CERN-TH.4053/84

ABOUT THE VACUUM STRUCTURE IN QCD AND HADRON-HADRON SCATTERING*)

José Cortés and Bernard Pire
CERN - Geneva

ABSTRACT

We critically analyse the possibility of taking into account the influence of the vacuum structure of QCD in the description of high-energy hadron-hadron reactions by including a correlation in spin and colour for the partons that initiate the process, as was recently proposed by Nachtmann and Reiter.

*) Work partially supported by the CAICYT.

1. - INTRODUCTION

The rich vacuum structure of non-Abelian gauge theories has been advocated^{1),2)} in order to lead to a picture where the QCD vacuum consists of domains of spatial size of the order of $1/\Lambda$, where Λ is the characteristic scale of the theory. In these domains, the chromomagnetic and chromoelectric fields are almost constant with fluctuations characterized by a frequency of the order of Λ . Such a picture leads us to question the validity of the usual perturbative approach to hadron-hadron hard scattering phenomena, in particular of the assumption of factorization of single parton distribution functions and of their universality (i.e., process independence). Indeed, taking as an example the Drell-Yan process for lepton pair production, the domain structure of the vacuum may lead to the existence of correlations between the incoming quark and antiquark quantum states, which might, for instance, prefer to align their spin or their colour space components. The asymptotic freedom property of QCD might, however, still be useful if one assumes a new kind of factorization between a long-distance dominated non-perturbative density matrix $\rho(a,b; A,B)$ and a short-distance perturbative subprocess matrix element, leading to a physical cross-section written as

$$d\sigma(AB \rightarrow \gamma^* X) = \rho(ab; AB) \otimes d\hat{\sigma}(ab \rightarrow \gamma^* X) \quad (1)$$

Such a picture has recently been discussed in a quite detailed way by Nachtmann and Reiter¹⁾. These authors consider in particular whether the well-known K factor between experimental data and lowest-order QCD predictions for the Drell-Yan cross-section may be explained by the colour structure of the density matrix. This was achieved by parametrizing the density matrix and looking at deviations from the usual picture you can get in this framework. It turns out that these deviations are quite severely constrained by positivity arguments.

Here we shall critically examine the consistency of this picture with regard to the applicability of perturbation theory to the determination of the subprocess cross-section $d\hat{\sigma}$. It is known that a crucial point for the consistency of the usual QCD improved parton description of hard processes is the possibility of defining an infra-red finite cross-section $d\hat{\sigma}$ after factorization of the collinear divergences into Q^2 -dependent parton distribution functions. A crucial problem is the cancellation of infra-red divergences. In the case of the Drell-Yan process at order α_s , the relevant diagrams are the vertex correction of Fig. 1a and the real gluon emission contribution of Fig. 1b. We shall show in

Section 2 that the requirement that such cancellations take place yields strong constraints on the colour structure of the density matrix. In Section 3, we will examine the spin dependence of the density matrix and its possible contribution to the K factor. Section 4 will state our conclusions.

2. - THE COLOUR DEPENDENCE OF THE DENSITY MATRIX

Let us concentrate for the moment on the colour properties of the density matrix. It is straightforward to see that the most general matrix in the quark-antiquark annihilation case can be parametrized as

$$\rho_{ij;i'j'} = \rho_1 \delta_{ij} \delta_{i'j'} + \rho_8 T_{ji}^a T_{i'j'}^a, \quad (2)$$

where the Bjorken x dependence has been omitted and ρ_1 (ρ_8) gives the probability that in the presence of the vacuum chromoelectric and chromomagnetic fields the quark-antiquark system is in a colour singlet (octet) configuration. The convolution of this density matrix with the matrix element corresponding to the order α_s correction to the Drell-Yan process gives the colour factors:

$$\begin{aligned} \rho_1 T_F N(N^2-1) & \quad \text{for the virtual correction of Fig. 1a;} \\ \rho_8 T_F^2 (N^2-1) & \quad \text{for the real emission diagram of Fig. 1b,} \end{aligned}$$

which multiply the QED matrix elements. The generic situation thus leads to a non-cancellation of infra-red divergences. This is reminiscent of the result obtained by Doria et al.³⁾ in the usual framework. Although that problem has not been solved up to now, its significance is lowered by the fact that those infra-red divergences seem to occur only in the higher twist contributions to the cross-section. The problem is more severe here, since we deal with the lowest twist and next-to-lowest order in α_s contribution. It thus seems mandatory to impose as a constraint on the density matrix that these divergences cancel, as in the QED case. We thus need:

$$\rho_1 N = \rho_8 T_F \quad (3)$$

and the normalized density matrix reads

$$\rho = \frac{1}{N} \delta_{ii'} \delta_{jj'} \quad (4)$$

which is the starting point of the (QCD-improved) parton model description of hard processes.

Such a constraint on the colour structure of an admissible density matrix is in fact quite general, and it follows from the necessary assumptions of the Kinoshita-Lee-Nauenberg (KLN) theorem⁴⁾ for the cancellation of infra-red divergences. The colourless nature of the hadron is transmitted to the requirement that the colour of the initial partons be averaged over, i.e., that they have a definite ratio of probabilities for being in a singlet or an octet state. The diagonal character of the density matrix is thus needed for any hard scattering process calculations to be finite, apart from the single logarithms attached to collinear bremsstrahlung, which can be absorbed by a Q^2 -dependent definition of structure functions.

