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CHERENKOV RING SEARCH USING A MAXIMUM LIKELIHOOD TECHNIQUE

P. Baillon

CERN, Geneva, Switzerland

ABSTRACT

With the realization of large two dimensional ultra-violet photon detectors of several squared metres, full acceptance particle identifiers became possible. They use the position of the detected cherenkov photons to measure the velocity of the particles and thus to identify their mass when their momentum is known. Those photons are generally few (between 10 and 20) and mixed with numerous photons coming from other sources. A technique of maximum likelihood was developed to do this identification. It appears to be feasible and powerful.

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1. INTRODUCTION

In the DELPHI detector to be built for the study of e^+e^- interactions in the large electron-positron ring (LEP) at CERN, a hadron identifier (RICH) using the Cherenkov radiation detected by a large area UV photon detector is foreseen [1].

The photon detector uses a photon sensitive molecule (TMAE) mixed with the normal drift chamber gas. Any photon above 5.5 eV (2000 Å) penetrating this gas is absorbed within 1.2 cm and has a good chance (10-40%) of liberating a free electron which then can be detected by the conventional technique of a long drift chamber (TPC) [2-4].

This hadron identifier (fig. 1) consists of 2×24 cells disposed as a barrel round the e^+e^- intersection, the axis being the e^+e^- axis. Each cell consists of a thin liquid radiator (1 cm) 170 cm long and 20 cm wide. A highly pure quartz window allows the UV Cherenkov radiation to escape from the radiator. The radiator is freon FC72 which is also UV transparent and has a refractive index around 1.22. A photon detector of about the same size and parallel to it is located at 12 cm larger radius. The thickness of the detector is 6 cm. Each photon is converted into an electron or absorbed in the 1.2 first cm of the detector. The efficiency of the conversion could reach 40% for UV radiation around 1700 Å. Both faces of the detector are of quartz and transparent to UV light. This allows a second Cherenkov counter to be incorporated with a gas radiator filled with freon FC87 which has an index round 1.0022. The UV light of the Cherenkov radiation from this second counter is reflected by parabolic mirrors onto the photon detectors. In such a detector, a high energy particle will emit Cherenkov photons from every transparent medium: Liquid (FC72), gas (FC87) or solid (quartz). Those photons are emitted with a well-defined angle with respect to the direction of the radiating particle. In the case of the liquid radiator, because of its small thickness, the photons are generated in a thin cone. This cone intercepts the detector along a curve which is of an elliptic, parabolic or hyperbolic shape. In the case of the gas radiator, the focussing mirrors achieve the same effect by concentrating the photons on a circle round the focus. The geometry of the various curves depends on the direction of the charged particle and its velocity, which determines the emission angle of the photons. Hence from the detected photon distribution, the velocity of the particle can be

deduced. Since its momentum is measured in other parts of the detector, its mass and flavour (π , K, P or even e and μ) can then be determined ([2] for more details).

The number of photons detected is small: between 10 and 20 and these are mixed with many photons coming from other sources and from other particles so an efficient algorithm is needed.

2. MAXIMUM LIKELIHOOD TECHNIQUE

The technique developed is similar to the maximum likelihood technique used extensively in high energy physics: it fits a set of parameters coming from a theory on a given set of events. The technique was extended because not only the relative distribution of the Cherenkov angle is known, but also the average number of photons. The unknown quantity is the shape and magnitude of the unassociated photon distribution. We may nevertheless assume that its distribution is flat on the detector surface and try to estimate the overall level by a maximum likelihood method. To ease the writing of formulae, we will consider here instead a flat background in a plane perpendicular to the radiating particle before any reflection and refraction. In the real program a flat background on the detector plane was used. Due to the focussing of the mirrors or to the small thickness of the liquid radiator, the position of the photon on the detector depends only slightly on its emission point. Thus we may assume the origin of the photon to be in the middle of the radiator liquid or gas. Since between two points there is only one light ray of a given wavelength joining them, the direction of the photon can be reconstructed at the assumed emission point for an average wavelength. The value of the Cherenkov angle with its error for any measured photon can then be obtained. The error on this angle is dependent on the measurement errors, the depth of the radiator and the variation of the indices of refraction with wave-length.

