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GRAVITON INDUCED COMPACTIFICATION IN THE LIGHT CONE GAUGE *

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ABSTRACT

We present the results of a light cone gauge computation of the graviton induced compactification of a $(4+N)$ -dimensional space-time into $(\text{Minkowski})_4 \times S^N$. We contrast the results for $N \leq 15$ with the ones obtained in harmonic gauge.

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The classical field equations of pure gravity or supergravity models in a $(4+N)$ -dimensional space-time admit non-trivial compactifying solutions with roughly the same magnitude of 4-dimensional and N -dimensional cosmological constants [1]. Recently there have been attempts [2-6] to flatten the first four dimensions by including the 1-loop quantum effects. In particular the pure gravity in a $(4+N)$ -dimensional space-time with odd N has been considered [3,4] and the computation of a 1-loop effective potential for the backgrounds $M_4 \times S^N$ has been carried out in the harmonic gauge. It was pointed out that in this gauge for $N \geq 3$ the Faddeev-Popov ghost contributes an imaginary term to V_{eff} . It was not clear whether this is a gauge artifact or an indication of the instability of the background $M_4 \times S^N$. Therefore it was necessary to perform the computation in another gauge, preferably in a ghost-free gauge. The light cone gauge is particularly convenient for this purpose. The present note is the report of these calculations.

Though not manifestly Poincaré invariant the light cone gauge has the advantage of dealing with truly propagating degrees of freedom only and therefore bypassing the difficulty of the Faddeev-Popov ghost term altogether. It also yields a bilinear action which is diagonal and thereby facilitates the computation of the Gaussian functional integrals.

We start from the action integral

$$S = - \frac{1}{16\pi G} \int d^4x \int d^N z \left(R + \lambda \right), \quad (1)$$

where G and λ are constants. Our aim is to compute the vacuum energy $V^{(1)}$ of the oscillations of the $(4+N)$ -dimensional graviton h_{AB} around the background geometry $M_4 \times S^N$, as a function of the radius a of the N -sphere S^N . If we perform harmonic expansion in S^N then

$$V^{(1)} = \frac{1}{2} \sum_{\text{hel}} \sum_{\ell} D_{\ell} \int \frac{d^N k}{(2\pi)^N} \left(k^2 + M_{\ell}^2(a, \lambda) \right)^{1/2}, \quad (2)$$

where ℓ is an $\text{SO}(N+1)$ label and M_{ℓ} denotes the tower of the Kaluza-Klein masses of fields of spins two, one and zero. D_{ℓ} represents the multiplicity of each mode [3] and $\sum_{\text{hel}}$ refers to a summation over different "helicity states" with the appropriate multiplicities.

The k integral in (2) is divergent. To render it meaningful we apply dimensional regularization and then perform the integration. The result is

$$V^{(1)} = -\frac{1}{2} \sum_{h \neq l} \sum_{\ell} D_l \frac{\Gamma(-\frac{n}{2})}{(4\pi)^{n/2}} \left(\frac{r^2}{r_l} \right)^{n/2} \quad (3)$$

where we have continued the space-time dimension from 4 to n . Despite the fact that each individual term in (3) diverges as $n \rightarrow 4$, it was shown in [3] that the sum is finite. This is a consequence of our assumption that N is odd.

Eq. (3) indicates that we need $M_k(a, \lambda)$ in order to be able to compute $V^{(1)}$. These are obtained most conveniently in the light cone gauge. To introduce this gauge first we define the light cone coordinates $(x^-, x^+, x^2, x^3, x^\mu, y^\mu, \mu = 1, \dots, N)$, where

$$x^\pm = \frac{x^0 \pm x^1}{\sqrt{2}}$$

and then impose the gauge conditions $h_{A+} = 0$, for all A . It turns out that the fields h_{A+} do not have independent propagation [7]. Eliminating them simplifies the bilinear action and we get

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left\{ \frac{1}{4} h_{ij}^t \left(\partial^2 + \nabla^2 - \frac{N(N-1)}{a^2} + \lambda \right) h_{ij}^t + \frac{N+2}{8N} h_{ii} \left(\partial^2 + \nabla^2 - \frac{(N-1)(N^2+2N-8)}{(N+2)a^2} + \lambda \right) h_{jj} + \frac{1}{4} h_{\alpha\beta}^t \left(\partial^2 + \nabla^2 - \frac{N^2-3N+4}{a^2} + \lambda \right) h_{\alpha\beta}^t + \frac{1}{2} h_{i\alpha} \left(\partial^2 + \nabla^2 - \frac{(N-1)^2}{a^2} + \lambda \right) h_{i\alpha} \right\}, \quad (4)$$

where $ij = 1, 2$ and $\alpha, \beta, \dots$ are the orthonormal frame indices on S^N . $h_{ij}^t = h_{ij} - \frac{1}{2} \delta_{ij} h_{kk}$ and $h_{\alpha\beta}^t = h_{\alpha\beta} - \frac{1}{N} \delta_{\alpha\beta} h_{\gamma\gamma}$.

