A method to determine the electro-weak mixing angle from the γ -distributions of semi-leptonic neutrino scattering.

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Abstract: A method to determine $\sin^2\theta$ from the combination of neutral-current and charged-current γ -distributions of semi-leptonic neutrino and antineutrino scattering is discussed. The dependence of $\sin^2\theta$ on quark model parameters can be reduced by this method. In particular, uncertainties introduced by the strange quark component of the nucleon can be removed.

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In the Glashow-Salam-Weinberg model of electro-weak interactions the coupling in all neutral-current processes depends on one single parameter, $\sin^2\theta$. This prediction is supported by a large amount of experimental data [1]. In the next to leading order corrections different processes are subject to different correction terms. Hence sufficiently precise measurements of $\sin^2\theta$, extracted from the data using simple zero-order approximations, should show differences for different reactions. A comparison of such measurements obtained in different reactions would provide a test of the ability of the theory to evaluate these corrections.

Measurements of semi-leptonic neutrino interactions have been an important tool to study the electro-weak interaction and to deduce the value of the electro-weak mixing angle. Precise measurements of the neutral-current (NC) to charged-current (CC) total cross-section ratio of neutrinos, R , can provide a sensitive measurement of $\sin^2\theta$. As the measurements of R improve, it is important to realise that the determination of $\sin^2\theta$ extracted from this semi-leptonic interaction is limited in precision to a significant level by the details of the quark-structure of the nucleon.

It was shown by Llewellyn-Smith that for isoscalar targets the major contributions to the NC and CC cross-sections are related by assuming isospin invariance alone [2]. The following equation for the differential cross-sections was derived:

$$\begin{aligned} d^2\sigma_{NC}^{v(\bar{v})}/dxdy &= (1/2 - \sin^2\theta + 5/9\sin^4\theta) d^2\sigma_{CC}^{v(\bar{v})}/dxdy \\ &+ (5/9\sin^4\theta) d^2\sigma_{CC}^{\bar{v}(v)}/dxdy \end{aligned} \quad (1)$$

This equation applies for nucleons containing only u and d quarks and for a zero Cabibbo angle. It was shown that the uncertainties introduced by higher twist effects are small, $\Delta\sin^2\theta < \pm 0.005$. Corrections to this simplified picture involve the contributions of strange and charm quarks and the quark mixing as described by the Kobayashi-Maskawa (K.M.) mixing matrix, U_{ij} . The largest of these so-called "theoretical uncertainties" are introduced by terms describing the strange quark contributions to the NC and CC cross-sections. These terms are:

1. In the CC sector a contribution proportional to $2s(|U_{us}|^2 + \xi_{sc}|U_{cs}|^2)$, where s is the momentum weighted strange quark component of the nucleon, and ξ_{sc} is a threshold function describing charm production.

2. In the NC sector a contribution proportional to $2s \cdot g_s^2$

where g_s^2 is the sum of the left-handed and right-handed coupling of the weak neutral-current to the s-quark.

The main problem arises from our ignorance of the element $|U_{cs}|$ in the K.M. matrix. Owing to these uncertainties the information obtained from neutrino induced prompt di-muon production does not determine the absolute value of the strange quark component of the nucleon with sufficient precision. The total "theoretical uncertainty" amounts presently to $\Delta \sin^2 \theta$ (theoretical) = ± 0.01 . These arguments were worked out in detail by Llewellyn-Smith [2]. This evaluation was based on values of the K.M. mixing matrix elements compiled by Pakvasa [3], and is not changed noticeably if one uses the values of the matrix elements obtained by Kleinknecht and Renk [4]. However, if further assumptions are made, such as:

- the off-diagonal terms of the K.M. matrix are small [2];
- the total number of quark flavours of any mass equals six [4];

then the absolute value of the momentum weighted strange quark component of the nucleon is much better constrained by the data. In such cases, one derives $\Delta \sin^2 \theta$ (theoretical, constrained) $< \pm 0.004$.

However, it can be argued that in measurements of fundamental constants such as $\sin^2 \theta$ these assumptions should not be made.

