ_R , and ν_R . This dependence, though $O(\alpha^{n+1})$, numerically can be arbitrarily large, and for a meaningful test the theory must be supplemented by some principle of choosing a "reasonable" renormalization scheme. Such a "principle of minimal sensitivity" has been formulated by Stevenson.² Unfortunately, it is not at all easy to follow in a theory as complicated as the GSW theory. We shall assume in this paper that similarly as in QED the most frequently used on-shell renormalization scheme is close enough to the "optimized" scheme to be considered a reasonable one.^{3,4} Meaningful theory tests cannot be sensitive to changes of one "reasonable" scheme into another one.

There is also another ambiguity which one encounters when comparing theory with experiment. Namely, to calculate a given M_4 one can use different sets of M_k , $k=1,2,3$ for extracting α_R , θ_R , and ν_R . The electric charge $e^2=4\pi\alpha$, $\alpha=137.03604(11)^{-1}$ and the muon lifetime τ_μ [or equivalently the Fermi constant $G_F=1.16637(2)\times 10^{-5}\text{GeV}^{-2}$] are known experimentally⁵ with a very high precision. It is convenient to use them always in the theory tests: $M_1=\alpha$, $M_2=G_F$ (thus, in our discussion α plays the role of the expansion parameter and of the input data).

Order by order in perturbation theory, after eliminating the parameters α_R , θ_R , and ν_R the model predicts a measurable quantity A in terms of e , G_F , and the third measurable quantity B (Ref. 6):

$$A_{(B)}^{(n)}(R) = f^{(n)}(e, G_F, B). \quad (1)$$

For R dependence and B dependence of the $A_{(B)}^{(n)}(R)$ we have, in general,

$$\begin{aligned} A_{(B)}^{(n)}(R_1) - A_{(B)}^{(n)}(R_2) &\sim \alpha^{n+1}, \\ A_{(B)}^{(n)}(R_1) - A_{(C)}^{(n)}(R_1) &\sim \alpha^{n+1}, \end{aligned} \quad (2)$$

since, eventually, for large n , $A_{(B)}^{(n)}(R)$ must converge to a B - and R -independent physical value A .

The standard strategy for testing a theory order by order in perturbation expansion is to exclude experimentally the results which are not allowed in a given order. Since, in general, the n th-order predictions are known up to the $O(\alpha^{n+1})$ corrections, the required experimental accuracy is also $O(\alpha^{n+1})$. Thus, to test $O(\alpha G_F)$ corrections one needs the accuracy $O(\alpha^2 G_F)$, well beyond the present experimental precision.

Fortunately, to test $O(\alpha G_F)$ corrections (but not higher-order corrections) one can also follow another strategy which is the subject of this paper. We observe first that for $n=0$ (and only for $n=0$) the first relation (2) is replaced by

$$A_{(B)}^{(0)}(R_1) = A_{(B)}^{(0)}(R_2), \quad (3)$$

i.e., obviously the zeroth-order prediction does not depend on the choice of the renormalization scheme. Therefore, if we define the correction

$$\delta A_{(B)}^{(1)} = A_{(B)}^{(1)} - A_{(B)}^{(0)} \quad (4)$$

then, using (3) and (2) for $n=1$, we get

$$\delta A_{(B)}^{(1)}(R_1) - \delta A_{(C)}^{(1)}(R_1) \sim \alpha, \quad (5)$$

$$\delta A_{(B)}^{(1)}(R_1) - \delta A_{(B)}^{(1)}(R_2) \sim \alpha^2. \quad (6)$$

Important for us is the fact that the $\delta A^{(1)}$ depends on the choice of the expansion parameter only at $O(\alpha^2)$ and for a "reasonable" choice of the renormalization scheme this dependence can be neglected as compared to the $O(\alpha)$ dependence on the input quantity B .

This is the very distinctive feature of the first-order corrections: higher-order corrections $\delta A^{(n)}(R) = A^{(n)}(R) - A^{(n-1)}(R)$ depend on the renormalization conditions already at $O(\alpha^n)$ [and not at $O(\alpha^{n+1})$ as for $\delta A^{(1)}$]. Thus, the difference $A^{(n)} - A^{(n-1)}$ is meaningless because it is of order α^n and its expansion parameter dependence is also $O(\alpha^n)$.

It is now clear that for testing $O(\alpha G_F)$ corrections a meaningful strategy is to test the first-order predictions against the zeroth-order approximation. Since this difference is of order αG_F , the experimental accuracy needed for such tests is also of the same order of magnitude. Of

course, in view of relation (5), for a given A the precision of experimental measurements of A and B needed for the first-order test depends on the choice of B . (The difference between $\delta A_{(B)}^{(1)}$ for various B 's are—as we will see—of the same order or larger than the correction itself.)

Imagine now that we measure A and B with some errors, $A \pm \Delta A$ and $B \pm \Delta B$. For a given uncertainty ΔB , the theoretical tree-level and the one-loop predictions $A_{(B)}^{(0)}$ and $A_{(B)}^{(1)}$, respectively, which follow from Eq. (1), are

$$\begin{aligned} A_{(B)}^{(0)} \pm \frac{dA_{(B)}^{(0)}}{dB} \Delta B &= A_{(B)}^{(0)} \pm \Delta A_{(B)}^T, \\ A_{(B)}^{(1)} \pm \frac{dA_{(B)}^{(1)}}{dB} \Delta B &= A_{(B)}^{(1)} \pm \Delta A_{(B)}^T, \end{aligned} \quad (7)$$

where $\Delta A_{(B)}^T$ is the error of our theoretical predictions for A caused by the experimental error of the input B ; we have assumed, as it happens within an approximation sufficient for us, that $dA_{(B)}^{(0)}/dB = dA_{(B)}^{(1)}/dB$. Then, the condition for the testability of the first-order corrections $\delta A_{(B)}^{(1)}$ can be formulated as

$$|A - A_{(B)}^{(0)}| \geq 2[(\Delta A_{(B)}^T)^2 + (\Delta A)^2]^{1/2}, \quad (8a)$$

$$|A - A_{(B)}^{(1)}| \leq [(\Delta A_{(B)}^T)^2 + (\Delta A)^2]^{1/2} \quad (8b)$$

(i.e., we require one standard deviation agreement with the first-order result and, simultaneously, exclude the tree-level approximation at the 95% confidence level).