Since (4) is diagonal we can readily identify M_k^2 . These are given by *)

*) We remark that the masses do not form patterns appropriate to the 4-dimensional Poincaré symmetry. This is due to the fact that $M_4 \times S^N$ is not a solution of the background equations. For more discussion of this point see Ref. [7].

$$M_k^2 = \frac{1}{a^2} \left[\left(k + \frac{N-1}{2} \right)^2 + Y_k(x) \right], \quad (5)$$

where $x \equiv \lambda a^2$. The values of Y_k which are different for $h_{ij}^t, h_{ii}, h_{\alpha\beta}^t$ and $h_{i\alpha}$ have been listed in Table I.

The next step is to substitute from (5) into (3) and carry out the summations over k . In [3] it was shown that this leads to a finite sum of poly-logarithmic functions $S_0(z)$. The integral or series representations of these functions can be used to compute $V^{(1)}$ numerically.

Our ultimate aim is to look for the stationary points of the effective potential

$$V_{\text{eff}} = V^{(0)}(a, \lambda) + V^{(1)}(a, \lambda) \quad (6)$$

$$V^{(0)} = \frac{1}{16\pi G_0} \left[-\frac{N(N-1)}{a^2} + \lambda \right], \quad \left(G_0 \equiv \frac{G}{V_0(S^N)} \right).$$

To facilitate the search of the extrema of V_{eff} it turns out to be convenient to define a new dimensionless function $F_N(x)$ by

$$V^{(1)}(a, \lambda) = \frac{\lambda^2}{x^2} F_N(x).$$

Then the stationary points of V_{eff} become identical with the zero's of the function $U(x)$ defined by

$$U(x) \equiv [N(N-1)(N+2) - (N+4)x] F_N(x) - 2[N(N-1)-x] x \frac{dF_N(x)}{dx}. \quad (7)$$

To obtain a flat Minkowski space in the first four dimensions we must fine tune the $(4+N)$ -dimensional cosmological constant λ to satisfy

$$\lambda = \frac{x_0 [N(N-1) - x_0]}{16\pi G_0 F_N(x_0)}, \quad (8)$$

where x_0 is a stationary point of V_{eff} .

From Eq. (5) and Table I it follows that in order for M_k^2 to be non-tachyons the variable x should be smaller than some upper bound. This corresponds to $x < 0$ for $N = 1$, $x < \frac{14}{5}$ for $N = 3$, $x < \frac{108}{7}$ for $N = 5$ and

$x < (N-1)(N-2) + 4$ for $N \geq 7$.

In Table II we have given the extrema of V_{eff} and the corresponding values of $F_N(x_0)$ for S^N up to $N = 15$. Figs. 1 to 8 give the plot of V_{eff} as a function of x .

As this table and the plots indicate there are minima for $N = 5, 7, 9, 11, 13$ and 15. Unfortunately the sign of the corresponding Newtonian constant G_0 which is related to the sign of $F_N(x_0)$ via Eq. (8) is negative. This is of course undesirable. However, to be conclusive the correction to the tree level G_0 ought to be computed. For comparison we have exhibited in Table III the results of the same computations in the harmonic gauge, where the imaginary contribution (for $N \geq 3$) of the Faddeev-Popov ghosts have been ignored. Note that in this gauge the 1-loop corrected Newtonian constant is negative for the cases considered.

We believe that in order to draw any viable conclusions, the question of the gauge dependence of the off-shell effective action should be further examined. This is particularly important since the prediction of the Yang-Mills coupling constants requires considerations of the quantum effects. However, if the only problem to be tackled is to flatten the first four dimensions there is an alternative approach already at the classical level, namely the addition of elementary matter fields to this higher dimensional action integral. These may be elementary scalar [8] or gauge fields [9]. The elementary gauge fields have an additional advantage that not only they induce stable compactification [10] they also solve the chirality problem [11].

