JET ACTIVITY ASSOCIATED WITH W AND Z⁰ PRODUCTION

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ABSTRACT

Predictions of lowest-order QCD perturbation theory are presented for the rate and properties of jet activity produced in association with Intermediate Vector Bosons at the CERN $p\bar{p}$ Collider. For W production, the rate of jet activity, the jet angular and transverse-momentum distributions and the W-jet invariant-mass distribution are shown to be in good agreement with the experimental data. Although the preliminary data suggest that the rate of jet activity in Z⁰ events may be higher than expected, the distributions are consistent with a bremsstrahlung interpretation for the jets.

1. INTRODUCTION

Weak boson production at the CERN $p\bar{p}$ Collider [1] is well described by the Drell-Yan mechanism of quark-antiquark annihilation. A necessary consequence of the QCD improved Drell-Yan mechanism is the associated production of additional quark and gluon jets at a rate determined by the strong coupling α_s (Fig. 1). The singularities associated with the collinear emission of gluons from approximately massless quarks imply that the majority of these initial-state bremsstrahlung quark and gluon jets tend to go in the directions of the beams and therefore escape detection. Sometimes, however, the jets are sufficiently energetic and at large enough angle to the beam directions for them to be detected. The corresponding production cross-section can be calculated in low-order QCD perturbation theory and can thus provide a quantitative test of the theory. To the extent that such QCD calculations can be made with some precision, any substantial deviations from our expectations would signal new physics phenomena; for example, supersymmetry, compositeness, etc.

In this paper we compare the recent UA1 data [2] on the jet activity associated with W and Z production at the CERN $p\bar{p}$ Collider with the predictions of perturbative QCD. We focus in particular on those events in which a single large- p_T jet accompanies a W boson. The transverse-momentum, angular, and invariant-mass distributions are shown to be in excellent agreement with theoretical predictions. Theoretical and experimental uncertainties can be minimized by comparing the (W + jet) events with a sample of three-jet events in which a third jet is produced in association with a large- p_T jet pair with an invariant mass of order m_W . Again we find good agreement between theory and experiment. The sample of Z^0 events is much smaller. Here the rate of jet activity appears higher than theoretical expectations. Nevertheless the jet distributions are again consistent with a bremsstrahlung interpretation.

The paper is organized as follows. In Section 2 we define the cross-sections and input parameters used in the analysis. The QCD predictions are compared with the (W + jet) data in Section 3. In Section 4 we discuss briefly the corresponding results for (Z + jet) production. Finally, in Section 5 we summarize our results and extrapolate our predictions to higher energy.

2. CROSS-SECTION DEFINITIONS

To leading order in ('renormalization group improved') QCD perturbation theory the total W production cross-section is given by

$$\begin{aligned}\sigma(W) &\equiv \sigma(p + \bar{p} \rightarrow W + X) \\ &= \int dx_1 dx_2 G_{a/p}(x_1, Q) G_{b/\bar{p}}(x_2, Q) \hat{\sigma}^{ab \rightarrow W},\end{aligned}\tag{1}$$

where a sum over the relevant parton types is understood. For definiteness we consider the production of W bosons; the label W refers always to a sum over W^+ and W^- . The corresponding Z cross-sections are obtained trivially by changing the weak couplings and the structure function combinations. The cross-section for (W + 1 jet) is given by

$$\begin{aligned}\sigma(W + J) &\equiv \sigma(p + \bar{p} \rightarrow W + J + X) \\ &= \int dx_1 dx_2 G_{a/p}(x_1, Q) G_{b/\bar{p}}(x_2, Q) \hat{\sigma}^{ab \rightarrow Wc}.\end{aligned}\tag{2}$$

We neglect the effects of hadronization, i.e. the jet four-momentum is identified with the underlying parton four-momentum. The subprocess cross-sections $\hat{\sigma}$ are calculated from the Feynman diagrams shown in Fig. 2b and are listed for example in Ref. [3].