The solution to the above inequalities provides the range within which the experimental value A should be found in order to test the radiative correction, as well as the required accuracies for A and B measurements. This can be illustrated, for instance, in the A - $\Delta A_{(B)}^T$ plane (Fig. 1). The meaning of Fig. 1 is that of a necessary condition: experimental points outside the shaded area cannot serve as a test of the first-order corrections. The more useful, sufficient condition independent of an actual experimental value A , can be formulated for the errors:

$$[(\Delta A_{(B)}^T)^2 + (\Delta A)^2]^{1/2} \leq \frac{1}{3} |A_{(B)}^{(1)} - A_{(B)}^{(0)}| = \frac{1}{3} |\delta A_{(B)}^{(1)}| \quad (9)$$

[provided that the inequality (8b) still holds, the number $\frac{1}{3}$ in (9) is the biggest one which implies the inequality (8b)] (Refs. 7 and 8).

Finally, let us comment on the nature of the one-loop corrections to weak processes. It is well known that the dominant corrections can be often attributed to the renormalization of α_{QED} from $q^2=0$ to $q^2=M_Z^2$ by the photon vacuum-polarization graphs. This correction enters

$$M_W^{(0,1)} = \frac{M_Z}{\sqrt{2}} \left[1 + \left[1 - \frac{4A}{M_Z^2} \right]^{1/2} \right]^{1/2} \left\{ 1 - \frac{A}{M_Z^2 \left[1 - \frac{4A}{M_Z^2} \right]^{1/2} \left[1 + \left[1 - \frac{4A}{M_Z^2} \right]^{1/2} \right]} \delta_\mu^{(0,1)} \right\}, \quad (10)$$

where

$$A = \frac{\pi\alpha}{\sqrt{2}G_F} = (37.281 \text{ GeV})^2,$$

$\delta_\mu^{(0)}=0$ and $\delta_\mu^{(1)}=\delta_\mu(\alpha, G_F, M_Z)$ is the radiative correction

FIG. 1. Experimental value of a quantity A vs the uncertainty of its theoretical estimation. Only experimental points within the shaded area can test the radiative correction $\delta A_{(B)}^{(1)} = A_{(B)}^{(1)} - A_{(B)}^{(0)}$.

weak cross sections in the GSW model through the structure of counterterms which are not independent for γ , W , and Z vacuum self-energies. In general, the presence of this correction would require a certain kind of unification. There is no reason for it, for instance, in models which reproduce the Born structure but are not renormalizable unified theories.⁹ Thus, the correction caused by the renormalization of α deserves experimental verification.

One can also consider the question of testing the remaining corrections defined as a difference of the full one-loop result $A^{(1)}$ and the tree-level formula $A^{(0)}$ in which $\alpha(q^2=0)$ has been replaced by $\alpha(q^2=M_Z^2)$ (Ref. 10). Eventual experimental testing of these “purely weak” corrections would distinguish between the GSW model and any other unified model of electroweak interactions.

II. EXAMPLES

A. $M_W = M_W(\alpha, G_F, M_Z)$

As the first application of Eq. (9), we consider the case when A and B are M_W and M_Z , respectively, measured directly in $p\bar{p}$ or e^+e^- colliders. We then obtain^{11,12}

to the muon-decay rate. Of course, δ_μ depends also on the quark masses and the Higgs-boson mass.

A number of authors have calculated δ_μ and found consistently $\delta_\mu \approx 0.07$ (Refs. 13–18) (the dependence on the Higgs-boson and quark masses is mild; we use

$m_H = M_Z, m_t = 36$ GeV as a reference choice for these parameters¹¹). One obtains therefore the results for $M_W^{(0)}$ and $M_W^{(1)}$ (in terms of M_Z) summarized in Table I. We note that radiative corrections decrease the value of M_W by 1 GeV or 1.2% ($\delta M_W \approx -1$ GeV).

In our present case ($A = M_W, B = M_Z$) with

$$\frac{dM_W^{(0)}}{dM_Z} \simeq 1.19$$

for $M_Z = 93$ GeV, one gets, from (9),

$$\Delta M_W \leq [(0.35 \text{ GeV})^2 - (1.2 \Delta M_Z)^2]^{1/2} \quad (11)$$

for the precision needed to test one-loop corrections by measuring two boson masses.

We note that the present experimental results are¹⁹

$$M_W = 83.5 \pm 1.1 \pm 2.8 \text{ GeV},$$

$$M_Z = 93.0 \pm 1.4 \pm 3.2 \text{ GeV},$$

from the UA1 Collaboration and (12)

$$M_W = 81.2 \pm 1.1 \pm 1.3 \text{ GeV},$$

$$M_Z = 92.5 \pm 1.3 \pm 1.5 \text{ GeV},$$

from the UA2 Collaboration. They are consistent with the theory predictions but as can be seen from Fig. 2, the errors are far too large for testing higher-order corrections.

From the experimental point of view, the precise measurement of M_W is rather difficult. On the other hand, it is expected that in $p\bar{p}$ colliders the ratio M_W/M_Z can be measured with better precision than M_W or M_Z , separately. This ratio is free from systematic error on the energy scale. One can expect a precision

$$\Delta(M_W/M_Z) = 2 \times 10^{-3}.$$

With independent measurements of M_Z from e^+e^- and M_W/M_Z from $p\bar{p}$ collider we will have a good chance to test one-loop radiative corrections. Indeed,

$$M_Z \frac{d}{dM_Z} \frac{M_W}{M_Z} \simeq 0.30, \quad (13)$$

where we have taken $M_Z = 93$ GeV. Observe that this

TABLE I. Zeroth- and first-order predictions for M_W as a function of M_Z . The values in the third column are strictly one-loop results and have been obtained with the use of Eq. (10). They do not include leading higher-order corrections [of order $\alpha^n \ln^n(m/M_Z)$] which are taken into account in column four (Ref. 11).

