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THE LOW ENERGY SUPERSYMMETRIC SPECTRUM

ACCORDING TO $N = 1$ SUPERGRAVITY GUTs

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A B S T R A C T

We reexamine the radiative $SU(2) \times U(1)$ breaking induced by broken $N = 1$ supergravity in the light of recent experimental results. Explicit formulae are given for the supersymmetric particle spectrum and the $SU(2) \times U(1)$ breaking condition. The experimental results (t-quark mass) constrain the supergravity parameters, particularly the bare Higgsino mass μ_0 and the "mixing" Higgs mass term μ^2 . Models with globally supersymmetric "observable" sectors (except for gaugino masses) are excluded in their simplest version. If the identity $B = A-1$ is assumed, further restrictions on the range of the supergravity parameters are found. The particle spectra leading to possible supersymmetric interpretations of the CERN missing p_T events are analyzed. We study in detail the following three scenarios: 1) $m_{\tilde{q}} \approx 80-140$ GeV, $M_{\tilde{g}} = 3$ GeV; 2) $m_{\tilde{q}} \approx M_{\tilde{g}} \approx 40$ GeV; 3) $M_{\tilde{g}} = 40$ GeV, $m_{\tilde{q}} \gg M_{\tilde{g}}$ and give the corresponding spectra of particles as a function of the A parameter.

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1. - INTRODUCTION

In the last couple of years, a great deal of effort has been dedicated to study grand unified theories (GUTs) coupled to broken $N = 1$ supergravity¹⁾. It has been shown that the breaking of $N = 1$ local supersymmetry may induce radiatively the breaking of the $SU(2) \times U(1)$ symmetry for a wide range of parameters²⁾⁻⁵⁾. In the beginning, it seemed, however, that one could only break the weak symmetry properly for a t-quark mass $m_t \gtrsim 60 \text{ GeV}$ ²⁾. Soon it was realized³⁾⁻⁵⁾ that one can easily obtain $SU(2) \times U(1)$ breaking even with arbitrarily light t-quarks. In particular, two of the authors presented a systematic study³⁾ of the constraints imposed on the free parameters of the supergravity models (including different m_t values) by requiring the appropriate Weinberg-Salam breaking at the M_W scale. Since then the UA1 collaboration has reported⁶⁾ some W decay events consistent with the process $W \rightarrow tb$ with $30 \lesssim m_t \lesssim 50 \text{ GeV}$. If $m_t \approx 40 \text{ GeV}$, there is one parameter less in the low energy supergravity models and one can find further constraints amongst the supergravity parameters. In this paper we study in detail these constraints and present supersymmetric spectra consistent with them. The plan of the paper is as follows. In Section 2 we fix the notation and conventions for the class of low energy supergravity models considered in the text. In Section 3, we give explicit (analytic) formulae for the evolution of the different masses and couplings in the approximation in which only the Yukawa coupling of the t-quark is considered. We present explicitly the dependence of these masses and couplings on the free parameters of the model at the GUT scale: m, M, μ_0, A, B . In this section we do not fix still $m_t \approx 40 \text{ GeV}$ but let the value of h_t free so that it can be fixed appropriately when a more precise value for m_t will be measured. In Section 4 we present numerical formulae for the masses of the s-particles as well as for the Higgs parameters after fixing m_t to the preferred experimental value $m_t \approx 40 \text{ GeV}$. We also study the restrictions obtained amongst the different parameters (m, M, μ_0, A, B) in order to be consistent with a stable radiative $SU(2) \times U(1)$ breaking. We consider general boundary conditions at the GUT scale as well as the case in which the relationship $B = A^{-1}$ holds.

Section 5 deals with the low energy supersymmetric spectrum consistent with the constraints presented in previous sections. We present spectra with squark and gluino masses which could be of interest in trying to give a supersymmetric explanation to the "monojet" events recently observed at the CERN collider⁷⁾. In this case (and assuming for simplicity $B = A^{-1}$) the possible spectra are

parametrized by a single parameter A . General features of the possible spectra are also discussed. After a short digression in Chapter 6 about some non-standard low energy supergravity scenarios, we present the final comments and conclusions in Section 7.

2. - LOW ENERGY SUPERGRAVITY: SOME NOTATIONS

We will closely follow (unless otherwise stated) the notation of Ref. 3) and will refer to formulae in that article by [IL-number]. Unexplained notations should also be clarified in this paper. We take as our low energy superpotential:

$$W = \varepsilon_{ij} h_t Q_i U H_{2j} + \mu \varepsilon_{ij} H_{1i} H_{2j} \quad ; \quad \mathcal{L}_{CH} = \int d\theta^2 W + h.c. \quad (1)$$

where only the part corresponding to the t -quark Yukawa is shown since, as we will discuss below, it is the only one relevant for the renormalization group equations. The breaking of $N = 1$ supergravity originates soft terms in the scalar potential:

$$V_{\text{soft}} = \sum_{i = \text{scalars}} m^2 |\phi_i|^2 + m h_t A_t \varepsilon_{ij} \tilde{Q}_i \tilde{U} H_{2j} + m \mu B \varepsilon_{ij} H_{1i} H_{2j} + h.c. \quad (2)$$

In the case of "minimal" $N = 1$ supergravity one has¹⁾, at the Planck scale, $B = (A-1)$, $A_t = A$ and $m = \text{gravitino mass}$. In the case of a general Kahler manifold, this will not be in general true. Notice that with this notation the relevant trilinear scalar terms in the potential will be:

$$-h_t \mu H_{1i} \tilde{Q}_i \tilde{U} + h_t m A_t \varepsilon_{ij} \tilde{Q}_i H_{2j} \tilde{U} + h.c. \quad (3)$$

