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NON-MINIMAL SUPERGRAVITY AT FINITE TEMPERATURES
AND COSMOLOGICAL APPLICATIONS

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ABSTRACT

We compute thermal one-loop corrections to the scalar potentials of two classes of non-minimal supergravities: the $SU(N,1)$ maximally symmetric supergravity and the generalized $SU(N,1)$ model that allows for cosmological inflation. For completeness, we give the result also for minimal supergravity, including contributions from the D-terms. We discuss the effects of temperature corrections in no-scale GUTs and in primordial maximally symmetric inflation. In particular we show that the latter satisfy trivially the condition for thermal stability.

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1. - INTRODUCTION

Locally supersymmetric¹⁾ theories appear to be promising candidates for grand unified theories that are free of the scale hierarchy problems as well as for models of early cosmological inflation²⁾. Supergravity, although being non-renormalizable by power counting, is believed to yield a renormalizable effective low-energy theory below the Planck mass M_P . However, in the context of primordial inflation^{3),4)} it has become evident that one should not neglect these non-renormalizable contributions, even though they are suppressed by inverse powers of M_P . In fact, these terms are usually crucial for obtaining a realistic⁵⁾⁻⁸⁾ inflationary scenario.

Recently, growing attention has also been devoted to phenomenological low-energy theories of non-minimal supergravity⁹⁾⁻¹¹⁾. These contain non-renormalizable interactions in the form of non-minimal kinetic energy terms of the superfields which, when properly normalized, will nevertheless lead to an effective low-energy theory that is renormalizable. A particularly promising class of such models are^{10),11)} the no-scale supergravity models based on the non-compact $SU(N,1)$ symmetry of the Kähler manifold of the scalar fields. They allow for a radiative determination of the electroweak scale M_W and have a tree-level cosmological constant that is strictly zero. They may¹¹⁾ also solve the strong CP problem by providing a mechanism whereby θ_{QCD} can dynamically relax to zero. It is also possible to determine the value of the gravitino mass $m_{3/2}$ dynamically through radiative corrections, which will typically yield a dependence on the electroweak scale that is of the form¹¹⁾ $m_{3/2} = O(M_W^p/M_P^{p-1})$, ($1 \lesssim p \lesssim 2$).

A detailed study of GUTs and inflationary models based on supergravity necessitates the knowledge of thermal corrections in these theories. Of particular importance will be the complications due to the non-minimality of the presently popular models, and this will be the main focus of the present paper.

The scalar potential of $N=1$ supergravity is¹⁾

$$V = e^{\mathcal{G}} \left[g^i (g^{-1})^j_i g_j - 3 \right] + \frac{1}{2} \text{Re} \int_{\alpha\beta}^{-1} D^\alpha D^\beta \quad (1)$$

where $\mathcal{G}$ is the Kähler potential, which is a real function of the chiral scalar fields y^i , and

$$g_i = \frac{\partial}{\partial y^i} g \quad ; \quad g^i = \frac{\partial}{\partial y_i} g \quad (2)$$

The D-term is given by

$$D^\alpha = g g_i (T^\alpha)^i_j y^j \quad (3)$$

where g is the gauge coupling constant and T^α is the generator of the gauge group; $f_{\alpha\beta}$ is an arbitrary analytic function of the chiral scalar fields y^i which transforms as the symmetric product of two adjoint representations. Finally, it should be noted that here, as well as in what follows, we will be working in units of $M = M_P/\sqrt{8\pi} = 2.4 \times 10^{18}$ GeV.

The finite temperature corrections to the potential (1) arise through loops of properly normalized scalar fields and their fermionic partners, of gauge vector bosons and their partners (gauginos), and of gravitinos. The one-loop thermal corrections are given by¹²⁾

$$\Delta V_T = -\frac{\pi^2}{24} (N+1) T^4 + \frac{T^2}{24} \left[\text{Tr } M_0^2 + \text{Tr } M_{1/2}^2 + \text{Tr } M_{3/2}^2 + 3 \text{Tr } M_1^2 \right] \quad (4)$$

where N is the total number of chiral superfields and M_j , $j = 0, \frac{1}{2}, 1, 3/2$, are the mass matrices of the spin- j fields. Equation (4) is valid provided that T is much larger than the mass of any particle.

A particular supergravity model is fixed once we choose the functions g and $f_{\alpha\beta}$. These will determine the boson mass matrices as well as the chiral fermion mass matrix which is related to the expression

$$\gamma^{ij} = g^{ij} + g^i g^j - g^k (g^{-1})^l_k g_l^{ij} \quad (5)$$

as we will see later on. To compute the contribution due to the gravitino, we use the gauge $\gamma^\mu \psi_\mu = 0$. There the gravitino decouples from the goldstino and the

effective action gives a contribution to the one-loop finite temperature correction as¹³⁾

$$\text{Tr } M_{3/2}^2 = -2e^G \quad (6)$$

where the minus sign is a reflection of the fact that we have separated the gravitino and goldstino modes. In fact, in the chiral fermion mass matrix (5) there is a term which corresponds to the would-be goldstino field, and which is eaten by the gravitino (the super-Higgs effect) at the minimum of the potential. However, for our purposes we need the mass matrices as functions of the fields rather than only at the global minimum of the potential which necessitates the separation of the gravitino and goldstino modes.

The contents of this paper are as follows. In Section 2, we will, for the sake of completeness, compute the finite temperature corrections in supergravity with minimal kinetic terms. Section 3 contains a similar computation, but this time for the non-minimal $SU(N,1)$ model. Section 4 is devoted to the calculation of thermal corrections in a class of supergravity models that are generalizations of the maximally symmetric $SU(N,1)$ models and which have recently been proposed^{14),5)-7)} for inflationary purposes. They appear to escape the problems of gravitino regeneration¹⁵⁾, baryon asymmetry generation¹⁶⁾ and low reheating which plague more conventional inflationary models. In Section 5, we will make an application of the general results of Section 3 to a particularly simple and interesting no-scale SUSY GUT model. In Section 6, we analyse inflationary models in the light of the results obtained in Section 4, paying special attention to the thermal stability of these models. Finally, Section 7 contains our conclusions.

2. - FINITE TEMPERATURE CORRECTIONS IN MINIMAL SUPERGRAVITY

Minimal supergravity has a flat Kähler metric $G_j^i = \delta_j^i$ and is defined by the Kähler potential

$$G = y^i y_i^* + F(y^i) + F^+(y_i^*) \quad (7)$$

where F is an analytic function of the chiral scalar fields, and

$$f_{\alpha\beta} = \delta_{\alpha\beta} \quad (8)$$

In this case, the scalar potential (1) reduces simply to

$$V = e^G [g^i g_i - 3] + \frac{1}{2} D^\alpha D_\alpha \quad (9)$$

Finite temperature corrections to (9) in the absence of the D-terms have recently been calculated by Olive and Srednicki¹⁷⁾ and Binétruy and Gaillard¹³⁾. Here we will also include the contributions due to D-terms.