It is useful to define the purely hadronic analogues of $\sigma(W)$ and $\sigma(W + J)$, where the W is replaced by a pair of large- p_T jets of the same invariant mass. Thus

$$\begin{aligned}\sigma(2J) &\equiv \sigma(p + \bar{p} \rightarrow J_1 + J_2 + X) \\ &= \int dx_1 dx_2 G_{a/p}(x_1, Q) G_{b/\bar{p}}(x_2, Q) \hat{\sigma}^{ab \rightarrow cd},\end{aligned}\quad (3)$$

$$\begin{aligned}\sigma(2J + J) &\equiv \sigma[p + \bar{p} \rightarrow (J_1 + J_2) + J + X] \\ &= \int dx_1 dx_2 G_{a/p}(x_1, Q) G_{b/\bar{p}}(x_2, Q) \hat{\sigma}^{ab \rightarrow cde},\end{aligned}\quad (4)$$

calculated from the diagrams of Figs. 2c and 2d. The subprocess cross-sections for Eqs. (3) and (4) are taken from Refs. [4] and [5], respectively. The kinematic restrictions for the 2J pair are that i) the 2J invariant mass is required to be close to the W mass ($m_{\min} < m_{jj} < m_{\max}$), and ii) the angle θ between the direction of the two jets and the average beam direction in the rest frame of the 2J system is required to satisfy $|\cos \theta| < \cos \theta_0$. In the present analysis the values $m_{\min} = 70 \text{ GeV}/c^2$, $m_{\max} = 90 \text{ GeV}/c^2$, and $\cos \theta_0 = 0.2$ are used unless otherwise stated. These requirements will enable us to compare our QCD predictions with the analysis presented in Ref. [2].

In QCD with massless quarks the cross-sections $\sigma(W + J)$ and $\sigma(2J + J)$ are singular when i) $p_T^J \rightarrow 0$ (soft) in the case where J is a gluon jet, and ii) the bremsstrahlung parton (quark or gluon) is collinear with either of the incoming partons. We will therefore require that the outgoing jets are at angles of $> 20^\circ$ with respect to the beam directions, and that they have transverse momenta larger than a cut-off value $p_T^{\min}$. Both singularities are then regulated. Two related factors determine the choice of $p_T^{\min}$: i) theoretically, $p_T^{\min}$ should be large enough so that low-order perturbation theory is applicable, and ii) experimentally, $p_T^{\min}$ should be large enough so that the jet algorithm can pick out the jet from the underlying event. The cross-section $\sigma(2J + J)$ has additional singularities at the parton level when the third jet is collinear with either of the other two. Following the experimental analysis we remove these potential problems by requiring that final-state partons (jets) are well separated in (η, ϕ) -space, where η and ϕ are the pseudorapidity and the azimuthal angle in radians, respectively. Following the method used by UA1 to reconstruct jets, we define

$$\Delta r \equiv (\Delta\eta^2 + \Delta\phi^2)^{1/2},$$

and require $\Delta r > r_{\text{cut}}$ for the parton pairs 1 + 3 and 2 + 3. The value of r_{cut} is again fixed by the jet algorithm. In the following the value $r_{\text{cut}} = 1$ is used unless otherwise stated.

With these cuts the cross-sections are finite in lowest-order perturbation theory. Not surprisingly, however, they depend quite sensitively on the values chosen for θ_0 , $p_T^{\min}$, and r_{cut} , and to a lesser extent on the choice of structure functions and Λ_{QCD} . It is therefore useful to define the ratios

$$\begin{aligned}R_W &\equiv \sigma(W + J)/\sigma(W), \\ R_J &\equiv \sigma(2J + J)/\sigma(2J),\end{aligned}\quad (5)$$

which measure the fraction of W and jet pair events containing an additional jet, and the ratio of ratios

$$R \equiv R^W/R^J. \quad (6)$$

As we will demonstrate explicitly in Section 3, many of the uncertainties disappear in the above ratios. The cross-sections $\sigma(W)$, $\sigma(W + J)$, $\sigma(2J)$, and $\sigma(2J + J)$ are $O(\alpha_s^0, \alpha_s^1, \alpha_s^2, \alpha_s^3)$, respectively. Thus R_W and R_J are both $O(\alpha_s)$, and we may therefore expect R to be relatively independent of

the value of α_s . In this respect it is important to note that we have not included any K-factors (i.e. finite higher-order perturbative corrections) in any of the above cross-sections. In fact only the correction to $\sigma(W)$ is known completely. In concentrating on the ratios R_W , R_J , and R we are tacitly assuming that the K-factors are of comparable magnitude and therefore tend to cancel. This is of course an approximation, but appears to be the only reasonable choice at the present time.