M_Z (GeV)	$M_W^{(0)}$ (GeV)	$M_W^{(1)}$ (GeV)	$M_W^{(\text{RGE})}$ (GeV)
90	79.5	78.4	78.26
91	80.7	79.7	79.53
92	81.9	80.9	80.79
93	83.1	82.1	82.03
94	84.3	83.4	83.26
95	85.5	84.6	84.43
96	86.7	85.7	85.68
98	89.0	88.1	88.06

number is 4 times smaller than dM_W/dM_Z . Thus, we get, instead of (11),

$$\Delta \left(\frac{M_W}{M_Z} \right) \times 93 \text{ GeV} \leq [(0.35 \text{ GeV})^2 - (0.30 \Delta M_Z)^2]^{1/2}. \quad (14)$$

It is clear that the measurements of the ratio M_W/M_Z can serve as a test of the first-order radiative corrections for less accurate measurements of M_Z than in the M_W measurement case. We have compared both conditions in Fig. 2.

The above arguments can also be used in more general cases. No matter what the choice of B is, we have

$$\frac{dM_Z}{dB} = \frac{dM_W}{dB} \frac{dM_Z}{dM_W} \simeq 0.85 \frac{dM_W}{dB} \quad (15)$$

and thus

$$\left| \frac{d}{dB} \frac{M_W}{M_Z} \right| \simeq \left| \frac{0.25}{M_Z} \frac{dM_W}{dB} \right| \simeq \left| \frac{0.30}{M_Z} \frac{dM_Z}{dB} \right|. \quad (16)$$

If $M_Z \delta(M_W/M_Z)_{(B)}^{(1)} \sim \delta M_Z \frac{(1)}{(B)} \sim \delta M_W \frac{(1)}{(B)}$, the choice $A = M_W/M_Z$ would be much more promising for testing the theory above the tree level than $A = M_Z$ or M_W . Notice also that Eq. (16) suggests that M_Z is a slightly better candidate for a quantity A than M_W .

In a similar way as the ratio M_W/M_Z , we can consider $A = M_Z - M_W$. From a purely theoretical point of view, the present choice is even better than the previous one since now

$$\left| \frac{d}{dB} (M_Z - M_W) \right| \simeq \left| 0.15 \frac{dM_W}{dB} \right| \simeq \left| 0.19 \frac{dM_Z}{dB} \right|. \quad (17)$$

However, the systematic scale uncertainties do not cancel

FIG. 2. Upper bounds for the experimental errors of the M_W , M_W/M_Z , $M_Z - M_W$, and M_Z measurements which allow us to test radiative corrections in the GSW model.

completely in $M_Z - M_W$ (although they are greatly reduced in comparison with the separate M_Z or M_W measurements). Because of that, it might be more difficult to achieve the required experimental accuracy for the radiative correction test with $A = M_Z - M_W$:

$$\Delta(M_Z - M_W) \leq [(0.35 \text{ GeV})^2 - (0.19 \Delta M_Z)^2]^{1/2} \quad (18)$$

than with $A = M_W/M_Z$. For completeness, the relation (18) is also shown in Fig. 2.

Let us now turn to the question of testability of purely weak radiative corrections in the weak-boson mass measurements, i.e., those remaining after subtraction of fine-structure-constant renormalization. This problem has been discussed in details by Bardin and Khovansky.²⁰ Their results can be summarized as follows: the purely weak correction is only a tiny part of δ_μ —from 0.0015 to 0.011 depending on the unknown values m_H and m_t . For our reference choice, we get, for $B = M_Z$,

$$\delta_{\text{weak}} M_W = 0.047 \text{ GeV} . \quad (19)$$

Thus, our testability condition takes the form (with $A = M_W$)

$$\Delta M_W \leq [(0.015 \text{ GeV})^2 - (1.2 \Delta M_Z)^2]^{1/2} . \quad (20)$$

If we wanted to test the correction (16) via the independent measurements of M_Z and the ratio M_W/M_Z , we would get, instead of (20),

$$\Delta \left[\frac{M_W}{M_Z} \right] \times 93 \text{ GeV} \leq [(0.015 \text{ GeV})^2 - (0.3 \Delta M_Z)^2]^{1/2} . \quad (21)$$

One can see from Eq. (21) that the test is possible if

$$\Delta M_Z \leq 40 \text{ MeV}$$

and

$$\Delta(M_W/M_Z) \leq 10^{-5} .$$

Such a precise measurement of the ratio M_W/M_Z is rather unlikely with the presently existing facilities.

$$\text{B. } M_{W,Z} = M_{W,Z}(\alpha, G_F, \sigma_{ve})$$

Another standard test of the GSW model is based on measuring a neutral-current cross section and the gauge-boson masses. For instance, let us express $M_W^{(0)}$ and $M_W^{(1)}$ in terms of e , G_F , and $\sigma_{\nu_\mu e}$. The result is given by inverting the relation¹⁷

$$\frac{\sigma_{\nu_\mu e}}{E_\nu} = \frac{\pi}{12} \alpha^2 m_e \left[\frac{16}{M_W^4} (1 + \delta_\nu) - \frac{12}{M_W^2 A} (1 - \delta_\mu + \delta_\nu) + \frac{3}{A^2} (1 - 2\delta_\mu + \delta_\nu) \right] \quad (22)$$

for $M_W^{(1)}$ and with $\delta_\mu = \delta_\nu = 0$ for $M_W^{(0)}$. [A and δ_μ are defined below Eq. (10); $\delta_\nu \simeq 0.121$ according to Aoki *et al.*¹⁷] Using²¹

$$\left[\frac{\sigma_{\nu_\mu e}}{E_\nu} \right]_{\text{expt}} = (1.55 \pm 0.26) \times 10^{-42} \text{ cm}^2/\text{GeV} ,$$

we get the following values:

$$\begin{aligned} M_W^{(0)} &= 77.5^{+6.6}_{-6.9} \text{ GeV}, \quad M_W^{(1)} = 81.2^{+7.1}_{-7.2} \text{ GeV}, \\ \delta M_{W(\sigma_{ve})}^{(1)} &= 3.7 \text{ GeV} . \end{aligned} \quad (23)$$

We notice that in the present case the radiative corrections to M_W are much larger than when expressed in terms of M_Z , thus giving us a clear evidence for their dependence on the choice of the input quantity B .