The spin-zero contribution is

$$\begin{aligned} \text{Tr } M_0^2 = & 2e^G \{ (g^{ij} + g^i g^j) (g_{ij} + g_i g_j) \\ & + (N-1)g^i g_i - 2N \} + 2g^2 C_2(R) |y^i|^2 \end{aligned} \quad (10)$$

where $C_2(R)$ is the second-order Casimir invariant of the representation R , and a sum over all representations is implied.

The fermion mass matrix is given by

$$M_{1/2} = \begin{pmatrix} m^{ij} & i\sqrt{2} D^{\alpha i} \\ i\sqrt{2} D^{\alpha j} & 0 \end{pmatrix} \quad (11)$$

where

$$m^{ij} = e^{G/2} (g^{ij} + g^i g^j) \quad (12)$$

One easily obtains

$$\text{Tr } M_{1/2} M_{1/2}^+ = e^G (g^{ij} + g^i g^j) (g_{ij} + g_i g_j) + 4g^2 C_2(R) |y^i|^2 \quad (13)$$

whilst the contribution from gauge vector bosons is simply given by

$$\text{Tr } M_1^2 = 2g^2 C_2(R) |y^i|^2 \quad (14)$$

Putting together Eqs. (6), (10), (13) and (14), we obtain for the thermal correction (4)

$$\Delta V_T = -\frac{\pi^2}{24}(N+1)T^4 + \frac{T^2}{24} \left\{ e^G \left[3(g^{ij} + g^i g^j)(g_{ij} + g_i g_j) + 2(N-1)g^i g_i - 2(N+1) \right] + 12g^2 C_2(R) |y^i|^2 \right\} \quad (15)$$

The number of chiral superfields is expected to be of the order of 10^2 ; for instance, in the minimal supersymmetric SU(5) model, we have $N = 80$: a singlet field z which is responsible for supersymmetry breaking, three families of ordinary matter, a Higgs Σ in the adjoint (24) representation and two Higgses H and $\bar{H}$ in the representations $\underline{5}$ and $\bar{\underline{5}}$. In this way, the large N limit of (15) simplifies to

$$\Delta V_T = \frac{T^2}{12} e^G N (g^i g_i - 2) \quad (16)$$

where we have kept only field-dependent terms in (15). This result agrees with those of Refs. 13) and 17). Note that the contribution from D-terms [the last term in (15)] is the subleading one in the large N limit because $C_2(R)$ grows as the dimension of the representation grows. Therefore, at the global minimum y_0^1 , the temperature-corrected potential to subleading order reads as

$$V = \frac{T^2}{12} \left[N m_{3/2}^2 + 6g^2 C_2(R) |y_0^i|^2 \right] \quad (17)$$

where we have used the fact that $m_{3/2} = e^{G/2}$. This may be taken as an indication that the minimum $y_0^1 = 0$ is preferred over other minima at high temperatures.

An application of (16) to inflationary models based on minimal supergravity has been made in Refs. 13) and 17). We will not dwell upon minimal supergravity any longer but, instead, turn to non-minimal models.

3. - FINITE TEMPERATURE CORRECTIONS IN MAXIMALLY SYMMETRIC SUPERGRAVITY

The Kähler potential of maximally symmetric supergravity is defined by^{18),14)}

$$G = G + F(\phi^e) + F^+(\phi_e^+) \quad (18)$$

where

$$G = -3 \log (f(z) + f^+(z^+) - \frac{1}{3} \phi_\ell^+ \phi^\ell) \quad (19)$$

We use, for scalar fields, the notation y^i ($i = 0, \dots, N-1$) with $y^0 \equiv z$ and $y^\ell \equiv \phi^\ell$ ($\ell = 1, \dots, N-1$). The function F in (18) is an analytic function of scalar fields, the superpotential. The material of this section has partly appeared in Ref. 19).

To compute the scalar potential (1) and the fermion mass matrix, we need to compute the inverse matrix $(g^{-1})^i_j$, where, using (18) and (19), we have

$$\begin{aligned} g_m^\ell &= G_m^\ell = \frac{1}{3} G^\ell G_m + e^{G/3} \delta_m^\ell \quad (\ell, m = 1, \dots, N-1) \\ g_0^i &= \frac{1}{3} G^i G_0 \quad ; \quad g_i^0 = \frac{1}{3} G^0 G_i \quad (i = 0, \dots, N-1) \end{aligned} \quad (20)$$

For that we can use Appendix A with

$$M_m^\ell = e^{G/3} \delta_m^\ell \quad (21)$$

A direct calculation shows that the scalar potential corresponding to (18) is given by

$$V = e^{\frac{2}{3}G} e^{F+F^+} F_\ell F^\ell + \frac{1}{2} \text{Re} f_{\alpha\beta}^{-1} D^\alpha D^\beta \quad (22)$$

and the contribution to the chiral fermion mass matrix, (5), is

$$\Upsilon^{ij} = \frac{2}{3} g^i g^j + F^{ij} + \frac{1}{3} F^i F^j \quad (23)$$

Now, because $g_j^i \neq \delta_j^i$, in order to compute the mass matrices we have to normalize properly the scalar and fermion fields which are contained in chiral supermultiplets. The kinetic term is given by

$$\mathcal{L}_{KE} = - G_i^j \partial_\mu y^i \partial^\mu y_j^+ + G_i^j \bar{\psi}_j \not{\partial} \psi^i \quad (24)$$

so that using (18) and (19) we obtain

$$G_i^j \partial_\mu y^i \partial^\mu y_j^+ = 3e^{\frac{2}{3}G} \partial_\mu z \partial^\mu z^+ + e^{G/3} \partial_\mu \phi^e \partial^\mu \phi_e^+ - e^{\frac{2}{3}G} \phi^e \partial_\mu z \partial^\mu \phi_e^+ - e^{\frac{2}{3}G} \phi_e^+ \partial_\mu z^+ \partial^\mu \phi^e \quad (25)$$

and a similar expression for the fermionic kinetic Lagrangian $G_{\bar{\psi}^j \psi^i}^{\bar{\psi}^j \psi^i}$. Therefore, and for the purpose of computing mass matrices in finite temperature corrections, we will redefine the normalized scalar fields

$$\begin{aligned} z &\rightarrow \sqrt{3} e^{G/3} z \\ \phi^e &\rightarrow e^{G/6} \phi^e \end{aligned} \quad (26)$$

and there is a similar redefinition for their fermionic partners coming from (24), as is needed by supersymmetry requirements.