Finally, all the above cross-sections depend implicitly on i) the parton structure functions and the associated value of Λ_{QCD} —the ‘mode’—and ii) the scale Q in $\alpha_s(Q)$ and $G_{a/p}(x, Q)$. The numerical results which follow use the two sets of structure functions given by Duke and Owens [6]: mode 1 with $\Lambda = 0.2$ GeV (soft glue) and mode 2 with $\Lambda = 0.4$ GeV (hard glue). We will take the difference between the results for the two modes as being representative of the uncertainties associated with the structure functions and Λ_{QCD} . The scale is chosen to be $Q = m_W$ everywhere.

3. RESULTS AND COMPARISON WITH DATA

For the two sets of structure functions we obtain the ‘total’ cross-sections:

$$\begin{aligned} \text{mode 1: } \sigma(W) &= 3.44 \text{ nb}, & \sigma(2J) &= 15.1 \text{ nb}, \\ \text{mode 2: } \sigma(W) &= 3.11 \text{ nb}, & \sigma(2J) &= 17.2 \text{ nb}. \end{aligned} \tag{7}$$

More detailed calculations of $\sigma(W)$ [7] suggest a K-factor of order 1.3 at collider energies. Increasing the above W cross-sections by such a factor and multiplying by the branching ratio for $W \rightarrow e\nu$ gives a result consistent with the experimental measurements [1]. Throughout this analysis we ignore the contribution to the jet pair cross-sections from the hadronic decays of the weak bosons into two light quark jets. This is a reasonable approximation—we estimate the contamination of the QCD-induced jet pair cross-section to be less than or of order 5%. It is shown in Table 1 that although the cross-sections $\sigma(2J)$ and $\sigma(2J + J)$ separately depend rather strongly on the angle θ_0 defined in Section 1, the ratio R_J does not.

In Fig. 3 the QCD prediction for R_W as a function of the threshold p_T^{min} is compared with the UA1 data. In the limit $p_T^{\text{min}} \rightarrow 0$ the theoretical prediction diverges, as discussed previously. In this region, higher orders in perturbation theory combine with non-perturbative smearing to regularize the singularity. Theoretical analyses of the W p_T distribution suggest that the low-order perturbation theory approximation breaks down somewhere below $p_T = 20$ GeV/c [8]. As Fig. 3 clearly demonstrates, our naïve prediction is in excellent agreement with the data for $p_T^{\text{min}} \geq 8$ GeV/c. We note that R_W is sensitive to the choice of p_T^{min} , especially at small p_T^{min} . The same is true for R_J . However, the ratio R of these ratios is remarkably stable (Table 2). Within our calculation error (of order ± 0.005) the value is constant over a broad range of p_T^{min} . For modes 1 and 2 we find $R = 0.73$ and $R = 0.69$, respectively. These results are in excellent agreement with the measured value [2] of $R = 0.7 \pm 0.2$. The dependence of our result on the structure functions appears to be due mainly to the different shapes used for the gluon distribution in the two modes. This shape dependence does not cancel in the ratio R_J , because $\sigma(2J)$ and $\sigma(2J + J)$ probe different values of the parton distributions:

$$\begin{aligned} \sigma(2J): & \quad \langle (x_1 x_2)^{1/2} \rangle = 0.14, \\ \sigma(2J + J): & \quad \langle (x_1 x_2)^{1/2} \rangle = 0.22, \end{aligned} \tag{8}$$

for mode 1 with $p_T^{\text{min}} = 10$ GeV/c and $r_{\text{cut}} = 1.0$. Thus the harder gluon distribution of mode 2 is felt more in $\sigma(2J + J)$ than in $\sigma(2J)$ and R is smaller.