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REFERENCES

- [1] P.C.O. Freund and M.A. Rubin, Phys. Lett. 97B, 233 (1980).
- [2] P. Candelas and S. Weinberg, Nucl. Phys. B237, 397 (1984).
- [3] M.H. Sarmadi, ICTP, Trieste, preprint IC/84/3 (1984).
- [4] A. Chodos and E. Myers, Ann. Phys. 156 (1984); Yale Univ., preprint YTF84-09 (1984).
- [5] P. Candelas and D.J. Raine, Texas, preprint (1983).
- [6] M.A. Rubin and C.R. Ordóñez, Univ. Texas, preprints UTTG-10-83 (1983) and UTTG-5-84 (1984).
- [7] S. Randjbar-Daemi, Abdus Salam and J. Strathdee, ICTP, Trieste, preprint IC/84/21 (1984) (to appear in Nuovo Cimento).
- [8] M. Gell Mann and B. Zwiebach, Univ. of California, preprint UCB-PTH-84/4 (1984).
- [9] J.F. Luciani, Nucl. Phys. B135, 11 (1978); S. Randjbar-Daemi and R. Percacci, Phys. Lett. 117B, 41 (1982).
- [10] S. Randjbar-Daemi, Abdus Salam and J. Strathdee, Nucl. Phys. B242, 447 (1984). A.N. Schellekens, Stony Brook, preprint ITP-SB-84-45 (1984).
- [11] Z. Horvath, L. Palla, E. Cremmer and J. Scherk, Nucl. Phys. B127, 57 (1977); P. Forgacs and N. Manton, Comm. Math. Phys. 72, 15 (1980); G.F. Chapline and R. Slansky, Nucl. Phys. B209, 461 (1982); S. Randjbar-Daemi, Abdus Salam and J. Strathdee, Nucl. Phys. B214, 491 (1983); Phys. Lett. 132B, 56 (1983); E. Witten, "Fermion quantum numbers in Kaluza-Klein theory", Princeton University, preprint (1983).

TABLE CAPTIONS

- Table I : The γ_k for different components of h_{AB} ($k = \frac{N-1}{2}$).
- Table II : The stationary points x_0 and the values of $F_N(x_0)$ together with the sign of G_0 in the light cone gauge.
- Table III: The stationary points x_0 and the values $F_N(x_0)$ together with the renormalized G in the harmonic gauge.

TABLE I

$SO(N+1)$ rep.	h_{ij}^t	h_{ij}^t	$h_{i\alpha}$	$h_{\alpha\beta}^t$
$(k, 0, 0, \dots, 0)$	$3k^2 + 2k - x$	$\frac{2k(4k^2 + 8k - 5)}{2k + 3} - 1 - x$	$3k^2 - 2k - x$	$3k^2 - 6k - x$
$(k, 1, 0, \dots, 0)$	-	-	$3k^2 - 1 - x$	$3k^2 - 4k - 1 - x$
$(k, 2, 0, \dots, 0)$	-	-	-	$3k^2 - 2k - x$

TABLE II

	Stationary points x_0	maximum or minimum	$F_N(x_0)$	Sign of G_0
s^1	-	-	-	
s^3	-	-	-	
s^5	13.19	min	-0.0363	neg.
s^7	27.22 29.84 33.97	max min. max	-0.0249 -0.0496 +0.1264	neg. neg. pos.
s^9	54.5 59.7	min max	-0.0537 +0.1982	neg. pos.
s^{11}	79.2 86.7 93.3	max min max	+0.0127 -0.0586 +0.3257	pos. neg. pos.
s^{13}	111 118 126 135	min max min max	-0.0006 +0.0128 -0.0653 +0.5192	neg. pos. neg. pos.
s^{15}	150 154 164 174 184	max min max min max	-0.0019 -0.0028 +0.0124 -0.0748 +0.7904	neg. neg. pos. neg. pos.

TABLE III

	Stationary points x_0	maximum or minimum	$F_N(x_0)$	β_N (defined by $\frac{1}{16\pi G} = \frac{\beta_N}{16\pi G_0}$)
s^1	- 0.1667	max	0.000378	-0.65
s^3	-	-	-	-
s^5	13.29	min	-0.1164	3.99
s^7	27.44 32.05	max min	-0.1893 -0.4863	3.82 2.49
s^9	48.06 50.16	min max	-0.289 -0.3225	7.6 5.68
s^{11}	85.85	max	-0.1099	3.48
s^{13}	103.18 131.74	min max	-2.176 -2.346	7.33 2.06
s^{15}	183.207	max	-3.785	2.05

FIGURE CAPTIONS

- Fig.1 - The effective potential $V_{\text{eff}}(x)$ for $N = 1$
 Fig.2 - The effective potential $V_{\text{eff}}(x)$ for $N = 3$
 Fig.3 - The effective potential $V_{\text{eff}}(x)$ for $N = 5$
 Fig.4 - The effective potential $V_{\text{eff}}(x)$ for $N = 7$
 Fig.5 - The effective potential $V_{\text{eff}}(x)$ for $N = 9$
 Fig.6 - The effective potential $V_{\text{eff}}(x)$ for $N = 11$
 Fig.7 - The effective potential $V_{\text{eff}}(x)$ for $N = 13$
 Fig.8 - The effective potential $V_{\text{eff}}(x)$ for $N = 15$

Fig.1

Fig.2

Fig.4

Fig.3

Fig.5

Fig. 6

Fig. 7

Fig. 8