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IN A UNIFORM FIELD IN A HALF SPACE

E.A.Akhundova, V.V.Dodonov, V.I.Man'ko

High Energies Electrons Laboratory

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THE QUASI-CLASSICAL PROPAGATOR OF A QUANTUM PARTICLE IN A UNIFORM FIELD IN A HALF SPACE

R.A.Akhundova, V.V.Dodenev, V.I.Man'ko

ABSTRACT

The quasi-classical formulas for the propagator of the Schrödinger equation and for the equilibrium density matrix are obtained for a quantum particle moving in a uniform force field in a half space over an ideal reflecting wall.

We consider the motion of a quantum particle in a uniform field with the potential $U(x) = -Fx$ in the half space $x > 0$, bounded by an impervious ideal reflecting wall. Solutions of the stationary Schrödinger equation $(\hat{H} - E)\psi = 0$ for this problem are given, for example, in (Flügge 1971) Green's functions of this equation are also known (Lukes and Somaratna 1969, Moyer 1973, Tachibana et al 1977). As to the propagator of the time-dependent Schrödinger equation, it is known in the case of the motion in the unbounded region $-\infty < x < \infty$ only (Kennard 1927):

$$G_{1/2}(x_2, x_1, t) = \left(\frac{m}{2\pi\hbar i t} \right)^{1/2} \exp \left\{ \frac{i}{\hbar} \left[\frac{m(x_2 - x_1)^2}{2t} + \frac{Ft(x_2 + x_1)}{2} - \frac{F^2 t^3}{24m} \right] \right\} \quad (1)$$

In the presence of a wall the only result available was an integral representation by the Airy functions derived by Moyer (1973). In the present paper quasi-classical formulas for the propagator and for the equilibrium density matrix are obtained.

In the quasi-classical approach the propagator has the form

$$G_f(x, y, t) = \exp \left\{ \frac{i}{\hbar} S(x, y, t) + iX(x, y, t) + i\hbar \varphi(x, y, t) + \dots \right\} \quad (2)$$

Introducing this expression to the Schrödinger equation (further $G_x \equiv \partial G / \partial x$, etc.)

$$i\hbar G_t = -\frac{\hbar^2}{2m} G_{xx} - Fx G_y = -\frac{\hbar^2}{2m} G_{yy} - Fy G_x \quad (3)$$

we obtain in the first approximation the classical Hamilton-Jacobi equation

$$S_t + \frac{1}{2m} S_x^2 - Fx = 0, \quad S_t + \frac{1}{2m} S_y^2 - Fy = 0 \quad (4)$$

The function $X(x, y, t)$ must satisfy the equations

$$X_t + \frac{1}{m} X_x S_x - \frac{i}{2m} S_{xx} = 0, \quad X_t + \frac{1}{m} X_y S_y - \frac{i}{2m} S_{yy} = 0 \quad (5)$$

It is not difficult to verify that the solution of these equations is the following function:

$$X(x, y, t) = -\frac{i}{2} \ln S_{xy} + \text{const} \quad (6)$$

The constant of integration can be found from the condition:

$$G(x, y, t) \rightarrow \delta(x - y) \quad \text{when} \quad t \rightarrow 0.$$

Thus we obtain the well-known Van Vleck formula (Van Vleck 1928, Berry and Mount 1972)

$$G(x, y, t) = \left[\frac{|S_{xy}|}{2\pi i \hbar} \right]^{1/2} \exp \left\{ \frac{i}{\hbar} S(x, y, t) \right\} \quad (7)$$

Taking into account eq. (6) we find the following equations for the next term of the propagator's phase expansion in powers of Planck's constant

$$\dot{\varphi} + \frac{1}{m} S_x \varphi_x + \frac{1}{8m} \left(\frac{S_{xxx}^2}{S_{xy}^2} - 2 \frac{S_{xyxx}}{S_{xy}^2} \right) = 0 \quad (8)$$

$$\dot{\varphi} + \frac{1}{m} S_y \varphi_y + \frac{1}{8m} \left(\frac{S_{yyy}^2}{S_{xy}^2} - 2 \frac{S_{xyyy}}{S_{xy}^2} \right) = 0$$

