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W'S, Z'S AND JETS

S. D. Ellis ^{*}), R. Kleiss and W.J. Stirling
CERN - Geneva

ABSTRACT

The process $p + \bar{p} \rightarrow W^{\pm}, Z^0$ plus 2 jets is discussed in the context of perturbative QCD. The magnitude of the expected rate for this process and the correlations anticipated between the jets are presented.

^{*})Permanent address: Dept. of Physics,
Univ. of Washington, Seattle, WA 98195,
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Now that the hadronic production of $W^\pm$'s and Z^0 's is being "regularly" observed¹⁾ at the CERN $p\bar{p}$ Collider, it is informative to study this process in more detail. In particular, it is of interest to consider the structure of the accompanying hadronic event, i.e., the structure of the inclusive state X in the process $p + \bar{p} \rightarrow W^\pm, Z^0 + X$. A natural way to characterize X is in terms of the number of hadronic jets. Such analyses will test both the perturbative framework thought to describe this process²⁾ and, perhaps, provide some basis from which to interpret the (possible) apparent differences³⁾ between the structure of the states X which accompany $W^\pm$'s and those which accompany Z^0 's.

While the definition of a jet is, in principle, very difficult to make precise⁴⁾, we shall treat it here as an experimental rather than theoretical issue. A straightforward application of the various experimental cuts to the perturbative final states appears²⁾ to work satisfactorily. In particular, a perturbative calculation²⁾ of the single jet configuration in which a jet is defined in terms of a parton with a minimum P_T separated from other partons by a minimum relative angle seems to adequately mimic the actual cuts applied to the full hadronic final states without the need of detailed Monte Carlo simulation of the complete final state. These cuts on the perturbative final state not only mimic the actual experimental hadronic cuts but also serve to control the infrared and collinear singularities inherent in the perturbative calculation. The angular cut controls the collinearity associated with "final state" bremsstrahlung configurations, while the requirement of a minimum jet P_T prevents the emission of partons which are arbitrarily soft or collinear with the incoming partons. With these cuts, the initial and final state partons are separated in phase space and perturbative cross sections are finite. In practice an additional polar angle cut (with respect to the beam direction) may be applied corresponding to a particular detector acceptance for jets.

The present note will address the production of two large P_T jets in the final state defined in the above fashion. In the perturbative context the desired process is described by the simple expression ($V = W^\pm$ or Z^0)

$$\begin{aligned} \sigma(V + 2 \text{ jets}) &\equiv \sigma(p + \bar{p} \rightarrow V + 2 \text{ jets} + X') \\ &= \int dx_1 dx_2 G(x_1, Q) G(x_2, Q) \sigma(ab \rightarrow V + ef, Q) \end{aligned} \quad (1)$$

where a sum over the relevant parton types is understood, i.e., a and b can be u , d , s , or c quarks or antiquarks or gluons (g) while e and f are summed over the same set plus b quarks. In the results presented below the scale Q is set

to M_V and the structure functions, G , are those of Duke and Owens⁵⁾ of type 1 (D01) with $\Lambda_{\text{QCD}} = 0.2$ GeV unless otherwise stated. The parton hard scattering cross section, σ , is calculated at order α_s^2 in the "tree" approximation. It is a straightforward, if tedious, task to evaluate the distinct perturbative 2 $\rightarrow$ 3 processes (252 in the case of the $W^\pm$, 69 for the Z°) which contribute to the perturbative cross section in Eq. (1). For example, Fig. 1 indicates the complete set of Feynman diagrams for the processes (a) $u\bar{u} \rightarrow Z^\circ + u\bar{u}$ and (b) $u\bar{u} \rightarrow Z^\circ + gg$. All other processes of interest can, in fact, be obtained from these by the appropriate crossing and changes in the weak coupling.