It might therefore appear that with more statistics the quantity R would serve to distinguish different gluon distributions. Some care is needed, however, because there is still quite a strong dependence on r_{cut} . Figure 4 shows R for r_{cut} values in the range 0.5–1.5. The increase with r_{cut} simply reflects the fact that to this order in perturbation theory $\sigma(2J + J) \rightarrow \infty$ (and hence $R \rightarrow 0$) as $r_{\text{cut}} \rightarrow 0$. We would expect the dependence on r_{cut} to be most strongly felt in the central region. This is illustrated in Fig. 5.

It has been suggested [9] that R is a measure of the relative probabilities for gluon bremsstrahlung off quarks and gluons, which in QCD is in some sense parametrized by the ratio $C_F/C_A = 4/9$. It is indeed true that i) the initial partons in events with high- p_T jets are more likely to be gluons than are the initial partons in W boson events [3], and ii) the ratios R_W and R_J receive *most* of their contribution from the region close to the beam directions where bremsstrahlung off the incoming partons dominates. However, the strong dependence of R on r_{cut} seems to rule out such a simple interpretation. This dependence indicates that the effect of final-state bremsstrahlung—controlled by r_{cut} —is not negligible.

We have already noted that the angular distribution of the jets accompanying the W should be strongly peaked in the beam direction, corresponding to initial-state bremsstrahlung (Fig. 1). To make this more quantitative, we consider

$$(d\sigma/d\cos\theta^*)(p + \bar{p} \rightarrow W + J + X), \quad (9)$$

where θ^* is the angle between the jet and the (Collins–Soper) average beam direction in the $W + J$ centre-of-mass frame. Figure 6 compares the θ^* distributions for $W + J$ and $2J + J$ with the UA1 data. For the former there is again excellent agreement over the complete angular range. For the $2J + J$ case the agreement is reasonable, especially since we recall that the shape of the angular distribution is rather sensitive to the value of r_{cut} (Fig. 5). A slight increase in this value from 1.0 would improve the fit. Note that the effect of final-state bremsstrahlung, which would appear as an enhancement around $\theta^* = 90^\circ$, has been suppressed by the jet separation cut.

A final check that the jet activity observed in W events is understood in the framework of low-order QCD perturbation theory can be made by considering the distribution in the $W + J$ invariant mass (Fig. 7). Once again there is good agreement between theory and experiment.

4. JETS IN Z EVENTS

The corresponding analysis for jets accompanying Z bosons goes through as above with very little quantitative difference in the theoretical predictions. For example with $p_T^{\text{min}} = 5 \text{ GeV}/c$ we get

$$\begin{aligned} \text{mode 1: } R_W &= 0.386, \quad R_Z = 0.390, \\ \text{mode 2: } R_W &= 0.422, \quad R_Z = 0.429. \end{aligned}$$

The small differences that do arise are due to the different shape of the up and down quark distributions in the proton. In contrast, the preliminary experimental measurement is $R_Z/R_W = 2.1 \pm 0.5$ [2]. A more apparent difference arises if the total number of jets rather than ‘events with jets’ are counted. In spite of this the transverse momentum and angular distributions of the jets do appear to be consistent with the QCD bremsstrahlung hypothesis [2]. For example, Fig. 8 shows the jet transverse-momentum distribution together with the theoretical prediction. The latter has been rescaled to reproduce the observed number of events with $E_T^J > 8 \text{ GeV}$. If with higher statistics the excess of jet activity produced in association with Z^0 bosons persists, it

cannot be understood in the framework of lowest-order QCD, strongly suggesting the existence of a new mechanism contributing to Z^0 production at the Collider.