From these equations one can see that if the action $S(x, y, t)$ is a quadratic form of coordinates, then formula (7) is exact. This situation takes place for systems with quadratic Hamiltonians in the absence of a wall, because for such systems solutions of the classical motion equations are linear functions of the coordinates of the initial point x_1 and the final point x_2 . Consequently, the action

$$S(x_2, t_2; x_1, t_1) = \int_{(x_1, t_1)}^{(x_2, t_2)} L(x(\tau), \dot{x}(\tau)) d\tau \quad (9)$$

(L - Lagrange's function) quadratically depends on x_1 and x_2 . Formula (7) allows to obtain also the exact result for the problem of a free particle in the half space. In this case there are two classical trajectories connecting the initial and final points.

The first one is the shortest trajectory

$$x(\tau) = \frac{1}{t} \left[x_1(t-\tau) + \tau x_2 \right], \quad \dot{x}(\tau) = \frac{x_2 - x_1}{t}; \quad 0 \leq \tau \leq t$$

Calculating for it the action by means of formula (9), and introducing this action to eq. (7) we obtain the propagator of a free particle

$$G_0(x_1, x_1, t) = \left(\frac{m}{2\pi i \hbar t} \right)^{1/2} \exp \left\{ \frac{i m (x_1 - x_1)^2}{2 \hbar t} \right\} \quad (10)$$

The second solution of classical equations corresponds to the motion with the absolutely elastic reflection from a wall at the moment of time

$$x(\tau) = \begin{cases} \frac{1}{t} [x_1(t-\tau) - x_2\tau], & 0 \leq \tau \leq t_* \\ \frac{1}{t} [x_2\tau - x_1(t-\tau)], & t_* \leq \tau \leq t \end{cases}$$

In the plane (t, x) this trajectory is obtained by means of connecting the points $(0, x_1)$ and $(t, -x_1)$ with the segment of a line and the following mirror reflection from the axis $x=0$ of that part of the segment which lies in the region $x < 0$. Therefore formula (7) leads to the function which differs from G_0 only by the sign before the coordinate x_2 :

$$G'_0(x_1, x_2, t) = \left(\frac{m}{2\pi i \hbar t} \right)^{1/2} \exp \left\{ \frac{i m (x_1 + x_2)^2}{2 \hbar t} \right\} \quad (11)$$

Since wave functions turn into zero on a wall, then the propagator must satisfy the boundary condition

$$G(x_2, 0, t) = G(0, x_1, t) = 0 \quad (12)$$

Therefore the propagator is the difference of functions G_0 and G'_0 :

$$G_0^{(w)}(x_2, x_1, t) = \left(\frac{m}{2\pi i \hbar t} \right)^{1/2} \left[\exp \left\{ \frac{im(x_2 - x_1)^2}{2\hbar t} \right\} - \exp \left\{ \frac{im(x_2 + x_1)^2}{2\hbar t} \right\} \right] \quad (13)$$

This result is exact. Note that the boundary condition is not the only possible. For example, exact propagator for the family of boundary conditions $\psi(0) = \beta \psi'(0)$, $-\infty < \beta < \infty$ were constructed in (Clark et al 1980). We shall consider however the boundary condition $\psi(0) = 0$ only, bearing in mind the physical meaning of the problem under study as the problem of the motion of a particle in the half space bounded by the infinite high potential wall, from which a particle is reflected in an absolutely elastic manner. Besides, the condition $\psi(0) = 0$ appears naturally, if equation (3) is considered as the equation for the radial part of the S -state wave function of the particle moving under the action of the spherically-symmetric potential $V(r) = -F/r$ (this potential, as known, has some relation to the problem of two quarks system).

