Contents

1	Introduction	1
1.1	Kicked oscillators	1
1.2	Poincaré sections	2
1.3	Crystalline symmetry	3
1.4	Stochastic webs	5
1.5	Normal and anomalous diffusive behavior	7
1.6	The sawtooth web map	9
1.7	Renormalizability	10
1.8	Long-time asymptotics	12
1.9	Linking local and global behavior	13
1.10	Organization of the book	13
	References	15
2	Renormalizability of the Local Map	17
2.1	Heuristic approach to renormalizability	18
2.1.1	Generalized rotations	18
2.1.2	Natural return map tree	19
2.1.3	Examples	21
2.2	Quadratic piecewise isometries	26
2.2.1	Arithmetic preliminaries	26
2.2.2	Domains	28
2.2.3	Geometric transformations on domains	30
2.2.4	Scaling sequences	31
2.2.5	Periodic orbits	33
2.2.6	Recursive tiling	33
2.2.7	Computer-assisted proofs	36
2.3	Three quadratic models	40
2.3.1	Model I	41
2.3.2	Model II	43
2.3.3	Model III	46

2.4	Proof of renormalizability	48
2.5	Structure of the discontinuity set	49
2.5.1	Model I	49
2.5.2	Model III	50
2.6	More general renormalization	51
2.7	The $\pi/7$ model	53
	References	61
3	Symbolic Dynamics	63
3.1	Symbolic representation of the residual set	63
3.1.1	Hierarchical symbol strings	63
3.1.2	Eventually periodic codes	65
3.1.3	Simplified codes for quadratic models	66
3.2	Dynamical updating of codes	66
3.3	Admissibility	68
3.3.1	Quadratic example	70
3.3.2	Models I, II, and III	73
3.3.3	Cubic example	75
3.4	Minimality	78
	References	78
4	Dimensions and Measures	79
4.1	Hausdorff dimension and Hausdorff measure	79
4.2	Construction of the measure	80
4.3	Simplification for quadratic irrational λ	81
4.4	A complicated example: Model II	83
4.5	Discontinuity set in Model III	84
4.6	Multifractal residual set of the $\pi/7$ model	85
4.7	Asymptotic factorization	86
4.8	Telescoping	91
4.9	Unique ergodicity for each $\Sigma(\mathbf{i})$	92
4.10	Multifractal spectrum of recurrence time dimensions	94
4.10.1	Auxiliary measures and dimensions	95
4.10.2	Simpler calculation of the recurrence time dimensions	100
4.10.3	Recurrence time spectrum for the $\pi/7$ model	102
	References	105
5	Global Dynamics	107
5.1	Global expansivity	107
5.1.1	Lifting the return map $\rho_K(0)$	107
5.1.2	Lifting the higher-level return maps	109
5.2	Long-time asymptotics	111
5.3	Quadratic examples	114
5.4	Cubic examples	119
5.4.1	Orbits in the $(0, k, 6^\infty)$ sectors	119

5.4.2	Numerical investigations	122
5.4.3	A non-expansive sector	125
5.4.4	Generic behavior	127
	References	129
6	Transport	131
6.1	Probability calculation using recursive tiling	132
6.2	Ballistic transport in Model I	134
6.3	Subdiffusive transport in Model II	138
6.4	Diffusive transport in Model II	142
6.5	Superdiffusive transport in Model III	143
6.6	Discussion	149
	References	150
7	Hamiltonian Round-Off	151
7.1	Vector field	151
7.2	Localization	154
7.3	Localization of the vector field and periodic orbits	155
7.4	Symbolic codes for walks	158
7.5	Construction of the probability distribution	159
7.6	Rotation number $1/5$	161
7.6.1	Recursive tiling for the local map	161
7.6.2	Probability distribution $P(\mathbf{x}, t)$	163
7.6.3	Fractal snowflakes	165
7.6.4	Substitution rules for lattice walks	168
7.6.5	Separating out an asymptotic walk	170
7.6.6	Asymptotic scaling	173
7.7	Model I	175
7.8	Model II	177
7.9	A conjecture	184
	References	184
Appendix A	Data Tables	185
A.1	Model I Data Tables, from Kouptsov et al. (2002).	185
A.1.1	Generating domain	185
A.1.2	Level-0 scaling sequence domains	185
A.1.3	Level-0 periodic domains	186
A.1.4	Miscellaneous periodic domains	186
A.2	Model II Data Tables, from Kouptsov et al. (2002)	186
A.2.1	Generating partition	186
A.2.2	Level-0 scaling domains, sequence A	187
A.2.3	Level-0 periodic domains, sequence A	188
A.2.4	Miscellaneous periodic domains, $j > 10$	188
A.2.5	Level-0 scaling domains, sequence B	189
A.2.6	Level-0 periodic domains, sequence B	189

A.2.7	Incidence matrices	190
A.3	Model III Data Tables, from Kouptsov et al. (2002)	191
A.3.1	Generating domain	191
A.3.2	Pre scaling level $L = -1$	191
A.3.3	Domains $D_j(L)$ for even L	192
A.3.4	Domains $D_j(L)$ for odd L	192
A.3.5	Domains $\Pi_j(L)$ for all L	192
A.3.6	Tiling data	193
A.3.7	Section of the discontinuity set	193
A.4	Cubic Model Data Tables, from Lowenstein et al. (2004)	194
A.5	Inadmissibility Tables for Models II and III	200
	References	200
Appendix B	The Codometer	201
Index		205
Color Figure Index		209