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Statistical Analysis Techniques in Particle Physics

Fits, Density Estimation and Supervised Learning

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