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THE CHARM OF GLUON JETS^{*)}

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A B S T R A C T

We investigate the charm content of gluon jets in perturbative QCD. The cross-section for $g+g \rightarrow g+c\bar{c}$ is calculated for a non-vanishing c -quark mass. Multiplicities as well as energy spectra closely resemble those of 1S_0 quarkonium decays to $g+c\bar{c}$, which can be presented in analytic form and thus provide a handy model adequate for charm production from gluon jets. The calculated rates are not quite as large as suggested by a recent experimental analysis, and the energy distributions are falling less steeply. The characteristics of 3P_J decays are distinctly different from S-wave decays.

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Production of charmed particles within jets has recently been observed in proton-antiproton collisions by the UA1 Collaboration. Under the trigger conditions stated in Ref. /1/ the average multiplicity per jet amounts to $N(D^{*\pm}) = 0.65 \pm 0.19 \pm 0.33$ for $z \geq 0.1$ and, furthermore, the momentum spectrum is steeply falling. In the energy region studied by this experiment, these jets are predicted in perturbative QCD to be dominantly gluon jets with a rather small admixture of quark jets. Thus it is the task to investigate theoretical pictures for the hadronization of gluons into heavy mesons /*1/.

As already expected on naive grounds, perturbative QCD is not able to account for the observed rates (when taken at face value) /*2/. A thorough study of these predictions is nevertheless mandatory - even if only to firmly fence in the area in which perturbative QCD can be applied /*3/. We therefore study the perturbative splitting of gluons into charmed quarks and charmed mesons in some detail and for a variety of reactions (including bottom quarks as a source of charmed hadrons). In particular we shall carefully take account of non-vanishing quark masses which set the scale for the probability of collinear gluon splitting $g \rightarrow Q\bar{Q}$ /*4/, and study their effect on the normalization and on energy distributions.

First we calculate the c-quark spectrum from heavy quarkonium decays, 1S_0 and $^3P_J \rightarrow g c\bar{c}$. These calculations, which of course include non-trivial dynamics and the correct phase space for a non-vanishing charm-quark mass, give analytic results for all quantities at the parton level. The spectra and multiplicities in 1S_0 decays can be taken as a handy approximation also for charm multiplicities and spectra in gluon jets produced in other reactions, where finite mass effects are still important but analytic expressions are too cumbersome to be reported. Such a lowest order Born term approach neglects higher-order multiple gluon bremsstrahlung that softens the final particle spectrum slightly. The error induced through this approximation is not expected to be large however, for jet energies up to a few 10 GeV.

The time required for a gluon to split into a charmed quark pair is $\lesssim E_g/(2m_c)^2$; the probability for additional multiple gluon radiation within this limit of a few 10^{-1} fm is expected to be small. At higher energies these radiation effects cannot be neglected any more. Keeping this limitation in mind, we have calculated in the most naive way the c-quark multiplicity and spectrum of the process $g + g \rightarrow g + c\bar{c}$ at $\frac{1}{2}\sqrt{s_{gg}} = 25$ GeV for c and $\bar{c}$ emitted into a narrow cone, Fig. 1. Elastic gluon scattering with a (virtual) final state gluon splitting into $c\bar{c}$ is the dominant production mechanism in this kinematical configuration. 3-parton configurations, in which the heavy quarks belong to different jets, can be characterized as radiative corrections to primordial c-jet production. The infrared singularities must, in this case, be absorbed by a complete treatment of radiative corrections, including loop corrections. In our case they are excluded by our jet definition. Since the quark masses are kept at their physical values, collinear singularities do not occur in the final state; initial state singularities are avoided by angular cuts. The calculation was also done for b quarks, decaying subsequently to charmed quarks. The quark distributions are folded with fragmentation functions to generate the final charmed-hadron distributions.

We start by calculating the multiplicities and energy distributions in quarkonium 1S_0 and $^3P_{0,1,2}$ decays. Bottomium decays into charmed mesons are interesting in their own right. In contrast to η_b which will be rather difficult to study experimentally, P states have already been observed [7]. In fact it seems possible to collect a rather clean sample of P-states due to the excellent γ resolution some experiments have accomplished. We discuss various limits and the reasons for the marked differences between the different quarkonium states. We then justify our claim that within perturbative QCD $^1S_0 \rightarrow g + g^* (\rightarrow c\bar{c})$ can be considered as a handy, adequate model for charm production in gluon jets.