There will also be gaugino and Higgsino mass terms (in Weyl notation):

$$\mathcal{L}_{FM} = -\mu \varepsilon_{ij} \tilde{H}_i \tilde{H}_j - M \lambda^a \lambda_a + h.c. \quad (4)$$

where a is a gauge index. The relevant Higgs scalar potential along the neutral direction is:

$$V_H = \frac{1}{8} (g^2 + g'^2) (|H_1|^2 - |H_2|^2)^2 + \mu_1^2 |H_1|^2 + \mu_2^2 |H_2|^2 - \mu_3^2 (H_1 H_2 + h.c.) \quad (5)$$

Notice that in the case of minimal supergravity one has, at the GUT scale:

$$\mu_3^2(0) = (1-A)\mu m \quad (6)$$

The potential is minimized for⁸⁾:

$$v^2 = v_1^2 + v_2^2 = 2 \frac{(\mu_1^2 - \mu_2^2 - (\mu_1^2 + \mu_2^2) \cos 2\theta)}{(g^2 + g'^2) \cos 2\theta} \quad (7)$$

where³⁾

$$\cos 2\theta \equiv (\omega^2 - 1)/(\omega^2 + 1) \quad ; \quad \sin 2\theta = 2\omega/(\omega^2 + 1) = 2\mu_3^2/(\mu_1^2 + \mu_2^2) \quad (8)$$

and $\omega \equiv v_2/v_1 = \cot \theta$. In principle, μ_3^2 may be both positive and negative (implying opposite signs for v_1 and v_2) and this may have some relevance in the "minimal" case in which the sign of μ_3^2 is related to the sign and size of A . Also M , μ and A may be both positive and negative, the correct sign on the fermion mass terms must only be imposed on the physical eigenstates. This is eventually possible through a redefinition of the fields.

In order to obtain the $SU(2) \times U(1)$ breaking at the right scale, one must impose $v^2 = 2M_W^2/g^2$ which can be written in terms of μ_1^2 , μ_2^2 and ω as follows³⁾:

$$(\mu_1^2 - \omega^2 \mu_2^2)/(\omega^2 - 1) = \frac{1}{2} M_Z^2 \quad (9)$$

Notice that this condition of symmetry breaking does not depend on the sign of ω (or $\sin 2\theta$) and hence applies to both positive and negative μ_3^2 .

A detailed numerical analysis of the condition (9) on the parameters A , m , M , m_t and $\mu_3^2(0)$ was presented in Ref. 3). We considered in that reference a negligible μ parameter (but did not neglect the bottom Yukawa coupling). Since then the UA1 collaboration has presented⁶⁾ evidence for the existence of a t -quark with a mass $30 \lesssim m_t \lesssim 50$ GeV, 40 GeV being apparently the favoured value. We analyze in this paper the further restrictions imposed by setting $m_t \approx 40$ GeV on the supergravity parameters. We also include a non-vanishing μ parameter in the equations.

3. - EVOLUTION OF THE COUPLINGS AND PARAMETERS FOR THE CASE OF A "LOW MASS" t-QUARK

As pointed out in Ref. 3), a low mass ($\lesssim 60$ GeV) t-quark necessarily means that at M_W one has $|\omega| \approx 1$ and hence $|v_1| \sim |v_2|$. In this situation, $h_b/h_t \approx m_b/m_t$ and one can neglect the effect of the h_b coupling in the renormalization group equations. This simplifies the formulae quite a lot and one can give some analytic expressions for the evolution of couplings and parameters. This we present in this section. Some other formulae were already given in Ref. 3). In Appendix I, we show for completeness the relevant renormalization group equations^{9),2)} in the case of a non-negligible μ .

Let us start with the t-quark Yukawa coupling. One has for the evolution of $Y_t = h_t^2/(4\pi)^2$:

$$Y_t(t) = Y_t(0) E(t) / (1 + 6 Y_t(0) F(t)) \quad ; \quad t = 2 \log(M_x/Q) \quad (10)$$

where $F(t) = \int_0^t E(t) dt$ and $E(t)$ is shown in Appendix II. Concerning the trilinear scalar coupling which corresponds to the t-quark Yukawa, one finds:

$$A_t(t) = \frac{A}{(1 + 6 Y_0 F)} + \frac{M}{m} \left(H_2 - \frac{6 Y_0 H_3}{(1 + 6 Y_0 F)} \right) \quad (11)$$

where $Y_0 \equiv Y_t(0)$ and $H_{2,3}$ are given in (II.4,5). For the evolution of the μ parameter, one has:

$$\mu^2(t) = \frac{\mu_0^2}{(1 + 6 Y_0 F)^{1/2}} (1 + \beta_2 t)^{3/b_2} (1 + \beta_1 t)^{1/b_1} \equiv \mu_0^2 \tilde{q}^2(t) \quad (12)$$