In the search of the associated photons, any photon inside a $\theta_{\min}$ to $\theta_{\max}$ range, which is the range of possible Cherenkov angles for p, K, π , μ , e within three standard deviations, is retained. We get a set of angles:

$$\theta_1 \dots \theta_i \dots \theta_n \quad \text{with} \quad \theta_{\min} < \theta_i < \theta_{\max}$$

For each mass hypothesis, the Cherenkov angle θ_c^h and the average number of photo-electrons is known. For any photon angle θ_i the probability density can be written;

$$P_i^h = \frac{N^h \cdot e^{-(\theta_i - \theta_c^h)^2 / (2\sigma_i^2)}}{\sqrt{2\pi}\sigma_i} + K^h \cdot \theta / \int_{\min}^{\max} \theta d\theta \quad (1)$$

$$\text{with } \int P_i^h(\theta) d\theta = N^h + K^h \quad (2)$$

The index i is for the number of a detected photon, θ_c^h is the calculated Cherenkov angle for each mass hypothesis, θ_i , σ_i are the measured Cherenkov angles and their errors, h is the index relative to the mass hypothesis, N^h is the number of expected photons evaluated by the program, K^h is the number of background photons (to be fitted) between $\theta_{\min}$ and $\theta_{\max}$. The density of that background is taken to be proportional to the angle. The formula (1) also allows the errors of the measurement on the radiating particle to be included: P_i^h may be averaged over the momentum error distribution which affects θ_c^h and over the angular distribution of the particle which affects θ_i .

The probability for one hypothesis h to produce the set of angles $\theta_1 \dots \theta_n$, can be broken down as follows:

- (a) The probability (a) to see a photon in each location $\theta_1 \dots \theta_n$ multiplied by a vanishing interval $d\theta_i$

$$Pr_a^h = \prod_i^n P^h(\theta_i) d\theta_i \quad (3)$$

- (b) Secondly the probability (b) of not seeing any photon elsewhere. This probability is independent of the location and the number of the photons seen since we can make the surrounding $d\theta_i$ as small as required:

$$\begin{aligned} Pr_b^h &= \prod [1 - P^h(\theta) \cdot d\theta] \\ \text{Log}(Pr_b^h) &= \sum \log(1 - P^h(\theta) \cdot d\theta) = -\sum P^h(r) \cdot d\theta \\ &= -\int P^h(\theta) \cdot d\theta = -N^h - K^h \end{aligned}$$

and

$$Pr_b^h = e^{-N^h - K^h} \quad (4)$$

Equ. (3) can be transformed into a density of probability by transforming the term $d\theta_1 \dots d\theta_n$ to an element of integration but in that case the labelling 1 ... n does not mean any order and the same term will occur $n!$ times so that the probability density should be written for (a) as

$$\Pr_a^h = 1./n! \prod_i^n P^h(\theta_i)$$

By multiplying the probability density (a) by the probability (b) we get the extended probability density for each hypothesis:

$$\Pr^h = 1./n! \{ \prod_i^n [P^h(\theta_i)] \} e^{-N^h - K^h} \quad (5)$$

By integrating (5) using (2), we get the Poisson distribution for n photons when $N^h + K^h$ is expected:

$$[\Pr^h d\theta_1 \dots d\theta_n] = 1/n! (N^h + K^h)^n e^{-N^h - K^h}$$

The technique of maximum likelihood can be used:

- (a) To maximise $\Pr^h$ for each mass hypothesis when K^h is left free (fit of K^h).
- (b) To give a relative probability between two possible hypotheses h_1, h_2 :

$$\Pr_{12} = \Pr^{h_1} / \Pr^{h_2}$$

This procedure allows the best hypothesis to be chosen.

In principle using (5) one can get also a confidence level for each hypothesis that is to say the probability of finding the probability density associated with $\theta_1 \dots \theta_n$ given by (5) smaller than the value found. In our case, this test is more a test of the validity of the uniform background hypothesis. Hence it is not really relevant.