For that purpose we compare various distributions from a numerical computation of the cross section $g + g \rightarrow g + c\bar{c}$ with our results for 1S_0 decays, and we find that they resemble each other closely. Compared to the data, however, they decrease less steeply and the normalization falls short by roughly one order of magnitude.

Starting from the matrix elements for the coupling of off-shell gluons to 1S_0 and 3P_J /8/ it is straight-forward to evaluate the distribution of $gc\bar{c}$ in the Dalitz plot. The result is finite for the S state but singular for all three P states in the limit $E_g \rightarrow 0$ /9,10/. This singularity can be removed by keeping the binding energy in the propagators. After integrating one of the Dalitz plot variables, the following c-quark energy distributions are obtained,

$$\frac{1}{\Gamma_{gg}} \frac{d\Gamma(0^{-+})}{dx} = \frac{\alpha_s}{\pi} \left\{ \frac{1}{2} [x^2 + (1-x)^2] J - x[1-x] I + \right. \quad (1)$$

$$\left. + 2 \sqrt{x^2 - \mu^2} [1-x] \right\}$$

$$\frac{1}{\Gamma_{gg}} \frac{d\Gamma(0^{++})}{dx} = \frac{\alpha_s}{3\pi} \left\{ \frac{3}{2} [x^2 + (1-x)^2 + \frac{2}{3} \mu^2] J + \left[\frac{4}{3} - 3x(1-x) + \mu^2 \right] I + \right.$$

$$\left. + \frac{2}{9} \sqrt{x^2 - \mu^2} \left[7(4-5x) + \frac{2+\mu^2}{1-x+b} \right] \right\} \quad (2a)$$

$$\frac{1}{\Gamma_{gq\bar{q}}} \frac{d\Gamma(1^{++})}{dx} = \frac{1}{6[-2\log b - 1]} \times \quad (2b)$$

$$\times \left\{ -3I + \sqrt{x^2 - \mu^2} \left[-1+5x + 2 \frac{2+\mu^2}{1-x+b} \right] \right\}$$

$$\frac{1}{\Gamma_{gg}} \frac{d\Gamma(2^{++})}{dx} = \frac{\alpha_s}{2\pi} \left\{ [x^2 + (1-x)^2 + \mu^2] J + \left[\frac{5}{6} - 2x(1-x) + \mu^2 \right] I + \right.$$

$$\left. + \frac{1}{18} \sqrt{x^2 - \mu^2} \left[73-89x + 10 \frac{2+\mu^2}{1-x+b} \right] \right\} \quad (2c)$$

$$I = \log \frac{2-x+\sqrt{x^2-\mu^2}}{2-x-\sqrt{x^2-\mu^2}} \rightarrow \text{const. for } x < 1, \mu^2 \rightarrow 0$$

$$J = \log \frac{2[1-x][x+\sqrt{x^2-\mu^2}]+\mu^2}{2[1-x][x-\sqrt{x^2-\mu^2}]+\mu^2} \rightarrow \log \frac{4}{\mu^2} \text{ for } x < 1, \mu^2 \rightarrow 0$$

x denotes the energy fraction $2E_c/M$, $\mu^2 = (2m_c/M)^2$, and b is the binding energy in units of M . Γ_{gg} is the lowest order gg decay width of the resonance while $\Gamma_{gq\bar{q}}$ is the partial width of 1^{++} for all light quarks u, d, s summed up. In S wave 0^{-+} decays (with a lowest-order 2-gluon decay mode) the standard result for the gluon splitting function $g \rightarrow c\bar{c}$ is reproduced when $M \rightarrow \infty$, and the leading behavior is given by

$$\frac{1}{\Gamma_{gg}} \frac{d\Gamma(0^{-+})}{dx} \rightarrow \frac{\alpha_s}{2\pi} [x^2 + (1-x)^2] \log \left(\frac{M}{m_c}\right)^2. \quad (3)$$

However, the approach is rather slow as will become evident from the discussion below. In 0^{++} and 2^{++} decays this leading log term will be overshadowed by $(b\bar{b}) \rightarrow c+\bar{c} + g_{\text{soft}}$ for all energies ever available.

