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HOW INFLATION MAY HAVE STARTED

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A B S T R A C T

Sufficient inflation can be obtained, while the inflaton field is close to the false vacuum for a renormalizable approximation to the full non-renormalizable potential to be valid; it is shown that this requirement is not compatible with a Coleman-De Luccia tunnelling mechanism. The spectrum of density fluctuations can, in this case, also be computed with the approximated potential. We list consistency conditions for cubic and quartic order expansions, which should be taken into account in model building

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It was originally suggested by Guth¹⁾ (see also Kazanas²⁾) that a period of exponential expansion (by a factor of e^{60} at least) of the scale factor R of the Universe could provide a solution to the horizon and flatness problems. The basic idea that the Universe should supercool in a "false vacuum", in order to produce this inflationary period, still remains. However, the exit from this period, proposed and criticized by Guth, consisting of the formation and coalescence of bubbles, does not lead to a uniform Universe. The new inflationary scenario of Linde, and Albrecht and Steinhardt³⁾, proposes that the whole observable Universe is contained in one bubble only, whose interior should, therefore, expand by a factor of at least e^{60} (in linear dimensions). This means that a bubble, or fluctuation region, should form when the Higgs field responsible for the inflationary transition attains a certain value on a very flat section of the potential. This is a value different from the one at the false vacuum but still very different from the value at the "true vacuum". Subsequently, the field rolls down towards the true vacuum, and it is in this period that the inflation inside the bubble takes place. The expansion rate (Hubble constant) $H = \dot{R}/R$, is determined by the usual Friedmann equation and while it is kept almost constant, the Universe is in a de Sitter phase. A very flat potential is required close to the false vacuum so that $H^2 = V(0)/3M^2$ ($M = m_{\text{Planck}}/\sqrt{8\pi} = 2.4 \times 10^8 \text{ GeV}$).

It appears quite clear by now that either the standard or the supersymmetric Coleman-Weinberg⁴⁾ $SU(5)$ ⁵⁾ phase transition cannot produce an acceptable inflationary scenario⁶⁾. The search for suitable mechanisms led people to consider singlet Higgs fields producing the inflation, mainly in the framework of $N = 1$ supergravity⁷⁾, in which the potentials are, in principle, non-renormalizable. However, the initial stages of inflation, which involve a supercooling mechanism (either a barrier in the zero temperature, $T = 0$, potential, or due to finite temperature effects) and a tunnelling (or transition to the flat section of the potential), should depend only on the details of the potential in the vicinity of the false vacuum. Therefore an expansion of the potential around the false vacuum up to cubic or quartic terms could be enough to account for the main features of inflation with the exception of reheating, which depends on the potential near the true vacuum. Moreover, the computation of finite temperature effects (to one-loop order), make sense only for renormalizable potentials. Therefore, it is almost a must in model building to produce sufficient inflation with a renormalizable expansion of the potential around the false vacuum. We will consider the $T = 0$ potential as a function of a single real field representing the direction in the multidimensional field space in which the inflation takes place. Temperature effects can only be computed when the full theory is known, while here we do not study the full potential.

We will be concerned here with the conditions that sufficient inflation and a suitable spectrum of density perturbations imply on the barrier crossing mechanism. The barrier should be close to the false vacuum (where we take the origin in field space). Thus we assume that a power expansion of the potential up to cubic or quartic terms in the field is sufficient to describe the barrier crossing and the subsequent roll-over (we do not treat the alternative possibility of inflation produced in a flat sector outside the range of validity of the renormalizable approximation).

Let us take as the characteristic time for inflation the e-folding period, τ , of the variation of the field from its value at the barrier. Then, surely a sufficient condition for inflation is

$$\tau H > 60 \quad (1)$$

The spectrum of density fluctuations is given at the moment the perturbations go out of the horizon during the de Sitter phase. This moment happens quite early, when the field is close to the false vacuum (unless the inflationary epoch is too long). Thus the almost scale independent value of the density contrast

$$\frac{\delta \rho}{\rho} \simeq 10^{-4} \quad (2)$$

should also be computed with the first orders of expansion of the potential.