3. RESULTS

The result of the method applied to the RICH detector of DELPHI may be seen in detail in ref. [2]. Fig. 2 extracted from [2] shows the separation obtained. The conditions were as realistic as possible:

all secondary interactions were introduced as well as the expected noise of the detector. The events came from Monte-Carlo generated Z^0 decays into two b quarks. Fig. 3 displays the analysis for two particles inside one of the two b jets. In fig 4, the improvement one may obtain by using a more adequate parametrisation of the unassociated photon distribution is shown: The top of fig 4 deals with a flat distribution for the background in a plane perpendicular to the particle and before making any refraction. A lot of unassociated photons can be seen to be localised near the total reflection limits. All those photons are detected far away from the particles. They generate a direction at grazing angle in the drift space following the radiator. This angle, after refraction, leads to an angle close to the total reflection angle in the liquid. The bottom of fig. 4 gives the result when the unassociated photon distribution was parametrised in the plane of the detector and the far away photons eliminated by a cut. In that case the behaviour near the internal reflection is well represented and the cut suppresses most of the unassociated photons and leaves untouched nearby all the associated photons.

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REFERENCES

- [1] DELPHI Technical Proposal, CERN/LEPC/83-3 LEPC/P 2, and DELPHI 83-66/1, 17 May 1983.
- [2] DELPHI Progress Report, CERN/LEPC 84-16, LEPC/PR 6, and DELPHI 84-60 GEN-11, 26 September 1984.
- [3] E. Barrelet et al., Nucl. Instr. & Meth. 200 (1982) 219-238.
- [4] L.O. Eek et al., CERN/EP 84-25.

FIGURE CAPTIONS

Fig. 1 DELPHI RICH cross section.

Fig. 2 Particle identification in the Barrel RICH (preliminary):

- (a) For pions: β is the percentage of correct answers for isolated pions and γ is for pions in $b\bar{b}$ jets. The jets were generated at $\theta = 75^\circ$ (angle to the beam axis). δ gives the probability of calling a pion something else, most of the time a K at high momentum. α indicates what was quoted in ref. [1] without full calculation.
- (b) For kaons: α and β give the percentage of correct answers for K in $b\bar{b}$ jets generated at 50 and 75° respectively; γ is a mean value, weighted by the solid angle; δ is the probability that a kaon is identified as a kaon or a proton.
- (c) For protons: ambiguous exclusively with K in the dip region.

Fig. 3 The pictures on the left hand side represent, for two generated tracks in a jet coming from $e^+e^- \rightarrow Z^0 \rightarrow b\bar{b}$, the reconstructed angles of the detected photon with respect to the particle in a polar representation. The distance from the origin is the sine of the Cherenkov angle, while the polar angle is the azimuth of the radiated photon. In such a representation the Cherenkov cone gives a circle. The two full circles represent the limits of accepted photons. The central circle corresponds to the generated hypothesis. The dotted curve (ellipse) gives the total reflection limit in the case of the liquid radiator. The points centred on a small ring are the genuine associated photons; triangle or squared points are the unassociated photons. The pictures at the right hand side represent the distribution of the measured Cherenkov angle with the origin at the angle predicted for an electron. P_L and P_T refer to the longitudinal and transverse momentum of the particle relative to the jet axis. The various curves are the expected distributions for π , K, p with their fitted background. The top figure (K 18 GeV/c gas Cherenkov) is an example where only the correct hypothesis gives a good probability $P(K) > 10^5 (P(\pi) + P(p))$ with 6 unassociated photons. In the bottom figure (K-6 GeV liquid Cherenkov) $P(K) \approx P(\pi)$ but p is excluded with $P(p) = 0.05(P(K) + P(\pi))$. The fitted background is 26 photons for the K hypothesis.

FIGURE CAPTIONS (Cont'd)

Fig. 4 Events in the liquid RICH with high background. The meaning of the curves is given in the captions to fig. 3. In the top drawings, one sees that the background is concentrated in the vicinity of total reflection since near this condition a small variation in the angle corresponds to a large change in the region explored in the detector. The background intercepted by the Cherenkov ring is therefore concentrated in two azimuthal regions which must be removed in the analysis as shown in the bottom drawings. In addition the background was fitted as uniform over the detector surface.

Fig. 2

Fig. 3

Fig. 4