The inclusion of a non-vanishing μ in the equations for the evolution of scalar masses leads to few changes. One has just to do the replacements $\mu^2 \rightarrow (\mu^2 - \mu^2)$ in Eqs. (II-38,44). Formulae (II-39) are still valid. Only the masses $m_Q^{1,2}$ and m_U of the third generation and μ_2^2 do depend on Yukawa couplings and it is enough to consider the evolution of one of them (e.g., μ_2^2) since the other masses are related to it

$$m_Q^2(t) = \frac{1}{2} m_0^2 - \frac{1}{2} m_4^2 + \frac{1}{2} m_U^2 \quad (13a)$$

$$m_U^2(t) = \frac{2}{3} \mu_2^2 - \frac{2}{3} \mu^2 - m_5^2 \quad (13b)$$

where m_4^2 and m_5^2 are given in (II-39a,b). The evolution of the $(\text{mass})^2$ of the

other scalars in the theory can be found in (IL-C). One can integrate the evolution equation for $\mu_2^2(t)$ to obtain:

$$\mu_2^2(t) = \mu_0^2 l(t) + M^2 e(t) + A m M f(t) + m^2 (h(t) - k(t) A^2) \quad (14)$$

where the explicit dependence on the soft parameters M , A , m and μ_0 at the GUT scale is shown. The corresponding coefficients are as follows:

$$\begin{aligned} l(t) &\equiv q^2(t) \quad ; \quad h(t) \equiv (3/D(t) - 1)/2 \quad ; \quad k(t) = 3Y_0 F/D(t)^2 \\ f(t) &\equiv -6Y_0 H_3/D(t)^2 \quad ; \quad D(t) \equiv (1 + 6Y_0 F) \\ e(t) &\equiv \frac{3}{2} \left[(G_1 + Y_0 G_2)/D(t) + (H_2 + 6Y_0 H_4)^2/3D(t)^2 + H_8 \right] \end{aligned} \quad (15)$$

where the functions H_2 , H_3 , H_4 , G_1 , G_2 and H_8 are given in Appendix II and do not depend on Y_0 but just on gauge coupling constants. Thus Eqs. (14) and (15) also show us the explicit dependence on the t -quark Yukawa coupling. We prefer to present here the results corresponding to arbitrary h_t since the precise experimental value for m_t is still not available. Moreover, the same formulae could be used in the case of the existence of a heavier fourth generation of fermions. Numerical values for all the functions in Eqs. (13)-(15) for the case $m_t \approx 40$ GeV will be given below. It is convenient to recall here the evolution equation for $\mu_1^2(t)$. It is given in (I.1) and one trivially gets, after integration:

$$\mu_1^2(t) = m^2 + \mu_0^2 l(t) + M^2 g(t) \quad (16)$$

where $g(t) = (3/2 \tilde{\alpha}_2(0)f_2(t) + \frac{1}{2}\tilde{\alpha}_1(0)f_1(t))$. Obviously one can check that $e(t) \rightarrow g(t)$ when $Y_0 \rightarrow 0$. The only other parameter whose evolution equation is of interest to us is μ_3^2 . One can integrate its evolution equation to obtain:

$$\mu_3^2(t) = q(t) \mu_3^2(0) + r(t) \mu_0 M + s(t) A m \mu_0 \quad (17)$$

where $q(t)$ was already defined and

$$s(t) \equiv 3q(t)Y_0 F/D(t) \quad ; \quad r(t) \equiv q(t) (3Y_0 H_3/D(t) - H_7) \quad (18)$$

H_7 , which does not depend on Y_0 is given in (II.9). Thus formulae (17) and (18) give us the explicit dependence of $\mu_3^2(t)$ on the parameters $\mu_3^2(0)$, μ_0 , M , m and A the GUT scale as well as on Y_0 .

The formulae obtained for $\mu_2^2(t)$ and $\mu_1^2(t)$ allow us to impose the $SU(2) \times U(1)$

breaking condition (9) which can also be rewritten as

$$\omega^2 = \frac{\mu_1^2 + \frac{1}{2} M_2^2}{\mu_2^2 + \frac{1}{2} M_2^2} = \frac{m^2 + \mu_0^2 l + M^2 g + \frac{1}{2} M_2^2}{m^2(h - \kappa A^2) + \mu_0^2 l + M^2 e + A m M f + \frac{1}{2} M_2^2} \quad (19)$$

implying a constraint amongst the parameters ω , m , μ_0 , M , A (for each given Y_0 , i.e., top quark mass). The appropriate $SU(2) \times U(1)$ breaking depends upon the ratios of the mass parameters at the GUT scale, M/m , μ_0/m as well as M_Z/m , as shown in Eq. (19). But, in fact, the dependence on M_Z/m is so tiny that for all possible values of m , we can neglect it, the effective resulting parameters being the mass ratios and, of course, A and B . With Eq. (19), one can reproduce, e.g., Figs. 4 and 5 of Ref. 3) (except for small effects due to a non-vanishing h_b). Notice that if m_t is ~ 40 GeV [implying $h_t(0) \approx 0.08$] μ_1^2 and μ_2^2 will be very similar and hence, from Eq. (19), $\omega \approx 1$. Thus we will approximately have $|\sin 2\theta| \approx 1$ at the M_W scale. From Eqs. (14)-(17) one can write:

$$\sin 2\theta(t) = \frac{2(g\mu_0^2(0) + r\mu_0 M + s\mu_0 A m)}{m^2(1+h-\kappa A^2) + 2\mu_0^2 l + M^2(g+e) + A m M f} \quad (20)$$

and hence, imposing $|\sin 2\theta(M_W)| \approx 1$ will give us a good approximation to the constraints amongst $\mu_3^2(0)$, μ_0 , M , m and A . Of course, $|\sin 2\theta| = 1$ is only obtained in the $h_t \rightarrow 0$ limit. A noteworthy feature of the above equation is that it is invariant under the simultaneous replacement $\mu_0 \rightarrow -\mu_0$, $M \rightarrow -M$ and $A \rightarrow -A$ implying that the $SU(2) \times U(1)$ breaking condition ($|\sin 2\theta(M_W)| \approx 1$) is invariant under that symmetry.