In Figs. 2a and 3a we present the normalized distributions for S -wave decays at $M = 50$ GeV and for the parabolonium η_b state at $M = 9.4$ GeV. The charmed quark mass is chosen at 1.8 GeV. Evidently the quark mass plays an important rôle for the shape of $d\Gamma/dx$. Even for $\mu^2 < 10^{-2}$ we are far from having reached the asymptotic leading log form (3). In Figs. 3b-d we show the distributions for P -wave 0^{++} , 1^{++} and 2^{++} bottonium decays. They are in fact markedly different in all four cases. Compared to 0^{-+} , the distributions for all P states are dominated by a hard component. This is a consequence of the singular behavior of the decay matrix element for $E_g \rightarrow 0$. It appears with equal rate in all three cases. Note that it is only the $x \approx 1$ region where the

binding energy is needed to regularize the distributions and which is thus sensitive to the choice of the binding energy E_B (~ 440 MeV). Since the charmed quark-antiquark pair has a large invariant mass the virtual gluon state is very short-lived and additional gluon bremsstrahlung is strongly suppressed. We can thus safely extrapolate those spectra into the toponium range, and we have checked explicitly that their shape is hardly altered.

To obtain the predictions for the charmed hadron distributions we fold the parton distributions with the fragmentation function /11/

$$D_Q^H(z) = \frac{N}{z \left[1 - \frac{1}{z} - \frac{\epsilon_Q}{1-z} \right]^2} \quad (4)$$

putting $\epsilon_c = 0.15$, a value adopted from the momentum spectrum in the 10 GeV range /12/, and $\epsilon_Q/\epsilon_c = (m_c/m_Q)^2$ in general. Although the shape of the energy distributions is changed appreciably, as shown in Figs. 2a and 3, the results are still different in the four cases. For 0^{-+} the spectrum is monotonically falling, for 0^{++} , 2^{++} , 1^{++} it is increasingly harder due to the rising relative weight of the singular component at $x \rightarrow 1$. Since the charmed hadron essentially retains the quark direction, various QCD predictions for angular distributions could be tested experimentally. We stress again that all these distributions could well be measured in currently ongoing experiments.

The total rate for events with a $c\bar{c}$ pair, normalized to the leading decay rate, is finally given by

$$N(0^{-+}) = \frac{\alpha_s}{3\pi} \left\{ \log \frac{1+\sqrt{1-\mu^2}}{1-\sqrt{1-\mu^2}} - \frac{2}{3} \sqrt{1-\mu^2} (4-\mu^2) \right\} \quad (5)$$

$$N(0^{++}) = \frac{\alpha_s}{27\pi} \left\{ 5 \log \frac{1+\sqrt{1-\mu^2}}{1-\sqrt{1-\mu^2}} + 2\sqrt{1-\mu^2} [2+\mu^2] \log \frac{4[1-\mu^2]}{\mu^2 b} - 2\sqrt{1-\mu^2} [8+3\mu^2] \right\} \quad (6a)$$

$$N(1^{++}) = \frac{1}{3[-2\log b-1]} \left\{ -2\log \frac{1+\sqrt{1-\mu^2}}{1-\sqrt{1-\mu^2}} + \sqrt{1-\mu^2} [2+\mu^2] \log \frac{4[1-\mu^2]}{\mu^2 b} - \sqrt{1-\mu^2} [1+\mu^2] \right\} \quad (6b)$$

$$N(2^{++}) = \frac{\alpha_s}{18\pi} \left\{ -4\log \frac{1+\sqrt{1-\mu^2}}{1-\sqrt{1-\mu^2}} + 5\sqrt{1-\mu^2} [2+\mu^2] \log \frac{4[1-\mu^2]}{\mu^2 b} - 11\sqrt{1-\mu^2} [1+\mu^2] \right\} \quad (6c)$$

For small μ^2 these formulae coincide with the asymptotic results of Ref. /9/, in particular $N(0^{-+}) \rightarrow (\alpha_s/3\pi) \log(M/m_c)^2$ as a consequence of equ. (3). The multiplicities for bottomonium and toponium decays ($M \sim 80$ GeV) are collected in Table 1. The binding energy is chosen at 440 MeV and 1460 MeV, respectively; the strong coupling constant α_s is set to 0.25; $n_q = 3$ light flavors normalize the 1^{++} rate. Note that these numbers also correspond to the total charm plus anticharm multiplicity per gluon jet.