In the same way, gauge vector bosons and gauginos have to be properly normalized if $f_{\alpha\beta} \neq \delta_{\alpha\beta}$, along with the gauge coupling constant. This normalization is discussed in Appendix B. Here we take the simplest non-minimal choice for $f_{\alpha\beta}$ in such a way that $\langle f_{\alpha\beta} \rangle$ is SU(5) invariant

$$f_{\alpha\beta} = \delta_{\alpha\beta} h(z) \quad (27)$$

This choice leads to a Majorana mass for gauginos which will break softly global supersymmetry and allows the SU(2) $\times$ U(1) radiative breaking and the dynamical determination of the electroweak scale:

$$m_{\tilde{V}} = \left| e^{G/2} \frac{G_0}{G_0} (\log \text{Re} h)_z \right| \simeq \mathcal{O}(M_W) \quad (28)$$

Using (26) and (B.7) we obtain for the normalized mass matrix M_0^2

$$\begin{aligned} \frac{1}{2} (M_0^2)_j^i &= e^{-G/3} (V_j^i + \tilde{D}^{\alpha i} \tilde{D}_j^\alpha) + \\ &+ \frac{1}{\sqrt{3}} e^{-G/2} \frac{f^i V_j^0 + f_j V_0^i}{|f^0|^2} + \frac{1}{3} e^{-\frac{2}{3}G} \frac{f^i f_j V_0^0}{|f^0|^4} \end{aligned} \quad (29)$$

where $\tilde{D}^\alpha$ is given by (B.8), with $\tilde{g} = g \text{Re} h^{-\frac{1}{2}}$ as in (B.6). A direct calculation shows

$$\begin{aligned} \text{Tr } M_0^2 = & 2e^{G/3+F+F^+} [F_{em} F^{em} + \frac{2}{3} g_e g^e F_m F^m + 2e^{G/3} F_e F^e + \\ & + \frac{1}{3} F^{em} F_e (2g_m + F_m) + \frac{1}{3} F_{em} F^e (2g^m + F^m) + \\ & + \frac{2}{3} e^{G/3} (N-1) F_e F^e] + 2\tilde{g}^2 e^{G/3} C_2(R) |\phi^e|^2 \end{aligned} \quad (30)$$

where, as in (10), a sum over representations is implicit in $C_2(R) |\phi^e|^2$.

The fermion mass matrix is given by

$$M_{1/2} = \begin{pmatrix} e^{\frac{1}{2}(F+F^+)} m_{ij} & i\sqrt{2} e^{-\frac{1}{6}G} \tilde{D}^{\alpha i} \\ i\sqrt{2} e^{-\frac{1}{6}G} \tilde{D}^{\alpha i} & m_{\tilde{V}}^{\alpha\beta} \delta^{\alpha\beta} \end{pmatrix} \quad (31)$$

where the chiral fermion mass matrix is

$$\begin{aligned} m^{en} &= e^{\frac{1}{6}G} \left(\frac{2}{3} g^e g^n + F^{en} + \frac{1}{3} F^e F^n \right) \\ m^{e0} &= -\frac{2}{\sqrt{3}} e^{\frac{1}{3}G} g^e \\ m^{00} &= 2e^{\frac{1}{2}G} \end{aligned} \quad (32)$$

From (31) and (32), we get

$$\begin{aligned} \text{Tr } M_{1/2}^2 = & e^{\frac{1}{3}G+F+F^+} \left\{ \frac{2}{3} g^e g^m + F^{em} + \frac{1}{3} F^e F^m \right\} \left\{ \frac{2}{3} g_e g_m + \right. \\ & \left. + F_{em} + \frac{1}{3} F_e F_m \right\} + \frac{8}{3} e^{\frac{2}{3}G+F+F^+} g^e g_e + 4e^G + \\ & 4e^{\frac{1}{3}G} \tilde{g}^2 C_2(R) |\phi^e|^2 + \dim(\text{Adj}) m_{\tilde{V}}^2 \end{aligned} \quad (33)$$

while the contribution from gauge vector bosons is, as in (14),

$$\text{Tr } M_1^2 = 2\tilde{g}^2 e^{\frac{1}{3}G} C_2(R) |\phi^e|^2 \quad (34)$$

The finite temperature correction potential (4) can be read off from (6), (30), (33) and (34). In particular, the large N limit gives

$$\Delta V_T \underset{N \text{ large}}{\simeq} \frac{T^2}{18} N e^{\frac{2}{3}G+F+F^+} F_\ell F^\ell \quad (35)$$

plus a field-independent T^4 -term.

We can thus state that, in the large N limit, the global minima of the total finite temperature potential are the same as those of the zero temperature potential, and they are given by the supersymmetric condition

$$F_\ell = 0 \quad (36)$$

However, as was the case in globally supersymmetric theories, there are often degenerate minima satisfying (36), and non-leading terms in N can break this degeneracy. In particular, once we impose condition (36), the global supersymmetric minimum is required to minimize the ϕ -dependent part of the total potential

$$\begin{aligned} V = \frac{T^2}{24} \left\{ e^{\frac{1}{3}G+F+F^+} \left[3F_{\ell m} F^{\ell m} + \frac{2}{3} e^{\frac{2}{3}G} (\phi^\ell \phi^m F_{\ell m} + \right. \right. \\ \left. \left. + \phi_\ell \phi_m F^{\ell m}) + \frac{4}{9} e^{\frac{4}{3}G} |\phi^\ell|^4 + \frac{8}{3} e^G |\phi^\ell|^2 \right] + \right. \\ \left. + \dim(\text{Adj}) m_V^2 + 2e^G + 12 e^{\frac{4}{3}G} \tilde{g}^2 C_2(R) |\phi^\ell|^2 \right\} \quad (37) \end{aligned}$$

where we have included the condition (36) and $D^\alpha = 0$.

Since the potential (37) is not a sum of positive definite terms, it is not evident that finite temperature corrections would prefer the symmetric phase $\langle \phi^\ell \rangle = 0$. For each particular model, this should be checked separately from (37), imposing the condition (36).

4. - FINITE TEMPERATURE CORRECTIONS IN GENERALIZED MAXIMALLY SYMMETRIC SUPERGRAVITY

In this section, we are going to compute finite temperature corrections in a class of supergravity models^{14),5)-7)} which generalize the maximally symmetric models studied in Section 3. This class of supergravity models has recently been used to construct realistic inflationary models.

The field content is the usual one (z, y^a) , $a = 1, \dots, N-2$, as in Section 3, and an additional gauge singlet ϕ - the inflaton field. The Kähler potential is given by

$$\mathcal{G} = G - aK(\phi, \phi^\dagger) + F(\phi, y^a) + F^\dagger(\phi^\dagger, y_a^\dagger) \quad (38)$$

where $K(\phi, \phi^\dagger)$ is a real function, a is a real positive constant, F is the superpotential function and

$$G = -3 \log (f(z) + f^\dagger(z^\dagger) - K(\phi, \phi^\dagger) - \frac{1}{3} y^a y_a^\dagger) \quad (39)$$

We observe that the $\mathcal{G}$ -function of maximally symmetric supergravity, (18), (19) is just a particular case of (38), (39) when $a = 0$ and $K = \phi\phi^\dagger/3$.