The actual calculation was performed by evaluating the matrix element on the helicity amplitude level using the spinor product method introduced in Ref. 6) and performing the phase space integrals using VEGAS⁷⁾. For the gluon polarization vectors we used a representation similar to that given by the CALKUL collaboration⁸⁾ which serves to considerably simplify the calculation of the three gluon vertex. The $W^\pm$, Z° polarization vectors were constructed explicitly from the 4-momenta of their decay products. The advantages of this method are several. First, the numerical evaluation of the amplitudes is very systematic and straightforward and therefore simple to check. Moreover, for a subset of the diagrams for the process of Fig. 1(a) we have compared the speed of the evaluation with an algebraic expression for the cross section obtained by the usual trace algebra methods. We conclude that the numerical evaluation of the expressions used here is at least as rapid as that for the more usual expressions. Finally the explicit representation of the $W^\pm$, Z° polarization vectors in terms of the lepton 4-momenta mentioned above enables us to directly implement cuts on the decay products of the weak bosons. Explicit results for the matrix elements and further calculational details will be given elsewhere⁹⁾.

The focus of the remainder of the present Letter will be on the results of the calculation. The simplest characterization of the results is in terms of the cross section of Eq. (1). This cross section is a function of the total energy (taken to be 540 GeV everywhere in this Letter) and of the experimental cuts used to define a jet. As noted above these are here taken as a minimum jet P_T and minimum angular separation. With $P_T^{\text{JET}} > P_T^{\text{min}} = 10$ GeV/c and $r = \sqrt{(\Delta\phi^2 + \Delta\eta^2)} > r_{\text{min}} = 1$ (where ϕ is the azimuthal angle and η is the pseudorapidity), the present calculation yields a cross section of 8.2 picobarns for $V = W^+ + W^-$ and 2.3 picobarns for $V = Z^\circ$ at $\sqrt{s} = 540$ GeV. However, a truly complete perturbative analysis of this process (and the 1 jet case) should also include contributions which are higher order in α_s [often referred to as the "K-factor"¹⁰⁾].

These have not yet been evaluated completely and will not be included here. Thus it seems more sensible to study ratios of cross sections so that such theoretical uncertainties tend to cancel out along with some of the experimental uncertainties. This procedure also tends to minimize the dependence on the specific choice of structure functions as noted below. In particular, it is useful to define the ratios

$$R_n = \sigma(V + n \text{ jets})/\sigma_0(V) \quad (2)$$

where R_1 has already been discussed in Ref. 2). The results for R_1 (at order α_s^1) and R_2 (at order α_s^2) for $\sqrt{s} = 540$ GeV and for $W^\pm$ and Z^0 production are plotted in Fig. 2 as functions of $P_T^{\min}$ for a fixed angular cut $r_{\min} = 1$. The numerical uncertainty in evaluating these quantities from Eq.(1) is of order 1%. However, as noted earlier (the "K-factor" and jet definition issues), the systematic theoretical uncertainties are considerably larger. Fig. 2 illustrates the expected falling behavior with increasing $P_T^{\min}$ of both ratios for $P_T^{\min} < M_V$; for very large $P_T^{\min}$ ($> M_V$ but $< \sqrt{s}/2$) one would expect an asymptotically constant ratio of ratios proportional to simply a number times the strong coupling constant α_s . Note the much more remarkable feature that, for this value of $r_{\min}$ and in the exhibited $P_T^{\min}$ range,

$$R_2 \sim (R_1)^2 \quad (3)$$

In detail, of course, these ratios depend on the angular cut $r_{\min}$. For R_1 this dependence occurs only in the higher order corrections and is presumably rather weak for $r_{\min}$ neither very large nor very small. For R_2 , however, there is an explicit dependence at leading order. Fig. 3 exhibits R_2 as a function of $r_{\min}$ for the range $0.5 < r_{\min} < 1.5$, a reasonable range experimentally. As $r_{\min} \rightarrow 0$ the perturbative collinear singularity becomes important and the cross section diverges logarithmically. As $r_{\min}$ becomes large the available phase space for the 2 jets becomes smaller and $R_2 \rightarrow 0$. For $P_T^{\min} = 10$ GeV/c the phase space limit corresponds to $r_{\min} = 8.3$. Thus it is clear that the relationship (3) cannot hold in detail for arbitrary $r_{\min}$. However, Fig. 2 does suggest that, for reasonable values of $r_{\min}$ and $P_T^{\min}$ and to within factors of 2, R_n can be easily estimated as a function of $P_T^{\min}$ from R_1 , i.e.,

