CERN-TH.4029/84

THE MASS OF THE t-QUARK FROM DISCRETE SYMMETRIES III

D. Grosser^{*)}

CERN - Geneva

ABSTRACT

Starting from the previously derived relation $m_t : m_b = m_c : m_s$ and the value $m_s(1 \text{ GeV}) = (110 \pm 10) \text{ MeV}$ recently obtained by Reinders and Rubinstein, we calculate $m_t(m_t)$. We compare this value with the invariant masses of six $l+\nu+\text{jet}$ systems which have been found by the UA1 collaboration at the $p\bar{p}$ collider at CERN and may originate from the decay of a top quark. The central values from experiment and from the calculation are in good agreement.

*) On leave from Institut für Theoretische Physik
der Universität Tübingen, D-7400 Tübingen

The relation

$$m_u : m_c : m_t : \dots = m_d : m_s : m_b : \dots \quad (1)$$

for current quark masses has been derived in Ref. 1) within the framework of the standard model exploiting an idea due to Glashow and Weinberg²⁾ which suggests that flavour-changing neutral Higgs couplings should be suppressed naturally, i.e., as a consequence of a, possibly discrete, symmetry. Equation (1) is, of course, not true for m_u and m_d . It is, however, only a lowest order relation and radiative corrections are expected to be significant for m_u and m_d . On the other hand, the relation

$$m_t / m_b = m_c / m_s \quad (2)$$

is expected to retain an approximate validity even after radiative corrections have been taken into account. Relation (2) was found earlier by Pakvasa and Sugawara³⁾ in an S_4 symmetric extension of the standard model in the limit $m_e = m_u = m_d = 0$.

In Ref. 1) a reliable value for m_t could not be deduced from (2) because m_s , m_c , and m_b were not known with sufficient accuracy at that time. In Ref. 4), $m_t(m_t)$ was calculated in the $\overline{MS}$ scheme using the then available knowledge about quark masses⁵⁾:

$$130 \text{ MeV} < m_s(1 \text{ GeV}) < 230 \text{ MeV} \quad (3)$$

$$m_c(1 \text{ GeV}) = 1.35 \pm 0.05 \text{ GeV} \quad (4)$$

$$m_b(m_b) = 4.25 \pm 0.10 \text{ GeV} \quad (5)$$

Although Eqs. (4) and (5) give fairly accurate values for m_c and m_b , Eq. (3) gives only a broad range for m_s . Therefore only upper and lower bounds for $m_t(m_t)$ could be deduced.

Recently Reinders and Rubinstein⁶⁾ analysed QCD sum rules taking the gluon condensate from heavy quark systems. Saturating these sum rules with the K^* and ϕ mesons they succeeded in deriving a much more accurate value for m_s :

$$m_s (1 \text{ GeV}) = 110 \pm 10 \text{ MeV} \quad (6)$$

In this note we use this new value for m_s to calculate $m_t(m_t)$ in the $\overline{\text{MS}}$ scheme. In this scheme the dependence of the running mass on the renormalization mass μ is given by

$$m_q(\mu) = \frac{\hat{m}_q}{\left(\ln \frac{\mu}{\Lambda} \right)^{4/\beta_0}} \quad (7)$$

where Λ is the QCD scale parameter, $\hat{m}_q$ is a constant independent of μ , and $\beta_0 = 11 - 2/3 n_f$ (n_f = number of quarks). It follows immediately from Eq. (7) that the ratio of two running masses is independent of μ . Therefore Eq. (2) makes sense without any specification of μ . If one defines $m_{q0} \equiv m_q(m_q)$ as the solution of the equation

$$m_q(m_{q0}) = m_{q0} \quad (8)$$

then one has from Eq. (2)

$$m_t(m_{b0}) = \frac{m_c}{m_s} m_{b0} \quad (9)$$

Eqs. (7) and (9) imply

$$m_t(\mu) = \left\{ 1 + \frac{\ln \frac{m_{b0}}{\mu}}{\ln \frac{\mu}{\Lambda}} \right\}^{4/\beta_0} \frac{m_c}{m_s} m_{b0} \quad (10)$$

In order to calculate m_{t0} from Eq. (10) we have to choose numerical values for Λ and β_0 . Bernreuther and Wetzel⁷⁾ have calculated the effective value of Λ which has to be used in the energy range between m_{b0} and m_{t0} if the influence of quarks heavier than the b quark is neglected. They give

$$\Lambda = 64^{+25}_{-17} \text{ MeV} \quad (11)$$

for $n_f = 5$. For $\mu \sim 40$ GeV, the right hand side of Eq. (10) does not depend very strongly on Λ in this range. For example, for $\mu = 40$ GeV, $m_{b0} = 4.25$ GeV, $m_c(1 \text{ GeV}) = 1.35$ GeV, and $m_s = 110$ MeV, it has the value 41.2 GeV for $\Lambda = (64+25)$ MeV and 42.3 GeV for $\Lambda = (64-17)$ GeV. In the following, we take $\Lambda = 64$ MeV.