Using the relation (10), we can express as well $M_Z^{(0)}$ and $M_Z^{(1)}$ in terms of $\sigma_{\nu_\mu e}$. The results read

$$\begin{aligned} \frac{4A}{M_Z^2} &= 1 - \frac{(1 + \sqrt{4b-3})^2}{16} + \frac{b}{4} \frac{1 + \sqrt{4b-3}}{\sqrt{4b-3}} \delta_\nu \\ &\quad - \left[\frac{3}{2} \frac{\sqrt{4b-3} + 1}{\sqrt{4b-3}} + \frac{1}{16} (3 - \sqrt{4b-3})^2 - \frac{3}{8} (3 - \sqrt{4b-3}) \frac{1 + \sqrt{4b-3}}{\sqrt{4b-3}} \right] \delta_\mu \end{aligned} \quad (24)$$

[where $b = (12A^2/\pi\alpha^2 m_e)(\sigma_{\nu_\mu e}/E_\nu)$],

$$M_Z^{(0)} = 88.4^{+5.5}_{-5.2} \text{ GeV}, \quad M_Z^{(1)} = 92.3^{+5.8}_{-5.5} \text{ GeV}, \quad \delta M_{Z(\sigma_{ve})}^{(1)} = 3.9 \text{ GeV} . \quad (25)$$

The precision needed for testing the first-order corrections is given by the relations

$$\begin{aligned} \Delta M_W &\leq \left[(1.2 \text{ GeV})^2 - \left[25.4 \frac{\text{GeV}}{10^{-42} \text{ cm}^2/\text{GeV}} \Delta \frac{\sigma_{\nu_\mu e}}{E_\nu} \right]^2 \right]^{1/2} , \\ \Delta M_Z &\leq \left[(1.3 \text{ GeV})^2 - \left[21.5 \frac{\text{GeV}}{10^{-42} \text{ cm}^2/\text{GeV}} \Delta \frac{\sigma_{\nu_\mu e}}{E_\nu} \right]^2 \right]^{1/2} , \\ \Delta \left[\frac{M_W}{M_Z} \right] \times 93 \text{ GeV} &\leq \left[(0.098 \text{ GeV})^2 - \left[5.4 \frac{\text{GeV}}{10^{-42} \text{ cm}^2/\text{GeV}} \Delta \frac{\sigma_{\nu_\mu e}}{E_\nu} \right]^2 \right]^{1/2} . \end{aligned} \quad (26)$$

We can see that the test would be possible if the experimental errors in $\sigma_{\bar{\nu}_\mu e}/E_{\bar{\nu}}$ measurements were pushed below

$$\begin{aligned} 0.05 \times 10^{-42} \text{ cm}^2/\text{GeV} , \\ 0.06 \times 10^{-42} \text{ cm}^2/\text{GeV} , \\ 0.02 \times 10^{-42} \text{ cm}^2/\text{GeV} , \end{aligned}$$

for M_W , M_Z , and M_W/M_Z , respectively.

Analogous considerations for $\sigma_{\bar{\nu}_\mu e}$ and M_W (or M_Z) give

$$\begin{aligned} \frac{\sigma_{\bar{\nu}_\mu e}}{E_{\bar{\nu}}} = \frac{\pi}{12} \alpha^2 m_e \left[\frac{16}{M_W^4} (1 + \delta_{\bar{\nu}}) - \frac{4}{M_W^2 A} (1 - \delta_\mu + \delta_{\bar{\nu}}) \right. \\ \left. + \frac{1}{A^2} (1 - 2\delta_\mu + \delta_{\bar{\nu}}) \right] , \end{aligned} \quad (27)$$

where $\delta_{\bar{\nu}} = 0.112$ (Ref. 17). With the experimental value²¹

$$\left[\frac{\sigma_{\bar{\nu}_\mu e}}{E_{\bar{\nu}}} \right]_{\text{expt}} = (1.58 \pm 0.30) \times 10^{-42} \text{ cm}^2/\text{GeV} ,$$

we get now

$$\begin{aligned} M_W^{(0)} = 71.4_{-4.6}^{+8.3} \text{ GeV} , \quad M_W^{(1)} = 73.1_{-4.6}^{+7.9} \text{ GeV} , \\ \delta M_W^{(1)}(\sigma_{\bar{\nu}_\mu e}) = 1.7 \text{ GeV} , \end{aligned} \quad (28)$$

and instead of relations (24) and (25) we get

$$\begin{aligned} \frac{4A}{M_Z^2} = 1 - \frac{3 - \sqrt{4\bar{b} - 3}}{16} + \frac{b}{4} \frac{-3 + \sqrt{4\bar{b} - 3}}{\sqrt{4\bar{b} - 3}} \delta_{\bar{\nu}} \\ - \left[\frac{-3 + \sqrt{4\bar{b} - 3}}{2\sqrt{4\bar{b} - 3}} + \frac{1}{16} (1 + \sqrt{4\bar{b} - 3})^2 - \frac{1}{8} (1 + \sqrt{4\bar{b} - 3}) \frac{-3 + \sqrt{4\bar{b} - 3}}{\sqrt{4\bar{b} - 3}} \right] \delta_\mu \left[\bar{b} = \frac{12A^2}{\pi\alpha^2 m_e} \frac{\sigma_{\bar{\nu}_\mu e}}{E_{\bar{\nu}}} \right] , \end{aligned} \quad (29)$$

$$M_Z^{(0)} = 83.7_{-3.2}^{+6.5} \text{ GeV} , \quad M_Z^{(1)} = 86.1_{-3.1}^{+6.1} \text{ GeV} , \quad \delta M_Z^{(1)}(\sigma_{\bar{\nu}_\mu e}) = 2.4 \text{ GeV} . \quad (30)$$