In the following discussion a method is described by which the absolute normalization of the momentum weighted strange quark component is not needed as input for the determination of $\sin^2 \theta$. The new method relies on comparing the NC and CC differential cross-sections following eq.(1), integrated over x . The uncertainties proportional to s can be removed by exploiting the additional information provided by a simultaneous measurement of the NC and CC y -distributions of neutrino and antineutrino scattering. The normalization of the ill-known terms can be introduced as free parameters. Corrections for the s-quark contributions to the NC cross-sections are treated by the substitution:

$$d\sigma_{NC}/dy \rightarrow d\sigma_{NC}/dy - (g_s^2 \cdot s) \cdot f(y) \quad (2)$$

where $(g_s^2 \cdot s)$ is a free parameter and $f(y)$ is obtained using a specific model, e.g. in the scaling limit

$$f(y) \rightarrow 2(1+(1-y)^2)$$

(3)

The free parameter ($g_s^2 \cdot s$) is interpreted as being the product of the coupling of the weak neutral-current to the strange quark, g_s^2 , and the strange quark content of the nucleon, s . This model can be improved by using QCD-type parametrizations. In the charged-current sector a similar correction is made, based on di-muon measurements:

$$d\sigma_{CC}/dy \rightarrow d\sigma_{CC}/dy - B^{-1} d\sigma_{di-\mu}/dy$$

(4)

in which B (the average branching ratio to muons of charm states produced in this process) is a free parameter. This procedure takes care of threshold effects and removes any need for a priori knowledge on the product of the element $|U_{cs}|^2$ and the absolute strange quark component in the nucleon. It is implicitly assumed that the average branching ratio of the mixture of charmed final states is independent of y , and the same for neutrino and antineutrino interactions. These assumptions can be tested by experiments, where charm production is recognized independently of the presence of leptonic decays, e.g. by tagging decays close to the interaction vertex.

Thus integrating equation (1) over x one obtains:

$$\begin{aligned} d\sigma_{NC}^{v(\bar{v})}/dy - (g_s^2 \cdot s) \cdot f(y) &= (1/2 - \sin^2\theta + 5/9 \sin^4\theta) \cdot (d\sigma_{CC}^{v(\bar{v})}/dy \\ &- B^{-1} d\sigma_{di-\mu}^{v(\bar{v})}/dy) + (5/9 \sin^4\theta) \cdot (d\sigma_{CC}^{\bar{v}(v)}/dy - B^{-1} d\sigma_{di-\mu}^{\bar{v}(v)}/dy) \end{aligned} \quad (5)$$

The remaining corrections to be applied to this equation involve the knowledge of the mixing elements $|U_{ud}|$, $|U_{us}|$, $|U_{cd}|$, and the contribution of charm quarks to the NC and CC cross-sections. According to Llewellyn-Smith [2] these corrections are calculable to within an uncertainty of $\Delta \sin^2\theta = \pm 0.004$. No assumptions on the shape of the parton distributions other than eq. (3) or its equivalent have to be made.

The statistical accuracy which can be obtained with this method was estimated using quark-parton model cross-sections with $(\bar{u}+\bar{d})/(u+d) = 0.1$, $2s/(u+d) = 0.05$, $B = 0.1$, and $\sin^2\theta = 0.22$. The statistical accuracies in the parameters $\sin^2\theta$ and $g_s^2 \cdot s$ which can be achieved in experiments with different statistics are given in Table (1). The errors for other event statistics can be

obtained by simply scaling with $\sqrt{N}$. In this analysis we assumed that experimental resolution effects can be neglected. Realistic resolution functions of a typical calorimeter exposed to a narrow-band beam raise these errors by less than 10%. The statistical uncertainty of $\sin^2\theta$, if derived from the NC/CC cross-section ratio in neutrino scattering, varies between 0.0023 and 0.0021 for the event numbers given in the table. In the latter case the antineutrino statistics contributes only through a small correction term proportional to the CC cross-section ratio of antineutrino and neutrino scattering.

Table 1: Statistical errors on the determination of $\sin^2\theta$.

ν CC events	$\bar{\nu}$ CC events	$\Delta(\sin^2\theta)$	$\Delta(g_s^2 \cdot s)/(g_s^2 \cdot s)$
100.000	1.000	± 0.015	± 0.53
100.000	10.000	± 0.006	± 0.46
100.000	100.000	± 0.004	± 0.30

This method improves on previous determinations of $\sin^2\theta$ from the differential cross-sections $d\sigma/dy$ because it uses the quark-parton model in correction terms only and, therefore, scaling violations and threshold effects are taken into account intrinsically. The influence of higher twist effects can be shown to be small ($\Delta\sin^2\theta < \pm 0.005$ [2]).

To conclude, it is shown that a measurement of the NC and CC y -distributions provides a way to avoid the largest theoretical uncertainties present in the determination of $\sin^2\theta$ from the ratio of the total cross-sections of neutral-current and charged-current neutrino interactions, R . Although the statistical accuracy is somewhat worsened, the reliability of the value of $\sin^2\theta$ can be improved.

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References

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