The metric for chiral scalars and fermions is given by

$$g_{ij}^{} = \frac{1}{3} G^i G_j + M_{ij}^{} \quad ; \quad i, j = z, \phi, a \quad (a=1, \dots, N-2) \quad (40)$$

where the matrix M_j^i is given by

$$\begin{aligned} M_{\phi}^{\phi} &= (3x - a) K_{\phi}^{\phi} \\ M_b^a &= x \delta_b^a \quad , \quad x = e^{\frac{1}{3} G} \end{aligned} \quad (41)$$

the other elements being equal to zero.

An application of the formulae contained in Appendix A leads to the scalar potential

$$V = \frac{x^3}{3x-a} e^{-aK+F+F^+} \frac{|F_\phi - aK_\phi|^2}{K_\phi} + x^2 e^{F+F^+-aK} F^a F_a + \frac{1}{2} \tilde{D}^\alpha \tilde{D}_\alpha \quad (42)$$

from which we see that V is positive semi-definite in the physical domain defined by

$$3x-a > 0 \quad (43a)$$

$$K_\phi > 0 \quad (43b)$$

From (41) we see that the scalar fields ϕ and y^a (and, correspondingly, their supersymmetric partners) have to be normalized as

$$\begin{aligned} \phi &\rightarrow [(3x-a)K_\phi]^{1/2} \phi \\ y^a &\rightarrow x^{1/2} y^a \end{aligned} \quad (44)$$

The minimization of the scalar potential (42) leads to $F_a = \tilde{D}^\alpha = 0$. If $V = 0$ at that minimum, the physical domain is $x > a/3$. However, if as in the case of inflation we want to have a metastable minimum with non-zero positive vacuum energy, we obtain

$$\begin{aligned} x &= a/2 \\ F_\phi - aK_\phi &\neq 0 \end{aligned} \quad (45)$$

This illustrates the crucial role played by a for inflationary models. Since $x = 0$ is outside the allowed physical domain, one must choose $a \neq 0$. Hereafter we will set for simplicity

$$a = 2 \quad (46)$$

so that $x = 1$ for all values of ϕ (except the global minimum), and therefore the matter fields y^a do not need any normalization.

We will separate in the thermal correction (4) the contributions from z and ϕ from that of y^a as

$$\begin{aligned} \text{Tr } M_0^2 &= \text{Tr } M_0^2(z, \phi) + \text{Tr } M_0^2(y^a) \\ \text{Tr } M_{1/2}^2 &= \text{Tr } M_{1/2}^2(z, \phi) + \text{Tr } M_{1/2}^2(y^a) \\ \text{Tr } M_1^2 &= \text{Tr } M_1^2(y^a) \end{aligned} \quad (48)$$

so that

$$\Delta V_T = \Delta V_T(z, \phi) + \Delta V_T(y^a) \quad (49)$$

with

$$\begin{aligned} \Delta V_T(z, \phi) &= \frac{T^2}{24} [\text{Tr } M_0^2(z, \phi) + \text{Tr } M_{1/2}^2(z, \phi) + \text{Tr } M_{3/2}^2] \\ \Delta V_T(y^a) &= \frac{T^2}{24} [\text{Tr } M_0^2(y^a) + \text{Tr } M_{1/2}^2(y^a) + 3\text{Tr } M_1^2(y^a)] \end{aligned} \quad (50)$$

We will concentrate first on $\Delta V_T(y^a)$. The contribution from scalar particles is given by

$$\begin{aligned} \frac{1}{2} \text{Tr } M_0^2(y^a) &= \left\{ \left(\frac{2}{3} G^a + F^a \right) \left(\frac{2}{3} G_a + F_a \right) F_b F^b + \left(\frac{2}{3} G^a + F^a \right) F_{ca} F^c + \right. \\ &+ \left(\frac{2}{3} G_a + F_a \right) F^{ca} F_c + \frac{2}{3} G_a G^a F_b F^b + \frac{2}{3} (N-2) F_b F^b + F_{ab} F^{ab} + \\ &+ \left. \frac{|F_\phi - 2K_\phi|^2}{K_\phi} [F^a F_a + \frac{2}{3} G^a G_a] \right\} e^{-aK + F + F^\dagger} + \tilde{g}^2 C_2(R) |y^a|^2 \end{aligned} \quad (51)$$

where we have assumed a completely additively separable form for the superpotential

$$F(\phi, y^a) = f_1(\phi) + f_2(y^a) \quad (52)$$

This assumption is made for simplicity only.

To compute the contribution from fermions, we need to know the chiral fermion mass matrix. Using Appendix A, one can find the general expression

$$\begin{aligned} \gamma^{ij} = & -\frac{1}{3} G^i G^j - 2K^{ij} + F^{ij} + (G^i + F^i)(G^j + F^j) - 2(G^i + F^i)K^j \\ & - 2K^i(G^j + F^j) + 4K^i K^j + \frac{F^\phi - 2K^\phi}{K_\phi^\phi} [H_\phi^j G^i + H_\phi^i G^j + 3H_\phi^{ij} - \\ & - 2K_\phi^{ij}] + [H_a^j G^i + H_a^i G^j + 3H_a^{ij}] F^a \end{aligned} \quad (53)$$

where

$$H = -K(\phi, \phi^\dagger) - \frac{1}{3} y^a y_a^\dagger \quad (54)$$

The different components of T^{ij} can be easily extracted from (53) as

$$\gamma^{ab} = \frac{2}{3} (G^a + F^a)(G^b + F^b) + F^{ab} + \frac{1}{3} F^a F^b \quad (55)$$

$$\gamma^{\phi b} = F^\phi F^b + 2K^\phi G^b \quad (56)$$

$$\gamma^{zb} = -2(G^b + F^b) \quad (57)$$

$$\gamma^{z\phi} = -6K^\phi \quad (58)$$

$$\gamma^{zz} = 6 \quad (59)$$

$$\begin{aligned} \gamma^{\phi\phi} = & (F^\phi + K^\phi)^2 + F^{\phi\phi} - 3(K^\phi)^2 - 2K^{\phi\phi} - \\ & - (F^\phi - 2K^\phi) \left(6K^\phi + \frac{K_\phi^{\phi\phi}}{K_\phi^\phi} \right) \end{aligned} \quad (60)$$

It is now clear from Eqs. (55)-(60) that the y -dependent contribution to $\text{Tr} M_{\frac{1}{2}}^2$ is given by

$$\begin{aligned} \text{Tr} M_{\frac{1}{2}}^2(y^a) = & e^{F+F^\dagger - 2K} \left[|\gamma^{ab}|^2 + \frac{2}{K_\phi^\phi} |\gamma^{b\phi}|^2 + \right. \\ & \left. + \frac{2}{3} |\gamma^{bz}|^2 \right] + 4\tilde{g}^2 C_2(R) |y^a|^2 \end{aligned} \quad (61)$$

where the normalizations (26) and (44) are used.