Thus, the precision required to test radiative corrections by $\sigma_{\bar{\nu}_\mu e}$ measurements is

$$\Delta M_W \leq \left[(0.57 \text{ GeV})^2 - \left[23.3 \frac{\text{GeV}}{10^{-42} \text{ cm}^2/\text{GeV}} \Delta \frac{\sigma_{\bar{\nu}_\mu e}}{E_{\bar{\nu}}} \right]^2 \right]^{1/2} , \quad (31a)$$

$$\Delta \frac{\sigma_{\bar{\nu}_\mu e}}{E_{\bar{\nu}}} \leq 0.02 \times 10^{-42} \text{ cm}^2/\text{GeV} ;$$

$$\Delta M_Z \leq \left[(0.8 \text{ GeV})^2 - \left[19.7 \frac{\text{GeV}}{10^{-42} \text{ cm}^2/\text{GeV}} \Delta \frac{\sigma_{\bar{\nu}_\mu e}}{E_{\bar{\nu}}} \right]^2 \right]^{1/2} , \quad (31b)$$

$$\Delta \frac{\sigma_{\bar{\nu}_\mu e}}{E_{\bar{\nu}}} \leq 0.04 \times 10^{-42} \text{ cm}^2/\text{GeV} ;$$

$$\Delta \left[\frac{M_W}{M_Z} \right] \times 93 \text{ GeV} \leq \left[(0.13 \text{ GeV})^2 - \left[4.9 \frac{\text{GeV}}{10^{-42} \text{ cm}^2/\text{GeV}} \Delta \frac{\sigma_{\bar{\nu}_\mu e}}{E_{\bar{\nu}}} \right]^2 \right]^{1/2} , \quad (31c)$$

$$\Delta \frac{\sigma_{\bar{\nu}_\mu e}}{E_{\bar{\nu}}} \leq 0.03 \times 10^{-42} \text{ cm}^2/\text{GeV} .$$

Now, we can ask about the size of the “purely weak” corrections to $\sigma_{\nu_\mu e}$ and $\sigma_{\bar{\nu}_\mu e}$. Once again they are much smaller than those coming from α renormalization. One finds that, for $\sigma_{\nu_\mu e}$,

$$M_W^{(0)}(\alpha(M_Z), \sigma_{\nu_\mu e}) = 80_{-7.1}^{+6.9} \text{ GeV} , \quad \delta_{\text{weak}} M_W = 1.2 \text{ GeV} , \quad (32a)$$

$$M_Z^{(0)}(\alpha(M_Z), \sigma_{\nu_\mu e}) = 91.3_{-4.4}^{+5.6} \text{ GeV} , \quad \delta_{\text{weak}} M_Z = 1.0 \text{ GeV} , \quad (32b)$$

and for $\sigma_{\bar{\nu}_\mu e}$

$$M_W^{(0)}(\alpha(M_Z), \sigma_{\bar{\nu}_\mu e}) = 73.7_{-4.8}^{+8.6} \text{ GeV}, \quad \delta_{\text{weak}} M_W = -0.6 \text{ GeV}, \quad (33a)$$

$$M_Z^{(0)}(\alpha(M_Z), \sigma_{\bar{\nu}_\mu e}) = 86.4_{-3.3}^{+6.7} \text{ GeV}, \quad \delta_{\text{weak}} M_Z = -0.3 \text{ GeV}. \quad (33b)$$

Using the above numbers, it is easy to write down our testability condition:

$$\Delta M_W \leq \left[(0.4 \text{ GeV})^2 - \left[25.4 \frac{\text{GeV}}{10^{-42} \text{ cm}^2/\text{GeV}} \Delta \frac{\sigma_{\nu_\mu e}}{E_\nu} \right]^2 \right]^{1/2}, \quad (34a)$$

$$\Delta \left[\frac{\sigma_{\nu_\mu e}}{E_\nu} \right] \leq 0.02 \times 10^{-42} \text{ cm}^2/\text{GeV};$$

$$\Delta M_Z \leq \left[(0.3 \text{ GeV})^2 - \left[21.5 \frac{\text{GeV}}{10^{-42} \text{ cm}^2/\text{GeV}} \Delta \frac{\sigma_{\nu_\mu e}}{E_\nu} \right]^2 \right]^{1/2}, \quad (34b)$$

$$\Delta \left[\frac{\sigma_{\nu_\mu e}}{E_\nu} \right] \leq 0.01 \times 10^{-42} \text{ cm}^2/\text{GeV};$$

$$\Delta \left[\frac{M_W}{M_Z} \right] \times 93 \text{ GeV} \leq \left[(0.09 \text{ GeV})^2 - \left[6.5 \frac{\text{GeV}}{10^{-42} \text{ cm}^2/\text{GeV}} \Delta \frac{\sigma_{\nu_\mu e}}{E_\nu} \right]^2 \right]^{1/2}, \quad (34c)$$

$$\Delta \left[\frac{\sigma_{\nu_\mu e}}{E_\nu} \right] \leq 0.014 \times 10^{-42} \text{ cm}^2/\text{GeV}.$$