We will analyze specifically which type of barrier crossing process is compatible with conditions (1) and (2). The barrier crossing can occur via bubble nucleation, studied in the presence of gravity by Coleman and De Luccia⁸⁾ (CD mechanism), via a "jump to the maximum of the barrier", as proposed by Hawking and Moss⁹⁾ (HM mechanism) or via quantum fluctuations when the barrier is small or does not exist at all ("spinodal" decomposition¹⁰⁾). The first two processes are tunnelling effects and the probability for the transition is calculated semi-classically. It is proportional to e^{-A} ¹¹⁾, where A is the difference between the classical Euclidean actions of the final and the initial field configurations. The initial configuration has the field, ϕ , at the origin $\phi = 0$ everywhere. In the CD mechanism, the final configuration consists of a bubble, outside which $\phi = \phi_1$ (a value on the false vacuum side of the barrier), inside which $\phi = \phi_2$ (a value on the true vacuum side of the barrier), and in whose wall ϕ takes all the intermediate values passing through ϕ_{top} , the value corresponding to the top of the barrier (see Figs. 1 and 2a). In the HM mechanism, the final configuration is $\phi = \phi_{\text{top}}$ everywhere (see Fig. 2b). The "spinodal" decomposition, on the other

hand, is a quantum effect in which the classical action is of order unity (in dimensionless units). This signals the breakdown of the semi-classical approximation.

Recently, Jensen and Steinhardt¹²⁾ have analyzed the conditions under which a CD or a HM transition occurs. Although their paper is devoted to a Coleman-Weinberg potential, their analytic proof is valid for any potential whose curvature, $|V''(\phi)|$, increases monotonically with increasing values of the field from $\phi = 0$ to ϕ_{top} . They consider the expansion of the potential around the top of the barrier and analyze the existence of a solution with the desired boundary conditions. The Euclidean versions of both the equation of motion for the field and Einstein's equation must be solved simultaneously when gravity effects cannot be neglected (that is when H becomes important with respect to the curvature of the potential). The Euclidean field equation for a rotationally invariant field is

$$\ddot{\phi} + 3H \cotg(H\xi) \dot{\phi} = V'(\phi) \quad (3)$$

where the solution of Einstein's equation for the scale factor in Euclidean space, $R_E = \sin(H\xi)$, has already been included (ξ is the radial co-ordinate in Euclidean space and $\dot{\phi} = d\phi/d\xi$). It is easily seen that there is no finite action solution (because ϕ diverges) unless $\dot{\phi} = 0$ when $\cot(H\xi) \rightarrow \infty$ (for $\xi = 0$ and $H\xi_{\text{max}} = \pi$). Thus the boundary conditions must be $\dot{\phi}(0) = \dot{\phi}(\xi_{\text{max}}) = 0$. These conditions determine uniquely the solution $\phi(\xi)$. In particular the asymptotic value of the field outside the bubble $\phi = \phi_1$ is fixed (and it need not be zero). $\phi = \phi_{\text{top}}$ is always a solution but its action is never smaller than that of the CD solution. Jensen and Steinhardt found that, as the modulus of the curvature at the top, $|V''(0)_{\text{top}}|$, approaches $4H^2$ from above (which is the same as saying that gravity becomes more and more important), both ϕ_1 and ϕ_2 approach ϕ_{top} , i.e., CD goes into HM. When the condition

$$-V''(\phi_{\text{top}}) \leq 4H^2 \quad (4)$$

is fulfilled, there is only a solution which is of the HM type.

In the following, we will use the two possible types of renormalizable potentials (at $T = 0$):

1) the cubic term is dominant beyond the barrier,

$$V_3 = \alpha + \beta \varphi^2 - \gamma \varphi^3 \quad (5)$$

2) the cubic term is absent or negligible

$$V_4 = \alpha + \beta \psi^2 - \delta \psi^4 \quad (6)$$

The constants α , β , γ and δ are positive numbers and ϕ and ψ are real fields. We will use ϕ when referring to the results of V_3 or valid for both cases (when specified) and ψ for those coming from V_4 . During the roll-over period after the tunnelling, the acceleration $\ddot{\phi}$ and the particle creation $\Gamma\dot{\phi}$ term (Γ is the total decay rate of ϕ) can be neglected: $\ddot{\phi} \ll 3H\dot{\phi}$, $\Gamma \ll 3H$, and the classical equation of motion becomes:

$$3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0 \quad (7)$$

The solutions in the two cases, in the range of validity of the expansions (5) or (6) to the actual potential, are

$$\varphi = \frac{\varphi_{top}}{1 - e^{t/\tau_3}} \quad , \quad \varphi_{top} = \frac{2\beta}{3\gamma} \quad (8)$$

and

$$\psi = \frac{\psi_{top}}{\sqrt{1 - e^{t/\tau_4}}} \quad , \quad \psi_{top} = \sqrt{\frac{\beta}{2\delta}} \quad (9)$$

The values ϕ_{top} , ψ_{top} of the fields correspond to the top of the barrier for V_3 and V_4 , respectively, and they are the solutions when $t \rightarrow \infty$. For $t \rightarrow 0$, the fields go to ∞ , which can be interpreted as going to the true vacuum. Thus the time is $t = 0$ at the end of inflation and $t < 0$ for any other previous moment. The characteristic time scales $\tau_3 = 3H/2\beta$ and $\tau_4 = 3H/4\beta$ are the e-folding periods of the variation of the fields with respect to the top. As a consistency condition, given the solutions (8) and (9), the second derivative term in the equation of motion can be neglected if $Ht \gg 1$, provided that $|t| > H^{-1}$.