If we take for $m_c(1 \text{ GeV})$ and m_{b0} the central values, i.e., $m_c(1 \text{ GeV}) = 1.35$ GeV and $m_{b0} = 4.25$ GeV, then m_{t0} is a decreasing function of $m_s(1 \text{ GeV})$. We find

$$38 \text{ GeV} < m_{t0} < 45.5 \text{ GeV} \quad (12)$$

for $120 \text{ MeV} > m_s(1 \text{ GeV}) > 100 \text{ MeV}$. For the central value of m_s , i.e., for $m_s(1 \text{ GeV}) = 110$ MeV, we obtain

$$m_{t0} = 41.6 \text{ GeV} \quad (13)$$

The top quark could be produced at the $p\bar{p}$ collider at CERN as a decay product of the W:

$$p\bar{p} \rightarrow W^\pm + X, \quad W^\pm \rightarrow b^\pm t^\pm, \quad t^\pm \rightarrow b^\mp \ell^\pm \nu \quad (14)$$

Such events would have the following signature: (a) Two high p_T jets, a high p_T charged lepton, and missing p_T . (b) The total invariant mass of these decay products equals the mass of the W. (c) Since the top quark must be quite massive (> 22 GeV), it would be rather slow. This would entail that the decay products are widely separated in space. (d) The two jets should be different, namely the one originating from the b quark which is directly produced in the W decay should be narrower, of higher energy, and the angle with the charged lepton should be bigger. (e) If several such events are observed, then the invariant mass of the not so narrow jet, the charged lepton, and the missing p_T should peak at some value > 22 GeV.

The UA1 collaboration, although they do not claim the identification of the top quark, have reported⁸⁾ the finding of six events, three with a muon and three with an electron, which seem to fulfill the above requirements. The invariant

masses (in GeV) of the not so narrow jet, the charged lepton, and the missing p_T are

$$\begin{aligned} \text{Muon events:} \quad & 41^{+11}_{-6}, \quad 41 \pm 9, \quad 38 \pm 5 \\ & (15) \\ \text{Electron events:} \quad & 40 \pm 4, \quad 42 \pm 5, \quad 48 \pm 7 \end{aligned}$$

The mean value of the central values is

$$\bar{m}_{inv.} = 41.7 \text{ GeV} \quad (16)$$

Even neglecting errors, the agreement between $\bar{m}_{inv}$ and m_{to} is too good to be true. First of all, the invariant masses given in (15) do not include the longitudinal momentum belonging to the missing p_T . So the real invariant mass should be somewhat higher than $\bar{m}_{inv}$. The difference should not be more than 2-3 GeV because the invariant mass of all the W decay products is also determined without the same longitudinal momentum and one finds (in GeV):

$$78^{+10}_{-7}, \quad 78^{+5}_{-4}, \quad 79 \pm 10, \quad 61 \pm 5, \quad 82 \pm 7, \quad 76 \pm 7 \quad (17)$$

Except for one value, these invariant masses differ from the W mass by 2-3 GeV. Secondly, the relation between m_{to} and the invariant mass of the decay products of a top quark is not quite clear. The difference, however, is expected to be of $O(\alpha_s)$ and hence may be small. With these reservations in mind we would like to conclude that the agreement between the range for m_{to} given in Eq. (12) and the measured values given in (15) is quite good.

Understanding fermion and in particular quark masses is one of the outstanding problems of particle physics. If Eq. (2) should turn out to be approximately valid, then it would indicate that quark masses are related in a very simple way. This may eventually lead to understanding their origin.

ACKNOWLEDGEMENTS

I am grateful to C.-E. Wulz and H. Rubinstein for discussions, to D.V. Nanopoulos for reading the manuscript, and to the CERN Theory Division for hospitality.

REFERENCES

- 1) D. Grosser, Phys. Lett. B83 (1979) 355.
- 2) S.L. Glashow and S. Weinberg, Phys. Rev. D15 (1977) 1958.
- 3) S. Pakvasa and H. Sugawara, Phys. Lett. B82 (1979) 105.
- 4) D. Grosser, Phys. Lett. B136 (1984) 437.
- 5) J. Gasser and H. Leutwyler, Phys. Rep. 87 (1982) 77, and references cited therein.
- 6) L.J. Reinders and H.R. Rubinstein, Phys. Lett. B145 (1984) 108.
- 7) W. Bernreuther and W. Wetzel, Nucl. Phys. B197 (1982) 228.
- 8) C. Rubbia, talk at Neutrino 84 conference, Dortmund (June 1984);
D. DiBitonto, talk at VIIth European Symposium on Antiproton Interactions,
Durham (July 1984).