The substitution $t = -i\beta \hbar$, where $\beta = (kT)^{-1}$, T - absolute temperature, k - Boltzmann's constant, reduces the propagator to the equilibrium density matrix

$$\int_0^{(w)} G_0(x_2, x_1, \beta) = \left(\frac{m}{2\pi \beta \hbar^2} \right)^{1/2} \left[\exp \left\{ -\frac{m(x_2 - x_1)^2}{2\beta \hbar^2} \right\} - \exp \left\{ -\frac{m(x_2 + x_1)^2}{2\beta \hbar^2} \right\} \right] \quad (14)$$

This formula was obtained by another method, for example, in (Braun et al 1974). The propagators of a free particle moving in the sector $r \geq 0$, $0 < \varphi < \alpha$, bounded by two ideal reflecting walls were considered by Grandall (1983). It is naturally to expect that in the presence of uniform field the

propagator is also the difference of two terms, the first of which considers with the propagator without wall (1), while the second one is given by formula (7) with the action calculated along the trajectory corresponds to the motion with the reflection from the wall at some moment of time τ_* . For this trajectory the action, due to its additivity (see formula (9)) is equal to

$$\begin{aligned} S_1(x_1, x_2; t, 0) &= S_f(0, x_1; \tau_*, 0) + S_f(x_2, 0; t, \tau_*) = \\ &= \frac{m}{2} \left(\frac{x_1^2}{\tau_*} + \frac{x_2^2}{t - \tau_*} \right) + \frac{F}{2} \left[x_1 \tau_* + x_2 (t - \tau_*) \right] - \frac{F^2}{24m} \left[\tau_*^3 + (t - \tau_*)^3 \right] \end{aligned} \quad (15)$$

(x_1 - initial point, x_2 - final point, S_f - action for the trajectory without reflection from a wall: it is given by the expression in square brackets in formula (1)). We have to calculate the quantity τ_* . The solution of the equation of motion with one reflection from the wall is as follows,

$$x(\tau) = x_1 + b\tau + \frac{F\tau^2}{2m}, \quad 0 \leq \tau \leq \tau_*$$

$$x(\tau) = b'(\tau - \tau_*) + \frac{F}{2m} (\tau - \tau_*)^2, \quad \tau_* \leq \tau \leq t$$

Constants b , b' and τ_* are obtained from conditions of the absolute elasticity of a blow. As a result for the parameter τ_* one arrives at the cubic equation

$$\frac{F}{m} \tau_*^3 - \frac{3F}{2m} t \tau_*^2 - \tau_* \left(x_1 + x_2 - \frac{Ft^2}{2m} \right) + x_1 t = 0 \quad (16)$$

To understand qualitatively solution's character of this equa-

tion let us consider the case of $\mathcal{X}_1 = \mathcal{X}_2 = \mathcal{X}$. Then one solution is obvious: $\mathcal{T}_*^{(1)} = \frac{t}{2}$, and it is not difficult to find two others:

$$\mathcal{T}_*^{(2,3)} = \frac{1}{2} \left[t \pm \left(t^2 + \frac{8m\mathcal{X}}{F} \right)^{1/2} \right] \quad (17)$$

If $F > 0$ (repulsive force), then both solutions are outside the interval $(0, t)$, so that in this case there exists a unique physically acceptable solution. If $F < 0$ (attractive force, i.e. the motion is finite), then a situation is rather complicated, because three solutions are possible, i.e. there exist three different classical trajectories with a blow. Really, when $F < 0$ then there are trajectories with any number of blows on a wall, i.e. the quantity of classical trajectories is numerable. Therefore one can expect that quasi-classical expansion of the Green function has the form

$$G = \sum_{n=0}^{\infty} G_n \alpha_n \quad (18)$$

where every function G_n corresponds according to eq. (7) to certain classical trajectory, and α_n are some constant coefficients. The situation is simplified, if the quantity F is small. If the condition $|F|t^2/m\mathcal{X} \ll 1$ is satisfied, then for $F < 0$ both solutions (17) become complex, so that the trajectory with one blow is unique. Under the same conditions trajectories with two and more blows are impossible. Indeed, if during the time t , N blows on a wall take place then the moment $\mathcal{T}_*$ of the first blow can be found from the equation (this equation is obtained in the same manner as equation (16))

$$N(N+1) \frac{F^2}{m^2} \tau_*^4 - (2N+1) \frac{F^2}{m^2} t \tau_*^3 - \left[(2N-1) \frac{2F}{m} x_1 + \frac{2F}{m} x_2 \right. \\ \left. - \frac{F^2}{m^2} t^2 \right] \tau_*^2 + \frac{2F}{m} (2N-1) x_1 t \tau_* + 4N(N-1) x_1^2 = 0 \quad (19)$$