As was already noticed in Ref. 3), a value $\sin 2\theta(M_W) \approx 1$ is only obtainable if at the GUT scale one has $0.6 \lesssim \sin 2\theta < 1$. One can also see that in order to get $\sin 2\theta(M_W) \approx -1$ (which is also a possible condition of symmetry breaking) one needs to have, at the GUT scale, $-1 < \sin 2\theta < -0.6$. Thus, in order to have $\sin 2\theta(M_W) \approx -1$, one needs to start from a negative $\mu_3^2(0)$. All this is especially relevant in the case of "minimal" $N = 1$ supergravity in which case one can put $\mu_3^2(0) = (1-A)\mu_0 m$. Then one has the following possible situations:

$$\left. \begin{array}{l} \text{a) } \mu_0 > 0, \quad A < 1 \\ \text{b) } \mu_0 < 0, \quad A > 1 \end{array} \right\} \mu_3^2(0) > 0$$

$$\left. \begin{array}{l} \text{c) } \mu_0 > 0, A > 1 \\ \text{d) } \mu_0 < 0, A < 1 \end{array} \right\} \mu_3^2(0) < 0 \quad (21)$$

Thus, in cases a) and b), one will get $SU(2) \times U(1)$ breaking by imposing $\sin 2\theta(M_W) \approx 1$ and in cases c) and d) it will only be possible if $\sin 2\theta(M_W) \approx -1$. However, notice that Eq. (20) is invariant under the simultaneous replacements (in the minimal case!) $\sin 2\theta(M_W) \rightarrow -\sin 2\theta(M_W)$ and $\mu_0 \rightarrow -\mu_0$ and hence the $SU(2) \times U(1)$ breaking constraint in cases a)-d) and b)-c) is the same. Thus we will only study the SUSY breaking conditions for $\mu_0 > 0$.

Before showing numerical formulae for all the above equations, let us first write down some relations for some of the masses of the model. Let us start with the Higgs sector. There are three neutral Higgses with masses⁸⁾:

$$m_c^2 = \mu_1^2 + \mu_2^2 \quad (22)$$

$$m_{a,b}^2 = \frac{1}{2} \left(m_c^2 + M_z^2 \pm \sqrt{(m_c^2 + M_z^2)^2 - 4 m_c^2 M_z^2 \cos^2 2\theta} \right) \quad (23)$$

and a charged Higgs with

$$m_{H^\pm}^2 = M_w^2 + m_c^2 \quad (24)$$

From previous equations, one gets:

$$m_c^2 = m^2(1+h - \kappa A^2) + M^2(e+g) + 2l\mu_0^2 + f m M A \quad (25)$$

In the limit $|\sin 2\theta| \approx 1$ one gets to first order in $\cos^2 2\theta$:

$$m_b^2 \approx \frac{m_c^2 M_z^2}{m_c^2 + M_z^2} \cos^2 2\theta = (M_z \cos 2\theta)^2 \left(\frac{1}{1 + M_z^2/m_c^2} \right) \quad (26)$$

and finally

$$m_a^2 \approx M_z^2 + m_c^2 - m_b^2 \quad (27)$$

Notice that the Higgs H_b mass is very small since for the case at hand, $|\omega| \approx 1$ and $\cos 2\theta = (\omega^2 - 1)/(\omega^2 + 1) \rightarrow 0$. Thus with a t-quark mass as low as ~ 40 GeV there does necessarily exist one very light neutral Higgs^{5),10)}. The other two

are quite heavier, $m_c^2 \gtrsim 2m^2$, $m_a^2 \gtrsim M_Z^2 + 2m^2$ and the same is true for the charged Higgs, $m_H^2 \gtrsim M_W^2 + 2m^2$.

Concerning the masses of squarks and sleptons, all of them (except for the t-squarks) have simple expressions depending only on m and M . Those relations are given in (IL-C) and numerical formulae will be given below. The masses for the t-squarks have a more complicated expression because the "off-diagonal" mass terms ($\tilde{t}_L \tilde{t}_R$ mixing terms) are non-negligible. The t-squark mass matrix is¹⁾:

$$\begin{pmatrix} \tilde{t}_L^* & \tilde{t}_R^* \end{pmatrix} \begin{pmatrix} m_{\tilde{t}_L}^2 & -m_t(A_t m + \mu/\omega) \\ -m_t(A_t m + \mu/\omega) & m_{\tilde{t}_R}^2 \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix} \quad (28)$$

where $m_{\tilde{t}_L}^2 \approx m_Q^2 + m_t^2$, $m_{\tilde{t}_R}^2 \approx m_U^2 + m_t^2$ since $\cos 2\theta \approx 0$. The physical eigenvalues corresponding to this matrix are:

$$m_{\tilde{t}_{h,l}}^2 = m_t^2 + \frac{1}{2} \left\{ (m_Q^2 + m_U^2) \pm \left[(m_Q^2 - m_U^2)^2 + 4(A_t m + \mu/\omega)^2 m_t^2 \right]^{1/2} \right\} \quad (29)$$

Notice that due to the off-diagonal term one of the eigenvalues may be lighter¹¹⁾ than the t-quark for A_t and/or μ large enough^{*}). We will comment on this possibility later on.