It is the dynamics of non-relativistic P states and not their quantum numbers that leads to the peculiar dependence on the binding energy and the resulting hard charm spectra. (We checked, for example, that a spin 0 bound state of scalar quarks with $\ell = 0$ gives the same distributions as our 0^{-+} case.) We therefore consider only S-wave 0^{-+} decays as a prototype for charm production in gluon jets. In Fig. 2b we present the charmed quark multiplicity $N(0^{-+})$ for $\alpha_s = 0.25$. (Note that the complete result differs from the leading log approximation by a large constant term.) At $M = 50$ GeV, corresponding to gluon jets of an energy of about 25 GeV, we find $(N_c + N_{\bar{c}})/\text{jet} \sim 11\%$. This is of course much less than the aforementioned experimental number (if taken at face value). Evidently the experimental and perturbative theoretical results investigated here, are only compatible if full use is made of the large statistical and systematic errors.

Finally we compare these analytic calculations with results from a numerical computation of charmed hadron production through the subprocess $g + g \rightarrow g + c\bar{c}$, together with the relatively small

contribution from $g + g \rightarrow g + b\bar{b}$ ($b \rightarrow c$). We calculate these subprocesses at fixed energy $\sqrt{s} = 50$ GeV. The cross section has been determined in Ref. /13/ with quark masses kept at their physical values. This is most important as collinear singularities are avoided so that we can calculate the gluon splitting $g \rightarrow Q\bar{Q}$ in a jet down to zero splitting angles. Only those events might therefore be considered where the opening angle (defined in the center-of-mass frame) between the two heavy quarks is less than a fixed value $\Delta\chi$ of 20° , 30° , 40° or 90° so that they can reasonably be assigned to a "gluon jet". We further require the angle between the gluon recoiling against the heavy quark pair, and the beam axis to lie between 45° and 135° . These cuts eliminate all infrared singularities. We normalize these events relative to the subprocess $g + g \rightarrow g + g$ subject to the same constraint on the gluon scattering angle. α_s is again chosen to be 0.25. Both charm and bottom hadron distributions are obtained after folding with the fragmentation functions defined in equ. (4). In Table 2 we list the multiplicities per jet for various choices of $\Delta\chi$ and $\sqrt{s} = 50$ GeV together with the multiplicities from 1S_0 quarkonium decays. For comparison we include also the numbers from a calculation for $\sqrt{s} = 100$ GeV even though multiple gluon bremsstrahlung might start playing some role at so high an energy. Evidently the multiplicities are rather insensitive to $\sqrt{s}$, and our results should thus apply also to a sample of jets of different energies. We have checked that the (normalized) distributions are quite insensitive to the choice of $\Delta\chi$ in the range under discussion. Their shape, Fig. 4, is remarkably similar to our analytic result from quarkonium decay (and so are the multiplicities). Both calculations give monotonically decreasing distributions for $z \gtrsim 0.1$. They are apparently less steeply falling than the experimental results /*5/.

Note that the ratio of charmed quark jets produced in $gg \rightarrow c\bar{c}$ relative to gluon jets is close to 1% so that gluon splitting is by far the dominant source of charmed particles. Due to the steeply falling transverse momentum distribution, the dominant contribution to prompt leptons at large $p_\perp$ comes nevertheless from charmed quark jets /2/.