Analogously, in the $A = \sigma_{\bar{\nu}_\mu e}$ case we get

$$\Delta M_W \leq \left[(0.2 \text{ GeV})^2 - \left[23.3 \frac{\text{GeV}}{10^{-42} \text{ cm}^2/\text{GeV}} \Delta \frac{\sigma_{\bar{\nu}_\mu e}}{E_{\bar{\nu}}} \right]^2 \right]^{1/2}, \quad (35a)$$

$$\Delta \left[\frac{\sigma_{\bar{\nu}_\mu e}}{E_{\bar{\nu}}} \right] \leq 0.009 \times 10^{-42} \text{ cm}^2/\text{GeV};$$

$$\Delta M_Z \leq \left[(0.1 \text{ GeV})^2 - \left[19.7 \frac{\text{GeV}}{10^{-42} \text{ cm}^2/\text{GeV}} \Delta \frac{\sigma_{\bar{\nu}_\mu e}}{E_{\bar{\nu}}} \right]^2 \right]^{1/2}, \quad (35b)$$

$$\Delta \left[\frac{\sigma_{\bar{\nu}_\mu e}}{E_{\bar{\nu}}} \right] \leq 0.005 \times 10^{-42} \text{ cm}^2/\text{GeV};$$

$$\Delta \left[\frac{M_W}{M_Z} \right] \times 93 \text{ GeV} \leq \left[(0.09 \text{ GeV})^2 - \left[7.4 \frac{\text{GeV}}{10^{-42} \text{ cm}^2/\text{GeV}} \Delta \frac{\sigma_{\bar{\nu}_\mu e}}{E_{\bar{\nu}}} \right]^2 \right]^{1/2}, \quad (35c)$$

$$\Delta \left[\frac{\sigma_{\bar{\nu}_\mu e}}{E_{\bar{\nu}}} \right] \leq 0.012 \times 10^{-42} \text{ cm}^2/\text{GeV}.$$

C. $M_W = M_W(\alpha, G_F, R_\nu)$

At present, the most precise information about the neutral-current interactions comes from deep-inelastic scattering of neutrinos on nuclear targets, namely, from the measurements of the quantity R_ν :

$$R_\nu = \frac{\sigma_{\text{NC}}^{\nu N}}{\sigma_{\text{CC}}^{\nu N}}.$$

Assuming a valence parton model for nucleon and ignoring scaling violation, we get^{22,23}

$$R_\nu = \frac{1}{2} \left[1 + \delta_R - 2 \frac{A}{M_W^2} (1 + \delta_\mu + \delta_{\overline{\text{MS}}} + \delta_R) + \frac{40}{27} \frac{A^2}{M_W^4} (1 + 2\delta_\mu + 2\delta_{\overline{\text{MS}}} + \delta_R) \right]. \quad (36)$$

Here, $\delta_{\text{MS}} = -0.006$ is the difference between $1 - M_W^2/M_Z^2$ and $\sin^2\theta_W$ defined in the modified minimal subtraction ($\overline{\text{MS}}$) renormalization scheme at a scale $\mu = M_W^2$ (Ref. 24). δ_R is experiment cut dependent. Its value varies from -0.0255 to -0.0319 for the existing data.^{22,25}

A comparison of the relation (36) with the actual experimental results is not straightforward: one has to take into account all the above-mentioned effects which are neglected in Eq. (36). The thorough analysis of Refs. 26, 22, and 25 gives

$$\begin{aligned} M_W^{(0)} &= 78.2_{-2.5}^{+2.7} \text{ GeV}, \quad M_W^{(1)} = 83.1_{-2.8}^{+3.1} \text{ GeV}, \\ \delta M_{W(R)}^{(1)} &= 4.9 \text{ GeV}, \\ M_Z^{(0)} &= 89.0_{-2.0}^{+2.2} \text{ GeV}, \quad M_Z^{(1)} = 93.8_{-2.2}^{+2.5} \text{ GeV}, \\ \delta M_{Z(R)}^{(1)} &= 4.8 \text{ GeV}. \end{aligned} \quad (37)$$

Using (36) to estimate $dM_W^{(0)}/dR$, we get

$$\Delta M_W \leq [(1.63 \text{ GeV})^2 - (258 \text{ GeV} \times \Delta R_\nu)^2]^{1/2}, \quad (38)$$

$$\Delta R_\nu \leq 0.006,$$

$$\Delta M_Z \leq [(1.63 \text{ GeV})^2 - (219 \text{ GeV} \times \Delta R_\nu)^2], \quad (39)$$

$$\Delta R_\nu \leq 0.0075,$$

instead of (26) and (31) (the choice $A = M_W/M_Z$ leads to a more stringent requirement on the accuracy ΔR_ν and we do not give it here).

The required precision of R_ν measurements, although very high, has already been obtained in the experiment [$\Delta R_\nu = 0.003$ (Ref. 27)] (Ref. 28). Moreover, the theoretical uncertainties in R_ν due to the higher-twist QCD contributions [$R_\nu \lesssim 0.0035$ (Ref. 29)] are still smaller. Thus, the choice $A = M_Z$, $\Delta B = R_\nu$ is very promising for testing full electroweak radiative corrections in the near future. On the other hand, the theoretical uncertainties are an order of magnitude larger than the accuracies needed to test the radiative correction remaining after subtraction of the electric charge renormalization.

D. Asymmetries in e^+e^-

Among the projected tests of the standard model at CERN LEP, the measurements of various asymmetries in the processes $e^+e^- \rightarrow \mu^+\mu^-$, $\tau^+\tau^-$ at the Z -boson resonance seem to be most promising.³⁰ Following Lynn *et al.*³⁰, we discuss here polarization asymmetries for longitudinally (A_{LR}) and perpendicularly ($A_\perp$) polarized electrons and the forward-backward asymmetry (A_{FB}). In each case, only $\mu^+\mu^-$ final state is investigated. We concentrate on the case when the second measured quantity is the Z mass.