Let us now turn to gauginos and Higgsinos. It is well known that the charged winos combine with the charged Higgsinos to form a couple of Dirac particles with masses:

$$M_{h,l}^2 = \frac{1}{2} \left\{ M_2^2 + \mu^2 + 2M_W^2 \pm \left[(M_2^2 - \mu^2)^2 + 4M_W^4 \cos^2 2\theta + 4M_W^2 (M_2^2 + \mu^2 + 2M\mu \sin 2\theta) \right]^{1/2} \right\} \quad (30)$$

This simplifies somewhat in the case at hand, in which $\cos^2 2\theta \approx 0$ and $|\sin 2\theta| \approx 1$. As in the case of the s-tops, there is one eigenvalue which may be light if $M(\mu) \rightarrow \infty$ with $\mu(M)$ fixed. We will comment on examples below. Finally, there are the neutral gauginos and Higgsinos which have a 4×4 mass matrix. We thus cannot give a simple expression for the physical eigenvalues but will calculate them numerically when required.

^{*}) Notice also that in this respect the relative sign of A_t and μ/ω is important.

Notice that the situations [a),d) and b),c)] discussed in Eq. (21) not only have the same numerical $SU(2) \times U(1)$ breaking condition but lead to the same "stop", "chargino" and "neutralino" spectra. In the case of the t-squark this is because going from a)-d) [or b)-c)] one changes the sign of both μ and ω and formula (29) remains unchanged. The same happens in the chargino case with μ and $\sin 2\theta$.

4. - SOME NUMERICAL RESULTS

Let us now assume that $m_t \approx 40$ GeV in the previous equations and see what restrictions we get on the spectrum and on the supergravity parameters. In fact, since the precise value for m_t is not yet known we are going to assume instead a fixed $h_t(0) = 0.08$ which gives rise to t-quark masses around 37~41 GeV corresponding to $v_2 \approx v/\sqrt{2}$. This is more simple from the computational point of view. The values of the functions introduced in the previous chapter for $h_t(0) = 0.08$ and $t = 2\ln(M_X/M_W) \approx 67$ are shown in Table 1. We get the following numerical formulae.

a) Squark and slepton masses

For all $\tilde{q}$'s and $\tilde{l}$'s (except t-squarks), one obtains from Eqs. (IL-C1):

$$m_{\tilde{u}_L}^2 = m^2 + 7.6 M^2 - 0.35 M_z^2 \cos 2\theta \quad (31a)$$

$$m_{\tilde{d}_L}^2 = m^2 + 7.6 M^2 + 0.42 M_z^2 \cos 2\theta \quad (31b)$$

$$m_{\tilde{u}_R}^2 = m^2 + 7.17 M^2 - 0.15 M_z^2 \cos 2\theta \quad (31c)$$

$$m_{\tilde{d}_R}^2 = m^2 + 7.14 M^2 + 0.07 M_z^2 \cos 2\theta \quad (31d)$$

$$m_{\tilde{e}_R}^2 = m^2 + 0.15 M^2 + 0.23 M_z^2 \cos 2\theta \quad (31e)$$

$$m_{\tilde{e}_L}^2 = m^2 + 0.53 M^2 + 0.27 M_z^2 \cos 2\theta \quad (31f)$$

$$m_{\tilde{\nu}}^2 = m^2 + 0.53 M^2 - 0.5 M_z^2 \cos 2\theta \quad (31g)$$

where we remind the reader that $\cos 2\theta \rightarrow 0$ in the case $m_t \approx 40$ GeV (one numerical-

ly finds $\cos 2\theta \approx 0.0-0.18$ for wide ranges of parameters). To obtain these numbers, we have taken

$$\alpha_c = 1/24 \quad ; \quad M_x = 2.8 \cdot 10^{16} \quad ; \quad \sin^2 \theta_w(M_w) = 0.23 \quad (32)$$

corresponding to $\alpha_3(M_w) = 1/8$. One has to add to Eqs. (31) the $(\text{mass})^2$ of the corresponding fermionic partner if it is relevant (i.e., m_c^2, m_t^2, m_b^2). Also, these are the values of the masses at the M_w scale, if $m_{q,\tilde{q}}^2 < M_w^2$ one has to let them evolve further till $m_{q,\tilde{q}}^2(Q^2 = m_{q,\tilde{q}}^2) = m_{q,\tilde{q}}^2$ which corresponds to the physical pole in the propagator. On the contrary, if $m_{q,\tilde{q}}^2(M_w) > M_w^2$ one has to stop the "running" before it reaches M_w . However, this subtlety does not have in general a great numerical influence. Thus, for example, in a case in which $m_{q,\tilde{q}}^2(M_w) \approx (40 \text{ GeV})^2$, the effect of ignoring the further evolution of $m_{q,\tilde{q}}^2$ leads to an underestimation of the physical $m_{q,\tilde{q}}$ mass by at most $\sim 15\%$. However, we already have this order of uncertainty by neglecting second-order corrections in the general renormalization group equations. In the case of sleptons, this effect is negligible.

b) Masses of t-squarks

These are given by Eq. (29). Thus we need to know the values of m_Q^2, m_U^2, A_t and μ at the weak scale. One finds for $h_t(0) = 0.08$ the following numerical formulae:

$$A_t(M_w) = 0.93 A + \frac{M}{m} 4.02 \quad (33)$$

$$\mu(M_w) = 1.38 \mu_0 \quad (34)$$

$$m_U^2 = (0.93 - 0.02 A^2) m^2 - 0.1 m M A + 6.86 M^2 \quad (35)$$

$$m_Q^2 = (0.97 - 0.01 A^2) m^2 - 0.05 m M A + 7.45 M^2 \quad (36)$$

In the $\cos 2\theta \rightarrow 0$ limit [and also $(m_Q^2 - m_U^2)/(m_t m A_t + \mu m_t/\omega) \rightarrow 0$], a good approximate formula is given by

$$m_{\tilde{t},h}^2 \simeq m_t^2 + m^2(0.95 - 0.015 A^2) + 7.2 M^2 - 0.07 m M A \mp [0.94 m A + 4.0 M + \eta 1.38 \mu_0] \quad (37)$$

where $\eta = 1(-1)$ for $\mu_3^2 > 0(<0)$.