5. CONCLUSIONS

From the results of Section 3 we can conclude that the jet activity associated with W bosons is consistent with the predictions of lowest-order QCD perturbation theory. We should mention an alternative approach to the jet analysis based on a Monte Carlo generation of the complete event, using multiparton bremsstrahlung in the leading log approximation [10]. Whilst such an approach undoubtedly gives more information on such quantities as the transverse energy flow in the W events, it is not clear that it can provide a correct prediction for the ratio $R = R_W/R_J$, which is approximately 'infra-red stable' and probes the full matrix element (as distinct from the leading log part) for each subprocess. Indeed the agreement between theory and experiment for the $p_T^{\min}$, angular, and W + J invariant-mass distributions, and the agreement for the value of R, seem to indicate that to a good approximation the various processes are controlled by the lowest-order matrix elements. Quantitatively we find a value for R close to but less than 1, reflecting the participation of more coloured partons in the subprocesses for jet production than in those for weak boson production. The extra radiation off the final-state partons in $\sigma(2J + J)$ contributes even in the forward/backward angular regions. Remarkably, the value of R is insensitive to the overall centre-of-mass energy $\sqrt{s}$. Table 3 lists the cross-sections and ratios in the range $\sqrt{s} = 0.54 - 5$ TeV. Although the cross-sections are changing by orders of magnitude R remains almost constant. Naïvely one might have expected R to *decrease* owing to the dominance of gluon scattering at large $\sqrt{s}$ in the jet cross-sections. However quark-gluon scattering also becomes a more important part of the W + J cross-section and the two effects appear to cancel. Note that in this extrapolation we have used the structure function parametrizations of Eichten et al. [11] which give a more accurate representation of the structure functions at small x. It should be remembered, however, that the production of fixed-mass final states at very large collision energies can be sensitive to $\alpha_s^n (\ln 1/x)^m$ corrections which modify the evolution of the structure functions in Eqs. (1)–(4). Nevertheless, the quantity R does appear to be a stable 'benchmark' with which to compare theory and experiment over a wide kinematic range.

To analyse W and Z^0 events with more than one jet requires an exact calculation of the matrix elements $ab \rightarrow Wcd$ where a...d stand for all possible combinations of quarks and gluons. The analysis is in progress [12]. Given that each additional distinct hard jet costs a factor of (α_s/π) , it would appear that the small fraction of (W + 2 jet) events observed [2] will be consistent with expectations.

Finally, assuming that the higher-order calculation predicts very little difference between the W and Z^0 jet activity, as was found for the one-jet case, it is hard to understand an observed excess of (Z + multijet) events in the framework of QCD. If this result is confirmed with higher statistics, it may indicate an exciting deviation from our current expectations.

REFERENCES

- [1] G. Arnison et al., Phys. Lett. **122B** (1983) 103; **126B** (1983) 398; **129B** (1983) 273; **134B** (1984) 469; **147B** (1984) 241.
 M. Banner et al., Phys. Lett. **122B** (1983) 476.
 P. Bagnaia et al., Z. Phys. **C24** (1984) 1; Phys. Lett. **129B** (1983) 130.
- [2] UA1 Collaboration, paper presented by S. Geer at the DPF Annual Meeting of the American Physical Association, Santa Fe, 1984 (preprint CERN-EP/84-160).
- [3] A. Ali et al., Proc. ECFA-CERN Workshop on the Feasibility of Hadron Colliders in the LEP Tunnel, Lausanne and Geneva, 1984 (report ECFA 84/85, CERN 84-10, Geneva, 1984), Vol. II, p. 469.
- [4] B.L. Combridge, J. Kripfganz and J. Ranft, Phys. Lett. **70B** (1977) 234.
- [5] F.A. Berends, R. Kleiss, P. de Causmaecker, R. Gastmans and T.T. Wu, Phys. Lett. **103B** (1981) 124.
- [6] D.W. Duke and J.F. Owens, Phys. Rev. **D26** (1982) 1600; Erratum in **D28** (1983) 357.
- [7] G. Altarelli, R.K. Ellis, M. Greco and G. Martinelli, preprint CERN-TH.3851 (1984).
- [8] F. Halzen, A.D. Martin and D.M. Scott, Phys. Rev. **D25** (1982) 754.
 P. Chiapetta and M. Greco, Nucl. Phys. **B221** (1983) 269; Phys. Lett. **135B** (1984) 187.
 P. Aurenche and R. Kinnunen, LAPP preprint TH-78 (1983).
 A. Nakamura, G. Pancheri and Y. Srivastava, Z. Phys. **C21** (1984) 243.
 G. Altarelli et al., reference [7].
 P. Minkowski, Berne preprint BUTP-83/22 (1984).
 C.T.H. Davies, W.J. Stirling and B.R. Webber, preprint CERN-TH.3987 (1984).
- [9] F. Halzen and F. Herzog, Univ. Wisconsin (Madison) preprint MAD/PH/195 (1984).
- [10] R. Odorico, preprint CERN-TH.3662 and 3744 (1983).
- [11] E. Eichten, I. Hinchliffe, K. Lane and C. Quigg, Rev. Mod. Phys. **56** (1984) 579.
- [12] S.D. Ellis, R. Kleiss and W.J. Stirling, Perturbative QCD predictions for two jets accompanying a W or Z boson, in preparation.