For $N=1$ this equation is reduced to eq. (16) if $x_1=x_2=x$ then equation (19) can be solved exactly. Four solutions are as follows,

$$\tau_*^{(1,2)} = \frac{t}{2(N+1)} \pm \left[\frac{t^2}{4(N+1)^2} + \frac{2(N-1)xm}{(N+1)F} \right]^{1/2} \\ \tau_*^{(3,4)} = \frac{t}{2N} \pm \left[\frac{t^2}{4N^2} + \frac{2mx}{F} \right]^{1/2} \quad (20)$$

As we see, if $F < 0$ then all solutions become complex for $F \rightarrow 0$. If $F > 0$, then for any F either τ_* or the time of final blow are outside the interval $(0, t)$. Thus, one can suppose that for $F \rightarrow 0$ only two terms remain in the series (18). First of them is given by formula (1), the second one must transform to (11) in the limit $F \rightarrow 0$. Evidently, the time of the blow τ_* for $F \rightarrow 0$ will be slightly different from the time of a blow in a free case, i.e. the solution of equation (16) can be found in a form of a series with respect to the parameter F . The first terms of the expansions of τ_* and S_1 are as follows,

$$\tau_* = \frac{x_1 t}{x_1 + x_2} + \frac{x_1 x_2 (x_1 - x_2) t^3}{2m (x_1 + x_2)^4} F + \frac{x_1 x_2 [(x_1 - x_2)^2 - 2x_1 x_2] t^5}{(2m)^2 (x_1 + x_2)^7} F^2 \quad (21)$$

$$S_1 = \frac{m(x_1+x_2)^2}{2t} + \frac{Ft(x_1^2+x_2^2)}{2(x_1+x_2)} + \frac{F^2t^3}{2m} \left[\frac{x_1^2x_2^2}{(x_1+x_2)^4} - \frac{1}{12} \right] + \dots \quad (22)$$

The following terms of action's expansion in powers of F can be found with the aid of equations (4). From formula (22) one can make the conclusion that the function S_1 has the following functional form:

$$S_1 = \frac{m(x_1+x_2)^2}{2t} \left[1 + \psi(\mu, \bar{x}) \right]; \quad \mu = \frac{Ft^2}{m(x_1+x_2)}; \quad \bar{x} = \frac{(x_1-x_2)^2}{(x_1+x_2)}$$

Taking the sum of equations (4) and introducing new variables we obtain the equation for the function $\psi(\mu, \bar{x})$

$$\psi + \mu \frac{\partial \psi}{\partial \mu} + \frac{1}{4} \left(2\psi - \mu \frac{\partial \psi}{\partial \mu} \right)^2 + \bar{x} (1-\bar{x}) \left(\frac{\partial \psi}{\partial \bar{x}} \right)^2 - \mu (1+\bar{x}) = 0 \quad (23)$$

Further, we present the function ψ in the form:

$$\psi(\mu, \bar{x}) = \sum_{k=1}^{\infty} \mu^k \psi_k(\bar{x})$$

For the function $\psi_1(\bar{x})$ one can find the expression

$$\psi_1(\bar{x}) = \frac{1+\bar{x}}{2} \quad (24)$$

which conforms with formula (22). Other functions are calculated from the recurrent formula

$$\psi_k(\bar{x}) = \frac{1}{4(k+1)} \sum_{j=1}^{k-1} \left[4\bar{x}(\bar{x}-1) \psi_j'(\bar{x}) \psi_{k-j}'(\bar{x}) - (j-2)(k-j-2) \psi_j(\bar{x}) \psi_{k-j}(\bar{x}) \right] \quad (25)$$