c) Higgs parameters and masses

Numerical formulae for the mass parameters in the Higgs potential are given by:

$$\mu_1^2 = m^2 + 1.9 \mu_0^2 + 0.53 M^2 \quad (37)$$

$$\mu_2^2 = m^2(0.9 - 0.03 A^2) + 1.9 \mu_0^2 - 0.15 m M A + 0.08 M^2 \quad (38)$$

$$\mu_3^2 = 1.38 \mu_3^2(0) - 0.72 \mu_0 M + 0.05 A m \mu_0 \quad (39)$$

With these formulae, one obtains, for the neutral H_c Higgs field

$$m_c^2 = m^2(1.9 - 0.03 A^2) + 0.61 M^2 + 3.8 \mu_0^2 - 0.15 m M A \quad (40a)$$

and the rest of the Higgs masses can be calculated from m_c^2 using Eqs. (23), (24) and (25). In particular, one gets for the lightest Higgs H_b the following approximate formula

$$m_{H_b}^2 \simeq \left(\frac{M_z}{20} \right)^2 \frac{(1 + 0.53 Y^2 + 1.9 X^2)(1 + 4.5 Y^2 + 0.3 A^2 + 1.5 Y A)^2}{(0.95 + 0.3 Y^2 + 1.9 X^2 - 0.015 A^2 - 0.075 Y A + M_z^2/2m^2)^3} \quad (40b)$$

d) Gaugino-Higgsino masses

As we discussed in the previous chapter, the charged winos combine with the charged Higgsinos to form two Dirac particles. In the approximation $\cos 2\theta \rightarrow 0$ one gets eigenvalues:

$$M_{h,e}^2 = \frac{1}{2} M_w^2 \left\{ \hat{M}^2 + \hat{\mu}^2 + 2 \pm \sqrt{(\hat{M}^2 - \hat{\mu}^2)^2 + 4(\hat{M} + \eta \hat{\mu})^2} \right\} \quad (41)$$

where $\hat{M} = 0.81(M/M_W)$, $\hat{\mu} = 1.38(\mu_0/M_W)$ and $\eta = +1(-1)$ for $\sin 2\theta > 0 (< 0)$. This approximation with $\cos 2\theta \rightarrow 0$ is good up to 5% for not too large M or μ_0 . Let us remark, however, that the particle spectra shown later on are calculated with the exact formula Eq. (30).

The result for the gluino mass is trivial:

$$M_{\tilde{g}} = \frac{\alpha_3(t)}{\alpha_G} M \quad (42)$$

This gives $M_{\tilde{g}}(M_W) \approx 3M$, $M_{\tilde{g}}(10 \text{ GeV}) \approx 3.4 M$. Concerning the neutral gaugino and Higgsino spectra we have to calculate it numerically. However, in some interesting limits ($M \rightarrow 0$) there is an approximate photino eigenstate with mass

$$M_{\tilde{\gamma}} \approx \frac{\alpha_e(M_W)}{\alpha_e(M_*)} M \approx \frac{8}{3} \frac{\alpha_e}{\alpha_G} M \approx 0.5 M \quad (43)$$

so that typically we will have $M_{\tilde{g}}/M_{\tilde{\gamma}} \approx 6-7$. This constraint is important when trying to obtain phenomenological predictions. In this same limit and if also μ is small compared to M_W there is a Majorana Higgsino eigenstate with mass:

$$M_{\tilde{H}^0} \approx \mu \approx 1.38 \mu_0 \quad (44)$$

and a Dirac zino-Higgsino eigenstate with mass $\sim M_Z$. Notice, however, that all these statements are not true for general values of the parameters M and μ_0 .

Let us consider now the condition for $SU(2) \times U(1)$ breaking. This is given by Eq. (19) after substituting Eqs. (37) and (38). We have studied numerically that condition as well as the possible supersymmetric spectra compatible with it. We will show these results later on. In all the wide range of the parameters ($\mu_0^2(0)$, μ_0, A, m, M) studied one can only get $SU(2) \times U(1)$ breaking for $0.98 \lesssim |\sin 2\theta(M_W)| < 1$ so that in fact one can approximately use for the $SU(2) \times U(1)$ breaking condition

$$|\sin 2\theta(M_W)| \approx 1 \quad (45)$$

instead of Eq. (19). Notice that this condition is the same as the one imposed in dimensional transmutation schemes so that numerically there will be practically no difference between our "tree level" approach and the one pursued in Ref. 5). Numerically condition (45) reads:

$$\sin 2\theta(M_W) = \pm 1 = \frac{2.76 \mu_3^2(0) - 1.44 \mu_0 M + 0.09 A m \mu_0}{m^2(1.9 - 0.03 A^2) + 0.6 M^2 + 3.8 \mu_0^2 - 0.15 m M A} \quad (46)$$

which can be rewritten as a scale-independent formula for $\sin 2\theta$ at the GUT scale:

$$\sin 2\theta(M_X) = \frac{\pm 1}{(1+x^2)} \left[0.69 - 0.01 A^2 + 1.38 x^2 + 0.22 y^2 \mp 0.03 A x - 0.05 A y \pm \frac{0.52 x y}{(47)} \right]$$

where $x \equiv \mu_0/m$ and $y \equiv M/m$. The $+(-)$ signs stand for the two possibilities $\sin 2\theta(M_W) = \pm 1$.