The condition (4) for a HM transition is $\beta < 2H^2$ for V_3 and $\beta < H^2$ for V_4 . It translates into $\tau H > 3/4$. Thus the sole condition (1) for sufficient inflation is more than enough to ensure that a HM transition occurs. It may also be that a spinodal decomposition is compatible with producing sufficient inflation in the range of validity of V_3 or V_4 , but a CD tunnelling can occur only if inflation is

produced outside this range (in a flat region of the potential further out from the origin).

To understand the physical meaning of the condition (1), let us note that at the end of the HM transition, the Universe is at the maximum of the potential, which is a stationary point. To get a value $\phi \neq \phi_{\text{top}}$ one needs an infinite time. Hence, only quantum fluctuations can move the field to either side of the barrier. Typical fluctuation regions will be of the size of the horizon, H^{-1} , and the scale for the quantum jump is also given by H since the Hawking temperature is $T_H = (H/2\pi)$. The solution of the equation of motion for small variations from the top of the barrier shows that the initial quantum departure $\Delta\phi \approx H$, is e-folded in a period τ . The total time for inflation computed with the equations (8) and (9), i.e., the time for ϕ to vary from $\phi_{\text{top}} + \Delta\phi$ to $\phi \rightarrow \infty$, is

$$\Delta t \approx \tau \ln\left(\frac{\phi_{\text{top}}}{\Delta\phi}\right) \approx \tau \ln\left(\frac{\phi_{\text{top}}}{H}\right) \quad (10)$$

Let us keep in mind this relation which will be quantified later. Note that $\Delta\phi \sim H \ll \phi_{\text{top}}$ should hold because quantum fluctuations should never drive the field over the flat part of the potential, which is of an order not much larger than ϕ_{top} . The actual condition for sufficient inflation within this range is then $(\Delta t - \tau)H > 60$, since for a time $t = -\tau$, the fields are still quite close to ϕ_{top} , $\phi = 1.6 \phi_{\text{top}}$ and $\psi = 1.26 \phi_{\text{top}}$. Remember that $t = 0$ corresponds to ϕ and $\psi \rightarrow \infty$. Thus the approximation breaks down within the last τ period before the end of inflation. In the following, we will quantify the condition (1) with a coefficient x

$$\tau H > x \cdot 10^2 \quad (11)$$

Let us examine now condition (2). Following the procedure of Guth and Pi¹³⁾, the density contrast is found to be

$$\frac{\delta\rho}{\rho} = \frac{1}{\sqrt{2}\pi^3} \frac{H^2}{\dot{\phi}(t^*)} \quad (12)$$

Here ϕ stands for both solutions (8) or (9), $\dot{\phi} = (\partial\phi/\partial t)$, and t^* refers to the moment of horizon crossing of the inhomogeneity during the de Sitter phase. Thus, when $t^* > -\tau$, we have for V_3 :

$$\left(\frac{\delta\rho}{\rho}\right)_3 \simeq \frac{1}{\sqrt{2\pi^3}} \frac{\sigma}{H} \left[\ln\left(\frac{H}{k_c}\right) \right]^2 \quad (13)$$

and for V_4 :

$$\left(\frac{\delta\rho}{\rho}\right)_4 \simeq \sqrt{\frac{16\sigma}{3\pi^3} \left[\ln\left(\frac{H}{k_c}\right) \right]^3} \quad (14)$$

where k_c is the comoving wave number of the inhomogeneity, taking $R(t=0) = 1$ for the scale factor at the end of inflation. At t^* , when the inhomogeneity of the comoving wavelength $\lambda_c = 1/k_c$ goes out across the horizon, we know that the physical wavelength is $R(t^*)\lambda_c = H^{-1}$. Let us call N the number of e-folds in R in the period from t^* to $t = 0$, $R(0) = e^N R(t^*)$, thus $N = \ln(H/k_c)$ and $t^* = -N/H$. For a wide range of reasonable values of H and λ_c corresponding to the size of galaxies, the value of N is now $N \simeq 0(50-60)$ ($\lambda_{\text{now}} = 10^{21} \text{m} = 5 \times 10^{36} \text{GeV}^{-1}$, thus $\lambda_c = \lambda_{\text{now}}/R_{\text{now}}$ with $R_{\text{now}} \simeq T(t=0)/T_{\text{now}}$).