$$R_n \sim (R_1)^n, \quad (4)$$

at least for n 's and $P_T^{\min}$'s such that overall energy conservation is not a severe constraint. This simple result also suggests that, for $P_T^{\min}$ and $r_{\min}$ values such that $R_1 \ll 1$ [as in the reported experimental results¹⁾], a fixed order perturbative description of the inclusive process is appropriate. With respect to the reported observation³⁾ that the average number of jets accompanying $W^\pm$'s is smaller than for Z^0 's, it is clear from Fig. 2 that perturbation theory predicts very similar (NOT different) accompanying final states as one would naively expect. It is also worth noting that, while R_1 is dominated by the quark-antiquark annihilation contribution (approximately 9 times the quark-gluon contribution), R_2 receives approximately equal quark-antiquark and quark-gluon contributions with a negligible gluon-gluon piece.

To obtain some feeling for the theoretical systematic error we have calculated R_1^W and R_2^W also for the second set of Duke and Owens⁵⁾ structure functions, DO2, with $\Lambda_{\text{QCD}} = 0.4$ GeV. These second structure functions differ from the first through the larger Λ value and a "harder" gluon distribution. The minimal dependence of the present results on these details is clearly indicated by comparing the two sets of curves in Fig. 4. The bulk of the difference is due simply to the larger value of α_s in the DO2 results and corresponds to a variation by about 10% in R_1^W and 30% in R_2^W .

A particularly simple way of characterizing the rate of multi-jet production is to consider the jet multiplicity distribution, i.e., the fraction f_n events containing n jets. In the present context we have

$$f_n = \frac{R_n}{1 + \sum_1 R_i}. \quad (5)$$

The UA1³⁾ data for $W^\pm$ and Z^0 are shown in Fig. 5 together with the QCD predictions. The results depend, of course, rather sensitively on the value of $P_T^{\min}$ (Fig. 2). We have chosen a value of 6 GeV/c, corresponding to the nominal experimental jet threshold of 5 GeV/c increased by 20%. [Although the Z^0 appear to differ from theoretical expectations, it must be remembered that the number of Z^0 events (9) is much smaller than the number of $W^\pm$ events (68) and it is therefore premature to draw any definite conclusion.]

Given the possibility of differences between the observed W^+ and Z^0 events, it is particularly interesting to describe these multi-jet final states in more detail in order to further test the perturbative picture. The simplest measure is the angular distribution of the jets. Following Ref. 2), we define the angle θ^* to be the angle between the jet and the average beam direction in the $V - 2$ jet centre-of-mass system. Fig. 6 illustrates the expected θ^* distribution of the jets for a range of $P_T^{\min}$ values. Note that, at least for small $P_T^{\min}$ values, this distribution exhibits the characteristic "initial state" bremsstrahlung like spectrum peaking for large rapidity (small angles with respect to beam the direction). However, it is clear also from the $r_{\min}$ dependence exhibited in Fig. 3 that this process is NOT dominated by "initial state" effects but also receives sizeable "final state" bremsstrahlung contributions as was also noted in Ref. 2). For a large $P_T^{\min}$ value the rapidly falling structure functions, which tend to minimize the total energy involved in the perturbative process, will serve to ensure that the jets are centrally produced. The expected correlation between the two jets in $\Delta\phi$ is illustrated in Fig. 7 for several $P_T^{\min}$ values. There is evidently little correlation at small $P_T^{\min}$ while P_T conservation leads to a ϕ correlation which peaks near 180° for large $P_T^{\min}$ values. The separation of the jets in phase space is quantified by the variable r defined above. The distribution in r is exhibited in Fig. 8 from which the $r_{\min}$ dependence of Fig. 3 is easily understood. Similar correlations are exhibited in a slightly different fashion in Fig. 9 where the calculated distribution in the $W + 2$ jet invariant mass is illustrated. Since the essentially identical structure is expected in both W^+ and Z^0 production, it will be informative to measure these distributions experimentally in the two cases.