In particular

$$\psi_1(x) = \frac{(x-1)^2}{16} - \frac{1}{12}; \quad \psi_3(x) = \frac{x(x-1)^2}{32}; \quad \psi_4(x) = \frac{1}{128} x(x-1)^2(2x-1) \quad (26)$$

It follows from formulas (24), (25) that for $k \geq 3$ the functions $\psi_k(x)$ have the form

$$\psi_k(x) = x(x-1)^2 P_k(x) \quad (27)$$

where the functions $P_k(x)$ are polynomials of the degree $k-3$.

One can obtain the following formula for the cross derivative:

$$\frac{\partial^2 S_1}{\partial x_1 \partial x_2} = \frac{m}{t} \left\{ 1 + \sum_{k=1}^{\infty} \int_0^1 \left[\frac{1}{2} (k-2)(k-1) \psi_k(x) + (2k-1)x-1 \right] \psi_k'(x) + 2x(x-1) \psi_k''(x) \right\} \quad (28)$$

Due to formulas (24-27) this derivative can be written in the form of

$$\frac{\partial^2 S_1}{\partial x_1 \partial x_2} = \frac{m}{t} \left\{ 1 + (x-1) \sum_{k=1}^{\infty} M^k \widetilde{P}_k(x) \right\} \quad (29)$$

where $\widetilde{P}_k(x)$ - polynomials of the degree $(k-1)$. On a wall (for $x_1 = 0$ or $x_2 = 0$) $x = 1$, therefore both the action S_1 and the modulus of its cross derivative are the same as for the action S_+ . So we find the following expression for the propagator:

$$G(x_1, x_2, t) = \left(\frac{m}{2\pi i t} \right)^{1/2} \exp \left\{ \frac{i}{t} \left[\frac{m(x_2 - x_1)^2}{2t} + \frac{Ft(x_1 + x_2)}{2} \right] \right\} \quad (30)$$

$$\begin{aligned}
 & - \frac{F^4 t^3}{24m} \Big] \Big\} - \left\{ 1 + (\chi^2) \left[\frac{\mu}{2} + \frac{\mu^2}{8} (5\chi - 1) + \frac{\mu^3}{32} (28\chi^2 - 17\chi + 1) + \right. \right. \\
 & + \sum_{k=4}^{\infty} \mu^k \tilde{P}_k(\chi) \Big] \Big\}^{1/2} \exp \left\{ \frac{i}{t} \left[\frac{m(x_1 + x_2)^2}{2t} + \frac{Ft(x_1^2 + x_2^2)}{2(x_1 + x_2)} + \right. \right. \\
 & + \frac{F^2 t^3}{24m} \left(12 \frac{x_1^2 x_2^2}{(x_1 + x_2)^4} - 1 \right) + \frac{8m x_1^2 x_2^2 (x_1 - x_2)^2}{t (x_1 + x_2)^4} \left(\frac{\mu^3}{32} + \right. \\
 & \left. \left. + \frac{\mu^4}{128} (3\chi - 1) + \sum_{k=5}^{\infty} \mu^k P_k(\chi) \right) \right] \Big\} \Big)
 \end{aligned}$$

As to the quantum correction $\varphi(x_1, x_2, t)$ (see formula (2)), it follows from equation (8) that it can be presented in the form of

$$\varphi = \frac{t}{m(x_1 + x_2)^2} \sum_{k=1}^{\infty} \mu^k \gamma_k(\chi)$$

(The expansion must begin with $k=1$, because formula (30) coincides with formula (13) for $F=0 \Rightarrow \mu$). The first two terms of the expansion are as follows:

$$\varphi = \frac{t}{m(x_1 + x_2)^2} \left[-\frac{3}{4}(\chi-1)\mu + \frac{\mu^2}{32} (11 + 58\chi - 69\chi^2) + \dots \right]$$