We plot condition (47) in Figs. 1a,b,c in which the lines corresponding to fixed $\sin 2\theta(M_X)$ are drawn in the A - X plane for several values of Y ($Y = -\frac{1}{2}, 0, \frac{1}{2}$). Notice that this plot is general and does not assume minimal kinetic terms for the scalars. These figures show us what range of the parameters A , x , y and $\sin 2\theta(M_X)$ is allowed by the conditions of symmetry breaking + stability of the minimum of the scalar potential. There is an allowed domain bounded by forbidden regions^{*} corresponding to $\sin 2\theta(M_X) > 1$ (scalar potential unbounded below, leading to $M_W \sim M_P$) and the conditions^(5),2):

$$A_t^2 m^2 \leq 3 (m_a^2 + m_u^2 + \mu_z^2) \quad (48)$$

$$A^2 \leq 3 (3 + x^2) \quad (49)$$

which forbid that charge + colour-breaking absolute minima appear. The second bound is just like the first one but taken at the GUT scale (the appearance of these undesired minima must be forbidden at any mass scale). Of course, it could well be that we live on an unstable local (charge+colour conserving) minimum¹²⁾ which will eventually decay into one of these undesired minima but we find this possibility contrived.

^{*}) In fact, the condition $|\sin 2\theta(t)| < 1$ must be fulfilled for any t and not only for $t=0$. Usually, $|\sin 2\theta(0)| < 1$ implies $|\sin 2\theta(t)| < 1$ except for values of $|\sin 2\theta(0)|$ very close to 1 in which case one may obtain values of $|\sin 2\theta(t)|$ slightly larger than 1 at intermediate t . Thus the real stability bound from $\sin 2\theta$ is slightly more stringent than the one shown in Figs 1-3 although numerically practically identical. We acknowledge discussions on this point with A. Bouquet and J. Kaplan.

One draws several interesting conclusions from these plots.

- i) The value of $\sin 2\theta(M_X)$ is forced to be in the range:

$$0.6 \lesssim |\sin 2\theta(M_X)| \lesssim 1 \quad (50)$$

which, in terms of the definition of this angle, means:

$$\mu_3^2(0) \simeq (0.6 - 1) (m^2 + \mu_0^2) \quad (51)$$

One can check that this is true for arbitrarily large values of M . Restriction (51) was already remarked in Ref. 3) and it is a rather puzzling result since it forces two independent parameters of the theory like μ_3^2 and (essentially) m^2 to be quite similar in size. This is surprising since μ_0 is just a free parameter in the superpotential whereas m is related to the supersymmetry breaking scale. The bound (50) rests entirely on the assumption of a relatively light ($\lesssim 60$ GeV) t -quark. In the case of a heavy t -quark $\mu_3^2(0)$ and m^2 may be widely different with no problem to get the appropriate $SU(2) \times U(1)$ breaking.

- ii) The bare Higgsino mass μ_0 cannot be arbitrarily large for a fixed $y = M/m$. Thus one gets, e.g.,

$$\begin{aligned} 0 &\leq \mu_0 \leq 0.8 m & (M/m = 0.5) \\ 0 &\leq \mu_0 \leq 1.2 m & (M = 0) \\ 0 &\leq \mu_0 \leq 1.5 m & (M/m = -0.5) \end{aligned} \quad (52)$$

so that the physical Higgsino mass in supersymmetric models cannot be made arbitrarily large (e.g., in order to get light charginos). For example, in models with $m_q \simeq m_g \simeq 40$ GeV (having $m \simeq 20$ GeV) one will have $\mu_0 \lesssim 40$ GeV whereas in models with $M \simeq 0$ one has just $\mu_0 \lesssim m_q$. Another feature shown in Fig. 1 is that the larger M/m is, the smaller μ_0/m has to be. This is due to the $\mu_0 M$ term of the evolution formula for $\mu_3^2(t)$ [Eq. (39)]. The smaller M is, the faster μ_3^2 increases at low energies. This allows $\sin 2\theta(M_X)$ to start at M_X from smaller values, the fast growing of μ_3^2 at low energies giving rise to the desired $\sin 2\theta(M_W) \simeq 1$. Starting at M_X from smaller values for $\sin 2\theta$ implies in general the possibility of having larger μ_0 's [since $\sin 2\theta(M_X) \sim 1/(m^2 + \mu_0^2)$]. This is the origin of the correlation between the allowed values of μ_0 and M .

In Fig. 1, the line corresponding to the minimal case [i.e., $B = A-1$,

$\sin 2\theta(M_X) = (1-A)x/(1+x^2)$ is also shown. Notice that one always has $\mu_0 \gtrsim 0.17m$. We will comment below in more detail about the minimal case.

In Fig. 1, we have only considered the case in which $\sin 2\theta(M_W) \approx +1$ (not -1) and also $\mu_0 > 0$. However, one may easily obtain the figures corresponding to the other cases using the symmetries of Eq. (46). Thus, for $\sin 2\theta(M_W) \approx -1$ one finds:

$$\begin{aligned} 0 \lesssim \mu_0 &\lesssim 1.5 m & (M/m = 0.5) \\ 0 \lesssim \mu_0 &\lesssim 1.2 m & (M/m = 0) \\ 0 \lesssim \mu_0 &\lesssim 0.8 m & (M/m = -1/2) \end{aligned}$$

The results for $\mu_0 < 0$ are equal to the ones for $\mu_0 > 0$ but with opposite A and y .