The relations (14) and (15) are valid in the case $N < \tau H$, since an expansion $e^{t^*/\tau} \simeq 1 + t^*/\tau + \dots$ is used to derive them. This gives a lower bound on τH . On the other hand, if τH is very large, the value of the field at t^* could fall outside the range of validity of the expansion (for $\tau H/N \gtrsim 10$, we have $\phi = (\tau H/N) \times \phi_{\text{top}}$ and $\phi = \sqrt{\tau H/N} \phi_{\text{top}}$). This upper bound depends on how far from the origin V_3 or V_4 approximate well the actual potential.

In the case of $t^* < -\tau$, i.e., $\tau H \lesssim N$, even though condition (1) is not respected, there can still be enough inflation if the more restrictive requirement $\tau H > 60 \ln(\phi_{\text{top}}/H)$ holds [see Eq. (10)]. An expansion of the exponential is not justified but we may neglect it with respect to 1 so that $\dot{\phi}(t^*) \simeq \exp(t^*/\tau) \times (\phi_{\text{top}}/\tau)$, which a factor $\frac{1}{2}$ to be included for $\dot{\phi}$. Thus

$$\left(\frac{\delta\rho}{\rho}\right)_3 \simeq \frac{H^2 \mathcal{Z}_3}{\sqrt{2\pi^3} \phi_{\text{top}}} \left(\frac{H}{k_c}\right)^{1/\mathcal{Z}_3 H} \quad (15)$$

or a factor of 2 larger for V_4 . Since $\tau H/\sqrt{2\pi^3} = 13 \times \tau H = 0(1)$ and also $(H/k_c)^{1/\tau H} \simeq 10^{20/\tau H} \gtrsim 0(1)$, we get $\delta\rho/\rho \simeq 10^{-4}$ only if $(H/\phi_{\text{top}}) \lesssim 10^{-4}$. Note that this condition ensures that quantum fluctuations are $\Delta\phi \ll \phi_{\text{top}}$. The expression (15) is scale-invariant even if τH is not very large, say $\tau H \gtrsim 10$. Therefore, the

range $10 \lesssim \tau H \lesssim N$ is phenomenologically acceptable [even if condition (1) is not fulfilled].

Let us return to the case $\tau H > N$. It is useful to parametrize N as

$$N^2 = \left[\ln\left(\frac{H}{k_c}\right) \right]^2 = y \cdot 10^3 \quad (16)$$

where y is of $O(1)$. The condition that quantum fluctuations should not drive the fields directly to the non-flat region of the potential then becomes

$$\frac{\varphi_{top}}{H} \simeq \frac{y}{x} \cdot 10^4 \gg 1 \quad (17)$$

and

$$\frac{\psi_{top}}{H} \simeq \frac{y^{3/4}}{x^{1/2}} \cdot 10^4 \gg 1 \quad (18)$$

The parameter x comes from Eq. (11), and Eqs. (13) and (14) are used with Eq. (2). Returning to Eq. (10), the total period of inflation can be estimated as

$$\Delta t \simeq z_3 (10 - \ln x) \quad \text{for } V_3 \quad (19)$$

and

$$\Delta t \simeq z_4 (10 - \frac{1}{2} \ln x) \quad \text{for } V_4 \quad (20)$$

It is instructive to express the HM action A_{HM} in terms of the above parameters, the phenomenologically meaningful quantities $\delta\rho/\rho$, τH , and $\ln(H/k_c)$. We know that when $A_{HM} \simeq 1$ (in units such that $\hbar = 1$), the semiclassical approximation breaks down, and "spinodal" decomposition occurs. We will show that $A_{HM} \gg 1$. We do not evaluate the CD action, because with V_3 or V_4 a CD mechanism is not compatible with the required features.

9)

Recall that

$$A_{H-M} \simeq 24 \pi^2 M^4 \frac{(V_{top} - V(o))}{V(o)^2} \quad (21)$$

Then, we have for V_3 and V_4 respectively,

$$A_{HM}^{(3)} \approx \frac{32 \pi^2}{9} \frac{\beta^3}{\gamma^2} \frac{M^4}{\alpha^2} \quad (22)$$

$$A_{HM}^{(4)} \approx 6 \pi^2 \frac{\beta^2}{\delta} \frac{M^4}{\alpha^2} \quad (23)$$