(it is essential that constants appearing in integrating of linear non-uniform equations (8) can be always chosen in such a way that the correction $\varphi(\mu, \chi)$ would be equal to zero on a wall, i.e. for $\chi=1$). Hence formula (30) is true under the conditions

$$\frac{F t^3 x_1 x_2}{m^2 (x_1 + x_2)^5} \ll 1; \quad \frac{F t^2}{m (x_1 + x_2)} \ll 1 \quad (31)$$

For the equilibrium density matrix one obtain the expression (we write $F = -f$, $f > 0$ because in the case discussed only the attraction field should be considered):

$$\begin{aligned} \rho(x_1, x_2, \beta) = & \left(\frac{m}{2\pi\beta\hbar^2} \right)^{1/2} \left\{ \exp \left\{ -\frac{m(x_1 - x_2)^2}{2\beta\hbar^2} - \frac{f\beta}{2}(x_1 + x_2) + \frac{f^2\beta^3\hbar^4}{24m} \right\} - \right. \\ & - \left[1 + (f-1) \sum_{k=1}^{\infty} \tilde{N}^k \tilde{P}_k(x) \right]^{1/2} \exp \left\{ -\frac{m(x_1 + x_2)^2}{2\beta\hbar^2} - \frac{f\beta}{2}(x_1 + x_2) - \right. \\ & - \left. \frac{f^2\beta^3\hbar^4}{24m} \left[12 \frac{x_1^2 x_2^2}{(x_1 + x_2)^4} - 1 \right] - \frac{8m x_1^2 x_2^2 (x_1 - x_2)^2}{\beta\hbar^4 (x_1 + x_2)^4} \sum_{k=2}^{\infty} \tilde{N}^k P_k(x) \right\} \Bigg\} \\ \tilde{N} = & \frac{f\beta^3\hbar^4}{m(x_1 + x_2)} \end{aligned} \quad (32)$$

Let us pay attention to the fact that for $x_1 = x_2 = x$ one obtains the closed expression for the function S_1

$$S_1(x, x, t) = \frac{3mx^2}{t} + \frac{Ft}{2}x - \frac{F^2 t^3}{96m} \quad (33)$$

In this case it is also possible to find the simple expression for the cross derivative. Introducing the variables $x = \frac{1}{2}(x_1 + x_2)$ and $\eta = x_1 - x_2$ we shall look for S_1 in the form of

$$S_1(x, \eta, t) = g_0(x, t) + \sum_{k=1}^{\infty} \eta^k g_k(x, t)$$

where the function $g_0(x, t)$ is given by formula (33). In order to find the functions $g_k(x, t)$ we write the difference of equations (4) in the new variables:

$$\frac{1}{m} \frac{\partial S}{\partial x} \frac{\partial S}{\partial \eta} - F \eta = 0 \quad (34)$$

From this we obtain

$$g_1 = 0, \quad g_2 = \frac{m F t}{F t^2 + 8 m x}$$

Thus

$$\left. \frac{\partial^2 S_1}{\partial x_1 \partial x_2} \right|_{x_2=x_1} = \left(\frac{1}{4} \frac{\partial^2 S_1}{\partial x^2} - \frac{\partial^2 S_1}{\partial \eta^2} \right) \bigg|_{\eta=0} = \frac{m}{t} \frac{8 m x - F t^2}{8 m x + F t^2}$$

Consequently, we obtain the following generalization of the classical Boltzmann's formula for the diagonal elements of the equilibrium density matrix:

$$\begin{aligned} p(x, x, \beta) = & \left(\frac{m}{2\pi\beta\hbar^2} \right)^{1/2} \left\{ \exp \left(-\frac{1}{2} \beta x + \frac{f^2 \beta^3 \hbar^2}{24m} \right) - \right. \\ & \left. - \left(\frac{8mx - f\beta^2 \hbar^2}{8mx + f\beta^2 \hbar^2} \right)^{1/2} \exp \left(-\frac{2mx^2}{\beta \hbar^2} - \frac{1}{2} f \beta x + \frac{f^2 \beta^3 \hbar^2}{96m} \right) \right\} \end{aligned}$$