The preceding discussion focused on inclusive V production with only a topological n -jet characterization of the accompanying final state. It is illuminating to consider the subset of events where the P_T of the V is also specified. This is because, for the decay $Z^0 \rightarrow \nu\bar{\nu}$, such events generally exhibit large missing transverse energy¹¹⁾. With the choice $P_T^{\min} = 6$ GeV/c the results for the distribution dR_n^Z/dP_T^Z are displayed in Fig. 10 for $n = 1, 2$. The areas under these distributions are just the corresponding points on the curves in Fig. 2. We note immediately that the dominance of $Z^0 + 1$ jet events ($R_1^Z(6) \gg R_2^Z(6)$) arises from rather low values of P_T for the Z^0 . In contrast, if we focus on the region $P_T^Z > 40$ GeV/c (approximately 20% of the total 2 jet events),

the two distributions in Fig. 10 are comparable. This, perhaps surprising, result suggests that simple, low order tree graph analyses of the jet structure in this kinematic regime are suspect. In particular, the fact that the magnitude of the order α_s^2 2 jet rate is comparable to the order α_s^1 1 jet rate for sizeable P_T of the V suggests that order α_s^2 corrections to the 1 jet cross-section (virtual correction diagrams plus real emission diagrams with the second parton outside the P_T and r cuts) may be important and similarly for the corresponding order α_s^3 corrections to the 2 jet cross-section, etc. Thus, while lowest order perturbation theory may be sufficient to describe the P_T distribution of the V in this regime (when no topological questions are asked and cancellations can occur between different topologies), it does not immediately follow that the P_T of the V is balanced by a single jet. Rather it appears that there are sizeable contributions from multi-jet configurations. A precise description clearly requires a more complete analysis⁹⁾.

So far we have described the production of weak bosons and 2 jets in terms of a single hard scattering of a pair of partons, one from each hadron. Another interesting mechanism which produces the same final state is double parton scattering in which there are TWO independent subprocesses -- one producing a weak boson, the other a pair of large P_T jets -- involving TWO partons from each hadron¹²⁾. Since the two parton distribution functions are proportional to $(\pi R^2)^{-1}$ where R is the effective radius of the hadron, the overall normalization of such processes is rather uncertain. With this in mind it is important to look for ways of distinguishing final states produced by single and double scattering. As an example, we can consider the distribution in the relative azimuthal angle of the two jets, $\Delta\phi$, as in Fig. 7. The characteristic feature of double scattering is that the jet transverse momenta are approximately equal and opposite. The $\Delta\phi$ distribution would therefore be expected to be sharply peaked around $\Delta\phi=180^\circ$, with a width proportional to $\langle k_T \rangle / P_T^{\min}$ where $\langle k_T \rangle$ is the average transverse momentum in a jet. In contrast, the single scattering distribution is very broad, being essentially flat at smaller values of $P_T^{\min}$. It appears, therefore, that the two processes can be readily distinguished in practice -- a more quantitative study is in progress⁹⁾.

In addition to providing tests of perturbative QCD per se, the production of a weak boson and two jets as described above also provides an important background to double weak boson production. To avoid rates that are too low at least one of the bosons must be detected by its hadronic (two jet) decays. Therefore we consider the two processes

$$q\bar{q} \rightarrow W(\rightarrow e\nu) W(\rightarrow q\bar{q}) \quad (6a)$$

and

$$q\bar{q} \rightarrow W(\rightarrow e\nu) q\bar{q} \quad (6b)$$

which have a similar final state when the invariant mass of the final $q\bar{q}$ pair in (6b) is of order M_W . More quantitatively, if we require $m_{q\bar{q}} > 70$ GeV then at $\sqrt{s}=540$ GeV the relevant cross sections are

$$\sigma(p\bar{p} \rightarrow WW) \sim 0(0.2) \text{ pb} \quad (7a)$$

$$\sigma(p\bar{p} \rightarrow W + 2 \text{ jets}) \Big|_{\text{Double Scattering}} \sim 0(1) \text{ pb} \quad (7b)$$

$$\sigma(p\bar{p} \rightarrow W + 2 \text{ jets}) \Big|_{\text{Single Scattering}} \sim 0(15) \text{ pb} \quad (7c)$$