The left-right asymmetry at the Z resonance is given by the formula

$$A_{LR}^{(0,1)} = -2 \frac{4s^2 - 1}{1 + (4s^2 - 1)^2} + \delta A_{LR}^{(0,1)}, \quad (40)$$

where one has to substitute s^2 with

$$s^2 = \frac{1}{2} \left[1 - \left[1 - \frac{4A}{M_Z^2} \right]^{1/2} + \frac{2A}{M_Z^2} \left[1 - \frac{4A}{M_Z^2} \right]^{-1/2} \right] \delta_\mu. \quad (41)$$

With $\delta A_{LR}^{(0)} = 0$, $\delta A_{LR}^{(1)} = -0.0242$ (Ref. 30) (for $M_Z = 94$ GeV) we get the following condition for the experimental accuracies:

$$\begin{aligned} \Delta A_{LR} &\leq [(0.05)^2 - (0.048 \text{ GeV}^{-1} \times \Delta M_Z)^2]^{1/2}, \\ \Delta M_Z &\leq 1 \text{ GeV}, \quad \Delta A_{LR} \leq 0.05. \end{aligned} \quad (42)$$

We can see that the requirements for the precisions are quite mild (especially that for M_Z). If we subtract the electric charge renormalization we will find, instead of (42),

$$\begin{aligned} \Delta A_{LR} &\leq [(0.008)^2 - (0.048 \text{ GeV}^{-1} \times \Delta M_Z)^2]^{1/2}, \\ \Delta M_Z &\leq 0.17 \text{ GeV}, \quad \Delta A_{LR} \leq 0.008. \end{aligned} \quad (43)$$

Even in this case, the test might be possible.

Consider now the asymmetry for perpendicularly polarized beams

$$A_\perp^{(0,1)} = \frac{(4s^2 - 1)^2 - 1}{(4s^2 - 1)^2 + 1} + \delta A_\perp^{(0,1)}, \quad (44)$$

where $\delta A_\perp^{(1)} = 0.0043$. For the full radiative correction the testability condition is

$$\begin{aligned} \Delta A_\perp &\leq [(0.013)^2 - (0.015 \text{ GeV}^{-1} \times \Delta M_Z)^2]^{1/2}, \\ \Delta M_Z &\leq 1 \text{ GeV}, \quad \Delta A_\perp \leq 0.013; \end{aligned} \quad (45)$$

whereas for the weak correction

$$\begin{aligned} \Delta A_\perp &\leq [(0.0014)^2 - (0.015 \text{ GeV}^{-1} \times \Delta M_Z)^2]^{1/2}, \\ \Delta M_Z &\leq 0.096 \text{ GeV}, \quad \Delta A_\perp \leq 0.0025. \end{aligned} \quad (46)$$

Since polarized beams will not be available in the early stages of LEP, it is of particular interest to consider the forward-backward asymmetry. The relevant formula is

$$A_{FB}^{(0,1)} = 3 \left[\frac{4s^2 - 1}{1 + (4s^2 - 1)^2} \right]^2 + \delta A_{FB}^{(0,1)}, \quad (47)$$

with $\delta A_{FB}^{(1)} = -0.0076$. Once again, the leading correction is due to the renormalization of α (and is hidden in the expression for s^2). Its value is about 0.09. Thus, we have

$$\begin{aligned} \Delta A_{FB} &\leq [(0.03)^2 - (0.021 \text{ GeV}^{-1} \times \Delta M_Z)^2]^{1/2}, \\ \Delta M_Z &\leq 1.3 \text{ GeV}, \quad \Delta A_{FB} \leq 0.03. \end{aligned} \quad (48)$$

To test the “purely weak” correction, one has to satisfy the following inequalities:

$$\begin{aligned} \Delta A_{FB} &\leq [(0.0025)^2 - (0.021 \text{ GeV}^{-1} \times \Delta M_Z)^2]^{1/2} \\ \Delta M_Z &\leq 0.12 \text{ GeV}, \quad \Delta A_{FB} \leq 0.0025. \end{aligned} \quad (49)$$

Equations (42)–(49) show that it might be relatively easy to test the full one-loop radiative corrections with the use of asymmetry measurements. These corrections are, as usual, dominated by the electric charge renormalization from $q^2 = 0$ to $q^2 = M_Z^2$. The remaining parts are very

tiny. To test them, one should probably wait for polarized beams at LEP and very-high-statistics data.

III. USE OF $\sin^2\theta_W$ (REF. 31)

As yet, we have been considering theory tests based on direct relations between two experimentally measured quantities. Another possibility, more often discussed in the literature, is to use an intermediate-step parametrization in terms, for instance, of the Weinberg angle. Then, instead of Eq. (1), we use

$$A^{(n)} = g^{(n)}(e, G_F, \sin^2\theta_W) \quad (50)$$

and the Weinberg angle is determined order by order in perturbation theory in terms of a directly measured quantity B (Ref. 32):

$$\sin^2\theta_W^{(n)} = h^{(n)}(e, G_F, B) . \quad (51)$$

Up to corrections $O(\alpha^2)$ we have an equality

$$\begin{aligned} \delta A_{(B)}^{(1)} &= A^{(1)}(B) - A^{(0)}(B) \\ &\simeq g^{(1)}(h^{(1)}(B)) - g^{(0)}(h^{(0)}(B)) \end{aligned} \quad (52)$$

and both methods are equivalent as long as we use the same input B on both sides of the above equation. However, with the use of the Weinberg angle one can ask the following legitimate question: can we improve our theory tests by using the value of the Weinberg angle obtained as an average from different experiments? This is indeed possible but with some caution. We notice first that for two quantities B and C we obtain

$$\sin^2\theta_W^{(0)}(B) - \sin^2\theta_W^{(0)}(C) \sim \alpha . \quad (53)$$

This is obvious because eventually, for large n , the $\sin^2\theta_W$ obtained from different measurements must converge to the same value. Actually, if we neglect terms $O(\alpha^2)$ we have

$$\sin^2\theta_W^{(1)}(B) = \sin^2\theta_W^{(1)}(C) \quad (54)$$

and Eq. (53) is a direct consequence of Eq. (54). The two equations [(53) and (54)] have certain implications which are worth noticing. It is convenient to define a new radiative correction

$$\delta\tilde{A}^{(1)} = g^{(1)}(h^{(1)}(B)) - g^{(0)}(h^{(1)}(B)) , \quad (55)$$

where we have used the value $\sin^2\theta_W^{(1)}$ of the Weinberg angle both in $g^{(1)}$ and $g^{(0)}$. Up to corrections $O(\alpha^2)$, which we neglect, the $\sin^2\theta_W^{(1)}$ is the “true” value of the Weinberg angle and can be taken as an average from different experiments, i.e., with a smaller experimental error.