Are there ways to ameliorate the discrepancy between this lowest-order perturbative approach and experiment (without just blaming error bars)? Reducing the charm mass from 1.8 GeV to the current quark mass value of ~ 1.2 GeV would increase the rates by $\sim 20\%$. Taking the extreme value $\alpha_s = 0.4$ would still give insufficient rates. Thus, if the experimental numbers pertain with reduced statistical and systematical errors we have to conclude that (hitherto unknown) nonperturbative phenomena must play a dominant role in the charm composition of gluon jets. In that case it is even more mandatory to shed light on this problem by comparing the fairly low energy bottomonium decays to charmed particles with the collider results at high energies.

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Table 1: Rates (in %) for bottonium and toponium decays with a $c\bar{c}$ or $b\bar{b}$ pair, normalized to lowest-order decay rates (parameters as specified in text).

	bottonium				toponium (M = 80 GeV)			
	0^{-+}	0^{++}	1^{++}	2^{++}	0^{-+}	0^{++}	1^{++}	2^{++}
$c\bar{c}$	2.3	7.9	32.1	17.2	13.1	20.2	33.5	33.2
$b\bar{b}$	-	-	-	-	7.7	14.8	33.4	27.7

Table 2: Charm and bottom quark multiplicities (%) in gluon jets, produced in gluon-gluon collisions at a total energy $\sqrt{s}$, and in S-wave 0^{-+} decays at an invariant mass M.

$g + g / 0^{-+}$	ΔX	charm	bottom	charm + bottom
$\sqrt{s} = 50 \text{ GeV}$	20°	1.8	0.2	2.0
	30°	2.7	0.5	3.2
	40°	3.7	0.7	4.4
	90°	7.5	2.2	9.7
$M = 50 \text{ GeV}$	all	10.6	5.3	15.9
$\sqrt{s} = 100 \text{ GeV}$	20°	3.8	1.0	4.8
	30°	5.3	1.1	6.4
	40°	6.7	1.8	8.5
	90°	12.0	5.7	17.7
$M = 100 \text{ GeV}$	all	14.2	8.9	23.1

FOOTNOTES

- /*1/ The experimentally observed multiplicity exceeds the number expected from primordial c-quark jets by about 2 orders of magnitude /2/.
- /*2/ Predictions of perturbative QCD /2/ for μ production from heavy flavor decays appear to be in accord with data at large transverse momentum /1A/.
- /*3/ The probability for spontaneous non-perturbative creation of $c\bar{c}$ pairs in gluonic flux tubes was estimated to be $O(10^{-11})$ /3/. Other non-perturbative mechanisms which could serve as a QCD motivated base for phenomenological discussions /4/ are presently not known.
- /*4/ Previous perturbative investigations were either based on leading log expansions /5/ or used the full kinematics, but matrix elements in the massless limit /6/. This technique cannot be applied to the case we are interested in, since $g \rightarrow Q\bar{Q}$ develops a collinear singularity if quark masses are neglected.
- /*5/ Additional gluon bremsstrahlung, effectively setting in at higher energies, would soften the spectrum somewhat. As discussed above, this effect cannot play a decisive role in the energy range relevant in this context.

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FIGURE CAPTIONS

- Fig. 1 Kinematics of the process $g + g \rightarrow g + c\bar{c}$.
- Fig. 2 1S_0 decays to $g + c\bar{c}$. (a) Energy spectrum of charmed quarks and hadrons for an invariant mass of 50 GeV. (b) Relative rate of events with $c\bar{c}$ pairs $\Gamma(gc\bar{c})/\Gamma(gg)$; this ratio can be interpreted as c-quark multiplicity per gluon jet for sufficiently high energies.
- Fig. 3 Energy spectra of charmed quarks and hadrons in bottomonium decays to $g + c\bar{c}$. (a) S-wave 0^{-+} decay; (b-d) P-wave 0^{++} , 1^{++} , 2^{++} decays.
- Fig. 4 Spectrum of charmed particles in the reactions $g + g \rightarrow g + c\bar{c}$ and $g + b\bar{b}$, the b quarks decaying weakly to c quarks; both quark spectra are folded with appropriate fragmentation functions.

- Figure 1 -

- Figure 2a -

- Figure 2b -

- Figure 3a -

- Figure 3b -

- Figure 3c -

- Figure 3d -

- Figure 4 -