When $\tau H > N$, Eqs. (22) and (23) become respectively

$$A_{HM}^{(3)} \approx \frac{2}{3\pi} \frac{[\ln(H/k_c)]^4}{(\tau_3 H)^3 (\delta\rho/\rho)^2} = \frac{2}{3\pi} \frac{\gamma^2}{x^3} \cdot 10^8 \quad (24)$$

$$A_{HM}^{(4)} \approx \frac{2}{\pi} \frac{[\ln(H/k_c)]^3}{(\tau_4 H)^2 (\delta\rho/\rho)^2} = \frac{2}{\pi} \frac{\sqrt{10} \gamma^3}{x^2} \cdot 10^8 \quad (25)$$

When $\tau H < N$, using Eqs. (11), (15) and (2), one gets, for Eq. (22)

$$A_{HM}^{(3)} \approx \frac{2}{3\pi} \tau_3 H \left[\frac{(H/k_c)^{1/\tau_3 H}}{(\delta\rho/\rho)} \right]^2 = \frac{2}{3\pi} x \cdot 10^{10} \quad (26)$$

and $A_{HM}^{(4)}$ is larger than (26) by a factor of 3.

We note that $A_{HM} \gg 1$ in all cases, unless x (i.e., τH) is extremely large ($x \gtrsim 10^4$, i.e., $\tau H \gtrsim 10^6$).

We have thus shown that the Hawking and Moss transition is the only mechanism compatible with sufficient inflation and an amplitude of density fluctuations $\delta\rho/\rho \sim 10^{-4}$, produced with the expansions V_3 and V_4 of a non-renormalizable potential around the false vacuum [Eqs. (5) and (6)]. The Coleman-de Luccia mechanism is not compatible with sufficient inflation. A "spinodal" decomposition is only possible if τH is very large, but in this case V_3 and V_4 may not be suited to compute $\delta\rho/\rho$, because at t^* the field may already be too far from its value at the false vacuum.

REFERENCES

- 1) A. Guth - Phys.Rev. D23 (1981) 347.
- 2) D. Kazanas - Ap.J. 241 (1980) L59.
- 3) A.D. Linde - Phys.Lett. 108B (1982) 389;
A. Albrecht and P.J. Steinhardt - Phys.Rev.Lett. 48 (1982) 1220.
- 4) S. Coleman and E.J. Weinberg - Phys.Lett. D7 (1973) 1888.
- 5) H. Georgi and S.L. Glashow - Phys.Rev.Lett. 32 (1974) 438.
- 6) A.D. Linde - Phys.Lett. 116B (1982) 335;
A. Vilenkin and L. Ford - Phys.Rev. D26 (1982) 1231;
A.A. Starobinski - Phys.Lett. 117B (1982) 175;
S. Hawking - Phys.Lett. 115B (1982) 295;
A. Guth and S.-Y. Pi - Phys.Rev.Lett. 49 (1982) 1110;
J. Ellis, D.V. Nanopoulos, K.A. Olive and K. Tamvakis - Phys.Lett. 118B (1982) 225;
J. Breit, S. Gupta and A. Zaks - Phys.Rev.Lett. 51 (1983) 1007.
- 7) J. Ellis, D.V. Nanopoulos, K.A. Olive and K. Tamvakis - Nucl.Phys. B221 (1983) 524.
For reviews see, for example:
D.V. Nanopoulos - Invited talk at the First ESO-CERN Symposium (November 1983) and references therein;
B.A. Ovrut and P.J. Steinhardt - Invited Talk at Inner/Outer Space, Fermilab (May 2-5, 1984).
- 8) S. Coleman and F. De Luccia - Phys.Rev. D21 (1980) 3305.
- 9) S.W. Hawking and I.G. Moss - Phys.Lett. 110B (1982) 35.
- 10) P.J. Steinhardt - La Jolla Supergravity Workshop (1983).
- 11) S. Coleman - Phys.Rev. D15 (1977) 2929.
- 12) L.G. Jensen and P.J. Steinhardt - Nucl.Phys. B237 (1984) 176.
- 13) A. Guth and S.-Y. Pi - Ref. 6).
- 14) J. Ellis, D.V. Nanopoulos, K.A. Olive and K. Tamvakis - Phys.Lett. 120B (1983) 331.

FIGURE CAPTIONS

Fig. 1 Schematic view of the barrier near the false vacuum at $\phi = 0$.

Fig. 2 The square represents the Universe and two different transitions are shown;

a) CD transition: the final configuration is a bubble, with $\phi_1 \neq 0$ in principle.

b) HM transition: the final configuration is $\phi = \phi_{\text{top}}$ everywhere. Note that if ϕ_1 and ϕ_2 both approach ϕ_{top} , a) and b) coincide.

Fig. 1

Fig. 2