3. - THE SPIN DEPENDENCE OF THE DENSITY MATRIX

Let us consider the quark-antiquark annihilation in their centre-of-mass system, with $\vec{p}$ ($-\vec{p}$) the momentum of the quark (antiquark) and $p = \vec{p}/|\vec{p}|$. After averaging over all field configurations present in the vacuum, the more general spin-dependent density matrix consistent with Lorentz- and parity-invariance is

$$\rho = \frac{1}{4} \delta_{\alpha\alpha'} \delta_{\beta\beta'} + \frac{1}{4} \sigma_{\alpha\alpha'}^i \sigma_{\beta\beta'}^j \left[\rho_T (\delta_{ij} - \hat{p}_i \hat{p}_j) + \rho_L \hat{p}_i \hat{p}_j \right] \quad (5)$$

where the constraint $\text{Tr } \rho = 1$ has been used to fix the first term and ρ_L and ρ_T parametrize the longitudinal and transverse parts of the density matrix. It was assumed in Ref. 1) that ρ_L vanishes, but here we shall take the general case. The positivity of the density matrix constrains the range of the parameters as:

$$|\rho_L| \leq 1 \quad ; \quad |\rho_T| \leq \frac{1 - \rho_L}{2} \quad (6)$$

The K factor⁵⁾ for the Drell-Yan process can then be straightforwardly calculated as

$$K = \frac{\text{Tr } \rho \hat{\sigma}}{\text{Tr } \hat{\sigma} / 4} \quad (7)$$

where $\hat{\sigma}$ is the hard scattering subprocess cross-section. One finds

$$K = 1 + \rho_L \quad (8)$$

so that $0 < K < 2$. This result can be easily understood as a consequence of chirality conservation. The two extreme cases $K = 0, 2$ correspond to the situation where the vacuum field configuration is such that the helicity of the quark-antiquark system is 0, ± 1 , while in the usual case the average over spin configurations gives a factor $\frac{1}{2}$ ($K = 1$). Although such a non-perturbative K-factor would be phenomenologically welcome, it does not help the fact that higher-order corrections are, in the Drell-Yan case, of the order of the Born term. Indeed, such an enhancement factor will also be present for higher-order contributions to $q\bar{q}$ annihilation, so that one would get

$$\frac{\text{Tr } \rho \hat{\sigma}^{(i)}}{\text{Tr } \rho \hat{\sigma}^{(0)}} \simeq \frac{\hat{\sigma}^{(i)}}{\hat{\sigma}^{(0)}} \simeq 1$$

where superscript i denotes the contribution at $O(\alpha_s^i)$ to the cross-section. Moreover, it seems difficult to believe in a picture where there is no colour correlation, but strong spin correlation between incoming partons in the QCD vacuum.

4. - CONCLUSION

In this note we have analysed in detail a recent suggestion to incorporate non-perturbative effects in the description of high-energy hadron-hadron scattering. We find that the requirement of factorization of perturbative and non-perturbative effects puts severe constraints on the possible departures from the standard QCD-improved parton model description. The idea that the complex vacuum structure of QCD would infer colour correlations between the two incoming partons is indeed quite natural and appealing. However, its phenomenological representation as a factorized colour density matrix turns out to be inconsistent with a perturbative treatment of the subprocess matrix element. It may be advocated that the domain structure of the vacuum implies the existence of an infra-red

cut-off which should be applied to the subprocess computation, replacing infrared divergent quantities by cut-off dependent finite quantities. The dominance of lowest-order terms in a perturbative calculation is, in that case, doubtful, and all the predictive power of the parton description is lost.

From a phenomenological point of view, one of the motivations for incorporating non-perturbative effects was the possibility of explaining the K factor in Drell-Yan as a non-perturbative effect, opening up at the same time the possibility of making higher-order corrections smaller than those in the standard framework. We find that the only way to incorporate the effects of the vacuum structure in the initial state is through a spin correlation that could give a non-perturbative K factor as large as 2, but still higher order corrections will be of the same magnitude as the leading term.

ACKNOWLEDGEMENTS

We acknowledge useful correspondence with Professor O. Nachtmann.

REFERENCES

- 1) O. Nachtmann and A. Reiter, Z. Phys. C24 (1984) 283.
- 2) G. 't Hooft, Cargèse Lectures, Acta Phys. Austr. Suppl. XXII (1980) 531.
- 3) R. Doria, J. Frenkel and J.C. Taylor, Nucl. Phys. B168 (1980) 93.
- 4) T. Kinoshita, J. Math. Phys. 3 (1962) 650;
T.D. Lee and M. Nauenberg, Phys. Rev. 133 (1964) B1549.
- 5) See, for instance:
Proceedings of the Moriond Workshop on Lepton Pair Production, Les Arcs, 1981 (J. Tran Thanh Van, ed.), éditions Frontières, Dreux (France).

FIGURE CAPTION

Fig. 1 : First order corrections to the Drell-Yan process.

(a)

(b)

Fig. 1