Finally, the contribution of gauge vector bosons is only dependent on y^a (non-singlets) and can be written as in (34)

$$\text{Tr } M_i^2(y^a) = 2\tilde{g}^2 C_2(R) |y^a|^2 \quad (62)$$

Now, from (51), (55)-(57), (61) and (62), the thermal correction $\Delta V_T(y^a)$ can be obtained. In particular, in the large N limit

$$\Delta V_T(y^a) \underset{N \text{ large}}{\simeq} \frac{T^2}{18} N F_a F^a e^{-2K+F+F^\dagger} \quad (63)$$

we recover the same expression as in normal maximally symmetric supergravity, Eq. (35). It turns out that, in the large N limit, the global minima of the total y -dependent potential are the same as those of the zero temperature potential, i.e., $F_a = 0$. From now on, we can follow the same steps as in Section 3 to deduce that, among all possible supersymmetric minima, the finite temperature corrections will actually prefer $\langle y^a \rangle = 0$. The D-term contribution $|y^a|^2$ in (51), (61) and (62) will help in this sense. Not only that, but since y^a are non-singlets, the superpotential F cannot contain linear terms and $F_a(y^c = 0) = 0$. If the superpotential does not contain explicit mass terms, then also $F_{ab}(y^c = 0) = 0$ and correspondingly $\Delta V_T(y^a = 0) = 0$. Because $\Delta V_T(y^a)$ is a sum of positive definite (mass)² terms, this shows that $y^a = 0$ is a global minimum of the potential. Indeed, it should be stressed that the condition that the superpotential does not contain explicit mass terms is actually sufficient for $y^a = 0$ to be a global minimum, though not a necessary one. In SU(5) GUTs, this condition is met provided we do not introduce an explicit $H\bar{H}$ -mixing term but rather let it be produced by loop diagrams. This can be achieved by imposing certain discrete symmetries on the superpotential, as in Ref. 16).

On the other hand, to have a tree-level breaking of SU(5) we need a term like $a_2 \Sigma^2$ with $a_2 \neq 0$, which does not satisfy $F_{\Sigma\Sigma} = 0$ for $\Sigma = 0$. However, as in the inflationary model of Ref. 7), a_2 can be a function of the inflaton field ϕ . Then the slow roll-over of ϕ serves as time for a_2 , which can be zero for $\phi = 0$

so that $F_{\Sigma\Sigma}(\Sigma = 0) = 0$ is indeed satisfied. In inflationary models of the maximally symmetric type, high temperature corrections are relevant only in the region $\phi \approx 0$, and do not affect the details of the SU(5) phase transition that takes place⁵⁾ during the slow roll-over when $\phi \gg$ the Hubble constant.

We now turn to the calculation of $\Delta V_T(z, \phi)$ as given by (50). Using the normalizations (26) and (44), we can write

$$\text{Tr } M_o^2(z, \phi) = \frac{2}{3} V_z^2 + \frac{2}{K_\phi^\phi} V_\phi^\phi \quad (64)$$

with

$$V_z^2 = 6 e^{F+F^\dagger-2K} \frac{|F_\phi-2K_\phi|^2}{K_\phi^\phi} \quad (65)$$

and, after some algebraic manipulations, one can obtain for V_ϕ^ϕ the following expression:

$$\begin{aligned} V_\phi^\phi = e^{F+F^\dagger-2K} & \left\{ |F_\phi-2K_\phi|^2 \left[\frac{|F_\phi|^2}{2K_\phi^\phi} + \frac{5|K_\phi|^2}{K_\phi^\phi} - \frac{2}{K_\phi^\phi} F_\phi K_\phi^\phi + \right. \right. \\ & + \frac{3}{K_\phi^\phi} K_\phi K_\phi^{\phi\phi} - \frac{(F_\phi+K_\phi)}{(K_\phi^\phi)^2} K_\phi^{\phi\phi} - \frac{1}{2} \frac{K_\phi^{\phi\phi}}{(K_\phi^\phi)^2} + \left. \frac{|K_\phi^{\phi\phi}|^2}{(K_\phi^\phi)^3} - 3 \right] \\ & + 2K_\phi^\phi + \frac{1}{2K_\phi^\phi} |F_{\phi\phi}-2K_{\phi\phi}|^2 + \frac{(F_{\phi\phi}-2K_{\phi\phi})(F_\phi^\dagger-2K_\phi^\dagger)^2}{K_\phi^\phi} - \\ & \left. - \frac{(F_\phi-2K_\phi)(F_\phi^{\dagger\phi}-2K_\phi^{\dagger\phi})}{(K_\phi^\phi)^2} K_\phi^{\phi\phi} \right\} + h.c. \quad (66) \end{aligned}$$

which is written in an explicitly real form.

The contribution of chiral fermions is provided by

$$\begin{aligned} \text{Tr } M_{1/2}^2(z, \phi) = e^{F+F^*-2K} \left\{ (K_\phi^\phi)^{-2} |\gamma^{\phi\phi}|^2 + \right. \\ \left. + 3(K_\phi^\phi)^{-1} |\gamma^{\phi z}|^2 + \frac{1}{9} |\gamma^{zz}|^2 \right\} \end{aligned} \quad (67)$$

and using Eqs. (58)-(60), and including the gravitino contribution, we get

$$\begin{aligned} \text{Tr } M_{1/2}^2(z, \phi) + \text{Tr } M_{3/2}^2 = e^{F+F^*-2K} \left\{ 2 + 12 \frac{|K_\phi^\phi|^2}{K_\phi^\phi} + \right. \\ \left. + \frac{1}{(K_\phi^\phi)^2} \left| (F^{\phi\phi} + K_\phi^\phi)^2 + F^{\phi\phi} - 3(K_\phi^\phi)^2 - 2K_\phi^\phi - \right. \right. \\ \left. \left. - (F^{\phi\phi} - 2K_\phi^\phi) \left(6K_\phi^\phi + \frac{K_\phi^{\phi\phi}}{K_\phi^\phi} \right) \right|^2 \right\} \end{aligned} \quad (68)$$

The thermal correction $\Delta V_T(z, \phi)$ can be read off from (50), (64)-(66) and (67)-(68). This completes our computation of the thermal corrections in generalized maximally symmetric supergravity. We turn now to applications of the results obtained in this and in the previous section.