where the first two numbers are taken from Ref. 13) and the third is from the present analysis. It would therefore appear that it will be very difficult to observe the production of two weak vector bosons in this way, just as it is difficult to identify single $W^\pm$ and Z^0 production via their hadronic decays. This becomes crucial in scenarios where a new heavy neutral ("Higgs") state H is expected to decay predominately into W^+W^- . For $M_H \sim 0(2 M_W)$ we have this same problem of the effect being mimicked by the process $q\bar{q} \rightarrow W^\pm q\bar{q}$. Indeed it is amusing to notice the apparent "resonant" peak around 150 GeV in the $M(W + 2 \text{ jet})$ distributions, Fig. 9. Of course the ultimate observability of such a state H will depend sensitively on its mass, width, production cross section, etc. A more detailed study is in progress⁹⁾.

In summary, a perturbative tree level analysis provides an adequate description of the inclusive process $p\bar{p} \rightarrow V + n \text{ jets} + X'$. Various details of the 2 jet final state have been enumerated here. In all cases the results are very similar, independent of whether the V is a $W^\pm$ or a Z^0 . The present study indicates, however, that more care is necessary if the V is required to have sizeable P_T . In this kinematic regime higher order and virtual corrections seem to be important for a detailed understanding of the accompanying multi-jet state where states containing more than one jet appear to play a sizeable role. The

multi-jet states arising from a single hard scattering are apparently readily distinguishable from those arising from double hard scattering. Both of these processes constitute a large background for the detection of double weak boson production.

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FIGURE CAPTIONS

Fig.1: Feynman diagrams for the processes (a) $u\bar{u} \rightarrow Z^0 + u\bar{u}$ and (b) $u\bar{u} \rightarrow Z^0 + gg$. The Z^0 is represented by the dashed line.

Fig.2: The ratios R_1 and R_2 for $V=(W^+W^-)$ (solid lines) and $V=Z^0$ (dashed lines) as functions of $P_T^{\min}$. The structure functions used were D01 and the jet angular cut was $r_{\min}=1$.

Fig. 3: The dependence of R_2^W on $r_{\min}$, with D01 structure functions and $P_T^{\min}=10$ GeV/c.

Fig. 4: The ratios R_1^W and R_2^W for the two sets of structure functions from Ref. 5: D01 with $\Lambda_{QCD}=0.2$ GeV (solid lines) and D02 with $\Lambda_{QCD}=0.4$ GeV (dashed lines).

Fig. 5: Jet multiplicity distributions for (a) W^+ and (b) Z^0 production. The data are from Ref. 3. The QCD predictions are indicated by the horizontal lines corresponding to $P_T^{\min}=6$ GeV/c, $r_{\min}=1$ and the two sets of structure functions D01 (solid lines) and D02 (dashed lines).

Fig. 6: Jet angular distributions (as defined in the text) accompanying a W^+ for different values of $P_T^{\min}$: (a) 10 GeV/c, (b) 20 GeV/c, (c) 30 GeV/c using D01 structure functions, $r_{\min}=1$ and $\sqrt{s}=540$ GeV.

Fig. 7: Distributions in the jet azimuthal angle difference for different values of $P_T^{\min}$: (a) 10 GeV/c, (b) 20 GeV/c, (c) 30 GeV/c. Other parameters as in Fig. 6.

Fig. 8: Distributions in the jet separation variable $r=\sqrt{(\Delta\eta^2+\Delta\phi^2)}$ for $P_T^{\min}$ values: (a) 10 GeV/c, (b) 20 GeV/c, (c) 30 GeV/c. Other parameters as in Fig. 6.

Fig. 9: Distributions in the $W^+ + 2$ jet invariant mass for $P_T^{\min}$ values: (a) 10 GeV/c, (b) 20 GeV/c, (c) 30 GeV/c. Other parameters as in Fig. 6.

Fig. 10: Distributions of the quantity dR_n^Z/dP_T^Z for $n=1$ (solid line) and $n=2$ (dashed line).

(a)

(b)

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5a

Fig. 5b

Fig. 6

Fig. 7

Fig. 8

Fig. 9

Fig. 10