Radiative corrections $\delta A_{(B)}^{(1)}$ and $\delta\tilde{A}^{(1)}$ are, of course, different but both can serve for testing the theory. We again see that a precise test of the $O(\alpha)$ corrections needs a clear specification of relations which are tested.

As an example of the use of a parametrization in terms of $\sin^2\theta_W$, we consider once again the formulas for M_W and M_Z :

$$M_W^{(0,1)} = \frac{\sqrt{A}}{\sin\theta_W} (1 + \tfrac{1}{2}\delta_\mu^{(0,1)}) , \quad (56)$$

$$M_Z^{(0,1)} = \frac{\sqrt{A}}{\sin\theta_W \sqrt{1 - \sin^2\theta_W}} (1 + \tfrac{1}{2}\delta_\mu^{(0,1)}) , \quad (57)$$

where the Weinberg angle is defined in such a way that the relation $M_Z = M_W / \cos\theta_W$ is exact.¹⁴ Using the experimental value of the $\sin^2\theta_W^{(1)}$, $\sin^2\theta_W = 0.227 \pm 0.009$ (Ref. 32) we get, for the weak-boson masses,

$$\begin{aligned} M_W^{(0)} &= 78.2 \pm 1.6 \text{ GeV}, \quad M_W^{(1)} = 81.0 \pm 1.6 \text{ GeV}, \\ \delta\tilde{M}_W^{(1)} &= 2.7 \text{ GeV}, \end{aligned} \quad (58a)$$

$$\begin{aligned} M_Z^{(0)} &= 89.0 \pm 1.3 \text{ GeV}, \quad M_Z^{(1)} = 92.1 \pm 1.3 \text{ GeV}, \\ \delta\tilde{M}_Z^{(1)} &= 3.1 \text{ GeV}. \end{aligned} \quad (58b)$$

Whereas for the precision needed to test the first order,

$$\Delta M_W \leq [(0.91 \text{ GeV})^2 - (184 \text{ GeV} \times \Delta \sin^2\theta_W)^2]^{1/2} , \quad (59a)$$

$$\Delta \sin^2\theta_W \leq 0.005 ;$$

$$\begin{aligned} \Delta M_Z &\leq [(1.0 \text{ GeV})^2 - (150 \text{ GeV} \times \Delta \sin^2\theta_W)^2]^{1/2} , \\ \Delta \sin^2\theta_W &\leq 0.0075 ; \end{aligned} \quad (59b)$$

$$\begin{aligned} \Delta \left[\frac{M_W}{M_Z} \right] \times 93 \text{ GeV} &\leq [(0.07 \text{ GeV})^2 \\ &\quad - (46 \text{ GeV} \times \Delta \sin^2\theta_W)^2]^{1/2} , \end{aligned} \quad (59c)$$

$$\Delta \sin^2\theta_W \leq 0.0015 .$$

For completeness, we can also find the accuracies required to test the “purely weak” correction. With $A = M_Z$, one gets

$$\begin{aligned} \Delta M_Z &\leq [(0.068 \text{ GeV})^2 - (150 \text{ GeV} \times \Delta \sin^2\theta_W)^2]^{1/2} , \\ \Delta \sin^2\theta_W &\leq 0.0005 . \end{aligned} \quad (60)$$

As a final remark, we notice that so far the experimental value of $\sin^2\theta_W$ is obtained mainly from deep-inelastic scattering data. Thus, a comparison of (59) with the similar results obtained for the weak-boson masses expressed in terms of R_ν [i.e., Eq. (37)] gives us a clear indication of differences between radiative corrections $\delta A^{(1)}$ and $\delta\tilde{A}^{(1)}$. To be explicit,

$$\delta M_{W(R)}^{(1)} = 4.9 \text{ GeV}, \quad \delta\tilde{M}_W^{(1)} = 2.7 \text{ GeV} , \quad (61a)$$

$$\delta M_{Z(R)}^{(1)} = 4.8 \text{ GeV}, \quad \delta\tilde{M}_Z^{(1)} = 3.1 \text{ GeV} . \quad (61b)$$

Relations (58) have a direct consequence: in order to test each correction by measuring $M_{W,Z}$ and R_ν , one has to achieve a different experimental precision. For the case of $\delta M_{W,Z(R)}^{(1)}$ corrections, the required accuracy is given by (38) and (39), whereas for $\delta\tilde{M}_{W,Z}^{(1)}$ we get

$$\begin{aligned} \Delta M_W &\leq [(0.91 \text{ GeV})^2 - (258 \text{ GeV} \times \Delta R_\nu)^2]^{1/2} , \\ \Delta R_\nu &\leq 0.0035 ; \end{aligned} \quad (62a)$$

$$\Delta M_Z \leq [(1.1 \text{ GeV})^2 - (219 \text{ GeV} \times \Delta R_\nu)^2]^{1/2},$$

$$\Delta R_\nu \leq 0.005. \quad (62b)$$

Because of the theoretical uncertainties in R_ν , $\delta \tilde{M}_{W,Z}^{(1)}$ is more difficult to test than $\delta M_{W,Z}^{(1)}$.

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³We regard as “reasonable” any renormalization scheme for which $\alpha_R \ll 1$ and such that the correction due to a change of the “minimal sensitivity” scheme to this scheme is in the n th order: $O(1) \times O(\alpha^n + 1)$.

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