This formula is true under the following conditions

$$\frac{f\beta^2 \hbar^2}{m x} \ll 1; \quad \frac{f\beta^3 \hbar^4}{m^2 x^3} \ll 1, \quad (36)$$

i.e. for relatively weak fields, high temperatures and far off the wall. For strong fields and low temperatures the quasi-classical approach is not applicable to this problem, because, firstly, the preexponential in formula (35) becomes complex and, secondly, for $\alpha_1, \alpha_2 \rightarrow 0$ the leading term in the formula for the classical action $\frac{1}{2} \beta^2 \hbar^2 / m$ can be written up a coefficient as $(E_0 / kT)^3$ where E_0 is the energy of a ground state (Flügge 1971) while for $T \rightarrow 0$ the temperature can enter to an equilibrium density matrix only in a combination like $\exp(-E_0 / kT)$. Practically, formula (35) can be interesting only provided it is true at least for the value of the coordinate of the order of de Broglie's wave length $\alpha_v = (\beta \hbar^2 / m)^{1/2}$ when the second exponent in eq. (35) is not too small. Introducing $\alpha = \alpha_v$ to the inequality (36) we obtain the following restriction on the strength of field:

$$\Phi = \frac{1}{2} \beta^2 \hbar^2 / m (kT)^3 \ll 1 \quad (37)$$

In a gravitation field condition (37) is always fulfilled: for an electron in the gravitational Earth's field we have $\Phi \sim 3 \cdot 10^{-28}$ for $T = 1 K$ (Note, that the term $\frac{1}{2} \beta^2 \hbar^2$ is also very small for all reasonable values of β and α in this case, i.e. the density matrix coincides practically with equation (14)). The strength of the electrical field under the same conditions should be sufficiently small than 300V/m. Formulas (35) and (32) can be simplified if the condition (37) is fulfilled:

$$\rho(\alpha_2, \alpha_1, \beta) \approx \left(\frac{m}{2\pi\beta\hbar^2} \right)^{1/2} \left\{ \exp \left[-\frac{m(\alpha_2 - \alpha_1)^2}{2\beta\hbar^2} - \frac{\beta\hbar^2}{2} (\alpha_1 + \alpha_2) \right] \right\} \quad (38)$$

$$\exp\left[-\frac{m(x_1+x_2)^2}{2\beta\hbar^2}-\frac{\frac{1}{2}\beta(x_1^2+x_2^2)}{(x_1+x_2)}\right]; \quad \frac{1}{2}\beta\hbar^2\frac{x_1x_2}{(x_1+x_2)^3} \ll 1$$

$$S(x, x, \beta) \approx \left(\frac{m}{2\pi\beta\hbar^2}\right)^{1/2} \left\{ \exp(-\frac{1}{2}\beta x) - \exp\left(-\frac{2mx^2}{\beta\hbar^2} - \frac{1}{2}\frac{1}{\beta x}\right) \right\} \quad (39)$$

$$\frac{\frac{1}{2}\beta\hbar^2}{mx} \ll 1.$$

R E F E R E N C E S

1. Berry M.V., Mount K.E. 1972 Rep.Progr.Phys. 35 315.
2. Brown J.S., Brown R.G., March N.H. 1974 Phys.Lett., 46A 463.
3. Glark T.E., Menikoff R., Sharp D.H. 1980 Phys.Rev. D22 3012.
4. Grandall R.E. 1983 J.Phys. A16 513.
5. Flugge S. 1971 "Practical Quantum Mechanics".
6. Kennard E.H. 1927 Es.Phys. 44 386.
7. Lukes T., Somaratna K.T.S. 1969 J.Phys. G2 586.
8. Moyer G.A. 1973 J.Phys. G6 1461.
9. Tachibana A., Yamabe T., Fukui K. 1977 J.Phys. B10 3175.
10. Van Vleck J.H. 1928 Proc. Nat. Acad. Sci. USA 14 178.

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