5. - APPLICATION OF THERMAL CORRECTIONS TO A NO-SCALE SUSY GUT

In this section we will apply the results of Section 3 to the phenomenologically interesting SU(5) no-scale GUT of Refs. 10) and 11). There, as always in no-scale models, the potential vanishes identically at the tree level, except for the D-terms. If we focus on the SU(5) adjoint fields Σ in the SU(3) $\times$ SU(2) $\times$ U(1) invariant direction, the latter will also vanish. However, in this model there is a non-trivial function $f_{\alpha\beta}(\Sigma)$ which provides us with gaugino masses and thus breaks supersymmetry. This breaking propagates in radiative corrections and gives rise to a radiative potential¹⁰⁾ for Σ :

$$\Delta V_{\text{rad}} = - \frac{1}{64\pi^2} m_{\tilde{V}}^2 |\Sigma|^4 \log |\Sigma|^2 \quad (69)$$

This expression is expected to be meaningful only when $|\Sigma| < M_P$, as discussed in Ref. 10).

The finite T dependence can be read off from (37). The resulting potential is¹⁹⁾, using $m_{3/2} = e^{G/2}$,

$$\Delta V_T = \frac{T^2}{12} \left[\frac{2}{3} m_{3/2}^{10/3} |\Sigma|^4 + \left(\frac{4}{3} m_{3/2}^{8/3} + 30 \tilde{g}^2 m_{3/2}^{2/3} \right) |\Sigma|^2 \right] \quad (70)$$

Therefore at high T the only minimum lies at $\Sigma = 0$, and consequently $SU(5)$ is not broken. However, as the temperature decreases, (69) and (70) become of the same order of magnitude, and phase transition from $\Sigma = 0$ to $\Sigma \approx O(M_P)$ becomes possible at $T = T_c$. In the model of Ref. 9), where $m_{3/2} \approx M_P$, this happens when

$$T_c \approx \theta(m_{\tilde{V}}) \quad (71)$$

However, since $m_{\tilde{V}}$ will be determined dynamically by $SU(3) \times SU(2) \times U(1)$ radiative corrections to be $O(M_W)$, this model is not realistic. The barrier height between the minima $\Sigma = 0$ and $\Sigma = O(M_P)$ is $m_{\tilde{V}}^2$, much larger than the energy density of thermal fluctuations at T_c , which are $O(m_{\tilde{V}}^4)$. Therefore the phase transition can never be completed.

Let us consider the possibility¹¹⁾ $m_{3/2} \ll M_P$. In this case, the temperature correction (70) is dominated for $|\Sigma| < M_P$ by the contribution coming from D-terms only [last term in (70)], and is thus a simple quadratic form. The critical temperature is then

$$T_c \approx \theta(m_{\tilde{V}} m_{3/2}^{-1/3}) \quad (72)$$

If the phase transition is to take place, the scale of thermal fluctuations, T_c , should not be much smaller than the scale of the potential. Thus from (72) we obtain

$$T_c^4 \gtrsim \theta(m_{\tilde{V}}^2) \Rightarrow m_{3/2} \lesssim \theta(m_{\tilde{V}}^{3/2}) \quad (73)$$

This falls roughly in the range allowed by cosmological and particle physics constraints as discussed in Ref. 11). For instance, compatibility with the observed anomalous magnetic moment of the muon would imply²⁰⁾ $m_{3/2} \gtrsim 10^{-7}$ eV. It is perhaps worth remarking that a light ($\ll M_W$) gravitino can also evade¹¹⁾ the

cosmological bound $m_{3/2} < 1$ keV if they weigh less than the photinos. Then their number density, produced by photino decay, will be greatly reduced because of photino-photino annihilations.

It is also interesting to note that the limit (73) on T_c coincides with the bound $T_c \gtrsim 10^{10}$ GeV coming from requirements for baryon asymmetry generation²¹⁾. Since the decay temperature of the Higgs bosons, $T_D \sim \Gamma_H^{1/2} \sim (\alpha_H M_H)^{1/2}$, is much smaller than the mass of the Higgs bosons when $\alpha_H \sim 10^{-3}-10^{-6}$ (the square of the Yukawa coupling), cosmological baryon asymmetry will, in this model, proceed through out-of-equilibrium decays of Higgs bosons into light matter. Finally, let us mention that even without inflation, $T_c \approx 10^{10}$ GeV is consistent with the Parker bound²²⁾ on the average monopole flux in the galaxy.

6. - APPLICATION OF THERMAL CORRECTIONS TO MAXIMALLY SYMMETRIC INFLATIONARY MODELS

In this section we will apply the results of Section 4 to a class of inflationary models^{14),5)-7)} based on the generalization of the maximally symmetric case, as outlined in Section 4. The zero temperature limits of these models have recently been studied in detail⁷⁾. They provide a model for the early cosmological inflation and the subsequent reheating that is consistent with such particle physics and cosmology requirements as the gravitino density, generation of baryon asymmetry and suppression of $d = 5$ proton decay rate. The latter especially is a severe problem for inflationary scenarios with low [$0(10^{11})$ GeV] reheating. This is because the production of baryon asymmetry at such low temperatures also requires relatively light Higgs bosons which, however, will also be capable of mediating catastrophically rapid proton decay²³⁾. In maximally symmetric inflationary models this problem is easily sidestepped by a novel mechanism⁵⁾, whereby the mass of the SU(5) adjoint field Σ depends on the inflaton field ϕ . This can result in a breakdown of SU(5) during the inflation and a high reheating temperature $T_R \sim 0(M_X) \approx 0(10^{15}-10^{16})$ GeV. At the same time, the problem of how to achieve the GUT phase transition in the first place is also solved.

In the present section we will consider the thermal stability of the maximally symmetric inflationary models. Thermal corrections in an inflationary supergravity model with non-minimal kinetic terms have recently been considered in Ref. 8). Although these models are the two-component models of inflation in

the sense that ϕ and Σ are directly coupled in the region $\phi \approx 0$ which interests us, this coupling can be taken to be weak and therefore Eq. (52) and subsequent discussion is valid.