Table 1

Dependence of the jet production rates on θ_0 , where the angle θ between the average beam direction and the 2J axis in the 2J rest frame is required to satisfy $|\cos \theta| < \cos \theta_0$. Results are for mode 1, $p_T^{\min} = 10 \text{ GeV}/c$.

$\cos \theta_0$	$\sigma(2J)$ (nb)	$\sigma(2J + J)$ (nb)	R_J
0.1	7.3	3.8	0.53
0.2	15.1	8.0	0.53
0.3	24.0	12.7	0.53

Table 2

Stability of R with the jet p_T threshold $p_T^{\min}$. Results using mode 1.

$p_T^{\min}$ (GeV/c)	R
5	0.73
10	0.74
15	0.73
20	0.72

Table 3

Stability of R with $\sqrt{s}$. Results are for $p_T^{\min} = 20 \text{ GeV}/c$ with structure functions from [11].

$\sqrt{s}$ (TeV)	$\sigma(W)$ (nb)	$\sigma(W + J)$ (nb)	$\sigma(2J)$ (nb)	$\sigma(2J + J)$ (nb)	R_W	R_J	R
0.54	3.01	0.158	15.6	1.11	0.0525	0.0716	0.73
0.63	3.88	0.259	24.6	2.21	0.0667	0.0900	0.74
1	7.18	0.860	80.2	13.0	0.120	0.162	0.74
2	14.6	3.36	334	103	0.230	0.308	0.75
5	32.7	13.6	1518	831	0.416	0.547	0.76

Figure captions

- Fig. 1 : The production of a bremsstrahlung gluon jet with a W boson.
- Fig. 2 : Feynman diagrams for the subprocess cross-sections defined in Eqs. (1) – (4).
- Fig. 3 : The ratio R_W , defined in Section 2, as a function of the jet p_T threshold $p_T^{\min}$. The theoretical curves correspond to the two sets of structure functions from Ref. [6]. The data are from UA1 [2].
- Fig. 4 : The dependence of R on the jet separation cut r_{cut} . The curves correspond to the two sets of structure functions from Ref. [6].
- Fig. 5 : The dependence of the $2J + J$ cross-section on $\cos \theta^*$, where θ^* is the angle between the jet and the average beam direction in the $2J + J$ centre-of-mass frame. The data points are the QCD predictions for two different values of the jet separation variable: $r_{\text{cut}} = 1.0, 1.5$. The results are for $p_T^{\min} = 5 \text{ GeV}/c$, mode 1.
- Fig. 6 : Distributions in the angle θ^* for (a) $W + J$ and (b) $2J + J$. The curves are the QCD predictions using mode 1, $p_T^{\min} = 5 \text{ GeV}/c$ and $r_{\text{cut}} = 1.0$. The data are from UA1 [2].
- Fig. 7 : Distribution in the $W + J$ invariant mass. The curve is the QCD prediction using mode 1, $p_T^{\min} = 5 \text{ GeV}/c$ and $r_{\text{cut}} = 1.0$, normalized to the total number of events observed. The data are from UA1 [2].
- Fig. 8 : Comparison of the shape of the QCD prediction for the jet E_T distribution in $(Z^0 + \text{jet})$ events with the UA1 data from Ref. [2]. The curve is normalized to give the observed number of events with $E_T^J \geq 8 \text{ GeV}$.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

a)

b)

Fig. 6

Fig. 7

Fig. 8