The Kähler potential is given by Eq. (38), and in the following we will consider this simple form of the K-function:

$$K(\phi, \phi^\dagger) = \phi\phi^\dagger - \frac{1}{4} \beta (\phi\phi^\dagger)^2 + \dots \quad (74)$$

where β is a real parameter, and we envisage that the expansion (74) contains terms which are higher order in $\phi\phi^\dagger$. We are not particularly interested in their form, and use β as the representative for these higher order terms. Observe that if $\beta > 0$, the positivity condition (43b) forces the inflaton ϕ to remain inside the disc $|\phi| < \beta^{-\frac{1}{2}}$. If $\beta < 0$, the condition (43b) is automatically satisfied and does not impose any restriction on the range of ϕ . The global minima of the inflaton potential

$$V(\phi) = e^{F+F^\dagger-2K} \frac{|F_\phi - 2K_\phi|^2}{K_\phi} \quad (75)$$

where $F = F(\phi)$ is the superpotential, given by

$$F(\phi) = \log m_0 + \xi\phi + \frac{1}{2}(2-\xi^2)\phi^2 + \frac{1}{6}\left(2\xi^2 + \xi^{-1}\left(\frac{m_R}{m_0}\right)^2 - \beta\xi\right)\phi^3 + \dots \quad (76)$$

are given in the complex ϕ -plane by

$$\begin{aligned} 1 - u + \left(1 - \frac{\beta}{2\xi^2}\right)(u^2 - v^2) + \frac{\beta}{\xi^4}(u^2 + v^2)u &= 0 \\ v \left[4 - \xi^2 + (2\xi^2 - \beta)u - \frac{\beta}{\xi^2}(u^2 + v^2)\right] &= 0 \end{aligned} \quad (77)$$

where $u = \text{Re}(\xi\phi)$ and $v = \text{Im}(\xi\phi)$. Here ξ and m_0 are parameters that determine the value of the potential at $\phi = 0$, and thus the Hubble constant, through $3H^2 = V(0) = \xi^2 m_0^2$. The particular form of the superpotential (76) follows from the requirement for the existence of a metastable local minimum at $\phi = 0$. The parameter m_R^2 is the curvature of the inflaton potential along the real axis, which we require to be much smaller than the curvature along the imaginary direction

$$m_I^2 = m_0^2 (16 - 12\xi^2 + 2\beta\xi^2) - m_R^2 \gg m_R^2 \quad (78)$$

to define the path of the inflaton field during inflation along the real direction.

The system (77) leads us to a qualitatively different picture when $v = 0$ compared with the case $v \neq 0$. If $v = 0$, the global minima lie on the real axis. However, it is then evident from (77) that for $\beta = 0$ there will be no solutions for u . Therefore, let us take $\beta \neq 0$. The potential will have a maximum $\phi_{\max}$ in the real direction which is situated close to the metastable minimum

$$\phi_{\max} = \frac{2\sqrt{2}}{3} \frac{m_R^2}{m_0^2} \quad (79)$$

where

$$C = -\frac{4}{3}\xi(3\xi^4 - 2\beta\xi^2 + 3\beta) \quad (80)$$

generating a barrier

$$\Delta V = \frac{4}{27} \frac{m_R^6}{m_0^4 C^2} \quad (81)$$

However, such a barrier might also be created by finite temperature effects alone. In that case, there would be no need to consider quantum fluctuations à la Hawking and Moss²⁴⁾, which will drive the inflaton over the barrier with the probability $P = e^{-B}$, where

$$B = \frac{32\pi^2}{9} \frac{m_R^6}{\xi^4 m_0^8 C^2} \quad (82)$$

However, we will not consider here any detailed scenario. Let us mention that the roll-over time scale is

$$H\tau = \left(\frac{m_0}{m_R}\right)^2 \xi^2 \quad (83)$$

and that the energy density perturbations are given by

$$\delta\rho/\rho = \frac{3c}{4\pi^{3/2}\xi^2} H (\log Hk^{-1})^2 \quad (84)$$

where k^{-1} is the scale relevant for galaxies ($Hk^{-1} \approx 10^{21}$).

The requirements for successful inflation are satisfied if $H\tau > 65$ and $\delta\rho/\rho \approx 10^{-4}$. However, we must also ascertain that high temperature effects do not carry the inflaton over the barrier directly to the global minimum without any inflation^{25),26)}. Let us therefore write

$$\Delta V_T(z, \phi) = \frac{T^2}{24} [\tilde{A} + \tilde{B}(\phi + \phi^*) + \theta(\phi^2)] \quad (85)$$

If ϕ rolls to some real, positive global minimum ϕ_0 , it is evident that inflation will start from $\phi \approx 0$ if

$$\tilde{B} > 0 \quad (86)$$

Otherwise, the second term in (85) will push the high T minimum of the potential towards ϕ_0 .

We can now use the general results of Section 5 to obtain B in maximally symmetric inflationary models. Applying (54)-(58) and (74), it is a matter of straightforward (but tedious) calculation to show that

$$\tilde{B} = 22\xi \quad (87)$$

(for the benefit of curious readers, $22 = 16 + 4 + 4 - 2$). Note the remarkable simplicity of the result (87): there is no dependence on β . The thermal stability is ensured if $\xi > 0$. It is thus trivial to construct maximally symmetric inflationary models where all constraints, including the thermal one (86), are satisfied. Unlike in models with minimal kinetic terms²⁶⁾, in maximally symmetric inflation the condition for thermal stability imposes no essential restrictions on the space of parameters.

Just to give an example, we will choose $\xi^2 = 1/10$ and $\beta = -1/50$. In that case, the domain of positive kinetic energy covers the whole complex ϕ -plane [see (43b)]. The requirement $\delta\rho/\rho \approx 10^{-4}$ translates to the following set of values:

$$\begin{aligned} H &= 7.0 \times 10^{12} \text{ GeV} \\ m_0 &= 3.8 \times 10^{13} \text{ GeV} \\ V(0) &= (5.4 \times 10^{15} \text{ GeV})^4 \end{aligned} \tag{88}$$

On the other hand, the requirement $H \tau > 65$ provides, using (83), the absolute upper bound

$$m_R < 1.5 \times 10^{12} \text{ GeV} \tag{89}$$

which already satisfies the condition $m_R < 2H$ for neglecting Coleman-De Luccia tunnelling²⁷⁾. The above values of ξ and β lead to a unique real solution of the system (76): $(u,v) = (0.759,0)$, which corresponds to the global real minimum at

$$\phi_0 = 2.4 M \tag{90}$$

Before concluding this section, we would like to stress again that we have chosen a working model where the inflaton has a unique global minimum on the positive real axis and follows a real path from the origin. In that case the thermal stability condition is trivially satisfied and the whole physical picture, including the SU(5) phase transition, is implemented as in Ref. 7). Under such conditions we have been led to exclude the simplest case $\beta = 0$. However, for this case there are global minima of the inflationary potential (75) in the complex ϕ -plane. When the inflaton rolls from the origin to one of these minima, it can produce sufficient inflation and satisfy thermal stability conditions if an appropriate superpotential is chosen. We do not wish to enter into detailed discussions about various possibilities, but only point out that the potential (75) describes a very general class of inflationary models exhibiting thermal stability.

7. - CONCLUSIONS

In the present paper we have calculated finite temperature corrections in SU(N,1) maximally symmetric supergravity and in a generalization of SU(N,1). Maximally symmetric SU(N,1) leads to no-scale models, where the electroweak breaking scale is determined dynamically. The generalization of SU(N,1) in the form discussed in Section 4 is needed to obtain cosmological inflation.

Because of the non-minimality of both of these models, mass matrices of fermions and bosons, and consequently the finite temperature corrections, are

rather complicated. Therefore it is surprising that the condition for thermal stability of the origin in the inflationary model of Section 6 turned out to be extremely simple [see Eq. (87)]. It is also interesting that the condition does not depend on the parameter β , which represents the effects of higher-order terms in the function K . This fact is also partially responsible for the triviality of the thermal stability condition, which is satisfied for all $\xi > 0$. In fact, also $\xi < 0$ can always be made consistent with the thermal constraint if supplemented by a discrete field transformation $\phi \rightarrow -\phi$. Indeed, it is easy to verify that $\xi \rightarrow -\xi$, $\phi \rightarrow -\phi$ leaves our inflaton potential (75) invariant.

It should perhaps be stressed that there is no obvious way to obtain the limit where maximally symmetric non-minimal supergravities would collapse to the minimal case. Therefore it is not surprising that complicated contributions from non-minimal terms proliferate in all thermal corrections.

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APPENDIX A

In this Appendix, we consider the inverse matrix of

$$G_j^i = \frac{1}{3} G^i G_j + M_j^i \quad (i, j = 0, \dots, N-1) \quad (A.1)$$

where

$$M_0^i = M_j^0 = 0 \quad (A.2)$$

and

$$\det(M_m^l) \neq 0 \quad (l, m = 1, \dots, N-1) \quad (A.3)$$

We will just give the final expression:

$$\begin{aligned} (G^{-1})_m^l &= (M^{-1})_m^l \\ (G^{-1})_0^l &= - (M^{-1})_m^l G^m / G^0 \\ (G^{-1})_m^0 &= - (M^{-1})_m^l G_l / G_0 \\ (G^{-1})_0^0 &= (3 + G_l (M^{-1})_m^l G^m) / G^0 G_0 \end{aligned} \quad (A.4)$$

From these will follow the useful identities

$$G_i (G^{-1})_j^i = -e^{-G/3} f_j / |f|^2 \quad (A.5)$$

$$\begin{aligned} (F_i - aK_i) (G^{-1})_j^i &= (M^{-1})_j^i (F_i - aK_i) + \\ &+ \frac{e^{-G/3}}{3} \frac{f_j}{|f|^2} G^k (M^{-1})_k^i (F_i - aK_i) \end{aligned} \quad (A.6)$$

APPENDIX B

In this Appendix, we will study the correct normalizations, corresponding to $f_{\alpha\beta} \neq \delta_{\alpha\beta}$, which appear in the gauge terms of non-minimal $N = 1$ supergravity.

The kinetic Lagrangian for gauge vector bosons and gauginos is given by¹⁾

$$\mathcal{L}_{KE} = -\frac{1}{4} \text{Re} f_{\alpha\beta} F_{\mu\nu}^{\alpha} F^{\beta\mu\nu} - \frac{1}{4} \text{Re} f_{\alpha\beta} \bar{\lambda}^{\alpha} \not{D} \lambda^{\beta} \quad (\text{B.1})$$

so that the correctly normalized kinetic terms are written as

$$\mathcal{L}_{KE} = -\frac{1}{4} \tilde{F}_{\mu\nu}^{\alpha} \tilde{F}^{\alpha\mu\nu} - \frac{1}{4} \tilde{\bar{\lambda}}^{\alpha} \not{D} \tilde{\lambda}^{\alpha} \quad (\text{B.2})$$

where

$$\tilde{F}^{\alpha} \equiv \text{Re} f_{\alpha\beta}^{1/2} F^{\beta} = \partial_{\mu} \tilde{A}_{\nu}^{\alpha} - \partial_{\nu} \tilde{A}_{\mu}^{\alpha} + i \tilde{g}^{\rho\sigma} \tilde{f}_{\alpha\beta\sigma} \tilde{A}_{\mu}^{\rho} \tilde{A}_{\nu}^{\sigma} \quad (\text{B.3})$$

and

$$\begin{aligned} \tilde{A}_{\mu}^{\alpha} &= \text{Re}(f^{1/2})^{\alpha\beta} A_{\mu}^{\beta} \\ \tilde{\lambda}^{\alpha} &= \text{Re}(f^{1/2})^{\alpha\beta} \lambda^{\beta} \end{aligned} \quad (\text{B.4})$$

are the normalized gauge boson and gaugino fields which are related to the original fields by the matrix transformation $\text{Re} f^{1/2\alpha\beta}$ in the Lie algebra. Similarly, the normalized structure constants $\tilde{f}_{\alpha\beta\sigma}$ are related to the original ones by the similarity transformation

$$\tilde{f}_{\alpha\beta\sigma} = \text{Re}(f^{1/2})^{\alpha\gamma} f_{\gamma\beta\sigma} \text{Re}(f^{-1/2})^{\rho\sigma} \quad (\text{B.5})$$

in the Lie algebra, and the normalized gauge coupling constant $\tilde{g}$ is defined as

$$\tilde{g}^{\alpha\beta} = g \text{Re}(f^{-1/2})^{\alpha\beta} \quad (\text{B.6})$$

The normalized covariant derivative $\tilde{D}$ is defined as the usual one, but with gA_μ^α replaced by $\tilde{g}^{\alpha\beta} \tilde{A}_\mu^\beta$. We can also write the D-term contribution to the scalar potential as

$$\text{Re } f_{\alpha\beta}^{-1} D^\alpha D^\beta = \tilde{D}^\alpha \tilde{D}^\alpha \quad (\text{B.7})$$

where

$$\tilde{D}^\alpha = \tilde{g}^{\alpha\beta} g_i(T^\beta)_j^i y^j \quad (\text{B.8})$$

We see from (B.3), (B.6) and (B.8) that $\tilde{g}^{\alpha\beta}$ is going to break the gauge invariance in the vacuum if $\langle \tilde{g}^{\alpha\beta} \rangle \neq 0$. So we need $\langle \tilde{g}^{\alpha\beta} \rangle$ to be proportional to $\delta^{\alpha\beta}$ along the directions of the unbroken gauge group, i.e., $SU(3) \times SU(2) \times U(1)$. For instance, the simplest non-trivial choice for $f_{\alpha\beta}$ is that of Ref. 9), where

$$f_{\alpha\beta} = \delta_{\alpha\beta} f(\Sigma) + d_{\alpha\beta\gamma} \Sigma_\gamma g(\Sigma) \quad (\text{B.9})$$

with f and g some invariant functions of the adjoint Higgs field Σ . Then, when $\langle \Sigma_{24} \rangle \neq 0$, $\langle f_{\alpha\beta} \rangle = \delta_{\alpha\beta} f(\langle \Sigma \rangle)$ for $\alpha, \beta = 1, 2, \dots, 8, 21, \dots, 24$, i.e., along the $SU(3) \times SU(2) \times U(1)$ direction.

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