Let us now assume the minimal coupling of chiral superfields to $N = 1$ supergravity in which we have "canonical" kinetic terms for the scalars in the theory. One has^{1),13)} in this case $B = A-1$ leading to

$$\sin 2\theta(M_X) = (1-A)x/(1+x^2) \quad (53)$$

We consider this minimal case for two reasons. First, it is the simplest Kahler manifold one can think of. Second it is likely that a proportionality of the type $\mu_3^2(0) \sim A\mu_0 m$ may be present in more complicated situations, in particular the dependence on μ_0 and m will probably be quite universal. Thus, assuming Eq. (53) may give us an idea of the type of constraint, one would get from more general versions of couplings to $N = 1$ supergravity.

With the above assumption, we are then left with three parameters (A , x and y) related by the symmetry breaking condition. In Fig. 2, we plot how A , x and y should be related in this minimal case in order to get the appropriate $SU(2) \times U(1)$ breaking. We show the lines corresponding to fixed $y = M/m$ in the A - x plane. The lower lines correspond to the condition $\sin 2\theta(M_W) \approx 1$ and the upper ones to $\sin 2\theta(M_W) \approx -1$. The shaded area below (above) corresponds to the forbidden region $\sin 2\theta(M_X) > 1$ ($\sin 2\theta(M_X) < -1$). These lines correspond to equations $\pm 1 = (1-A)x/(1+x^2)$. Notice also that the lines of fixed y are sometimes cut by a short-barred line. This indicates the stability bounds for not getting charge-colour breaking minima from a large (A).

Let us comment on the numerical results for this minimal case starting by the $\sin 2\theta(M_W) \approx 1$ case. Several features are noteworthy:

- (i) the lines for fixed y have a similar shape as the boundary $\sin 2\theta(M_X) = 1$ line. This is simply a reflection of the fact that one always has values of $\sin 2\theta(M_X)$ not far away from the boundary ($0.6 \sim 1.0$);
- (ii) only values $A \lesssim -1$ are allowed. This is quite a strong restriction since, as shown in Fig. 1, A could in principle be any number between ≈ -3 and $+3$;
- (iii) the possible values for μ_0 are restricted. Thus one has, e.g.:

$$\begin{aligned}
 0.15 m &\lesssim \mu_0 \lesssim m & (y=0) \\
 0.17 m &\lesssim \mu_0 \lesssim 1.2 m & (y=-1/2) \\
 0.17 m &\lesssim \mu_0 \lesssim 1.5 m & (y=-1) \\
 0.22 m &\lesssim \mu_0 \lesssim 0.5 m & (y=1/2)
 \end{aligned}
 \tag{54}$$

- (iv) the values for M are also restricted. Thus, one can check that for $y \gtrsim 3/4$, the $\sin 2\theta(M_W) < 1$ bound is violated. As pointed out in Ref. 3) and contrary to statements in Ref. 5), the gaugino mass M may be arbitrarily small with no problem at all.

In the case $\sin 2\theta(M_W) \approx -1$, the parameters are even more restricted:

- (i) only values $2.6 \lesssim A \lesssim 3$ are allowed (for moderate y 's):
- (ii) the possible μ_0 values are restricted. One has, e.g.,

$$\begin{aligned}
 0.4 m &\lesssim \mu_0 \lesssim 1.2 m & (y=1/2) \\
 0.3 m &\lesssim \mu_0 \lesssim 1.6 m & (y=1) \\
 0.5 m &\lesssim \mu_0 \lesssim 0.9 m & (y=0)
 \end{aligned}
 \tag{55}$$

and, for example, the possibility $y \lesssim -1/2$ is already excluded.

Some of these features are more clearly seen in Fig. 3 in which we plot the

allowed values of the parameters $x = \mu_0/m$, $y = M/m$ and A in the x - y plane^{*)}. We insist that Figs. 2 and 3 assume $B = A-1$. Note that for $x \gtrsim 1$ gauginos cannot be massless. Another important point is that one may have large values of x , y and A , but only by letting them all increase simultaneously. Thus the usual assumption that $|A| \lesssim 3$ is only true for small values of x and y . In all the above cases we took $\mu_0 > 0$. However the results for $\mu_0 < 0$ are identical to these, as we stated in Chapter 3. All the above restrictions have their reflection on the allowed supersymmetric spectra as we will discuss in detail in the following section.

Before discussing the SUSY spectra, let us make some general comments on the relevance of the above restrictions on general $N = 1$ supergravity models. A first general statement is that any $N = 1$ sugra model (with minimal low energy content) in order to be consistent with a t -quark with $m_t \approx 40$ GeV needs to verify Eq. (50), i.e., $\mu_3^2(0) \approx (0.6-1)(m^2 + \mu_0^2)$ and thus $\mu_3^2(0)$ cannot be arbitrarily small. This is for example relevant for the recently proposed class of models¹⁴⁾ with an $SU(n,1)/SU(n) \times U(1)$ symmetry in the scalar sector of the theory. These models are characterized by having an "observable sector" which is globally supersymmetric (i.e., $m = A = B = 0$) except for gaugino mass terms. However, $B = 0$ implies $\mu_3^2(0) = 0$ violating condition (50). Starting with a vanishing $\mu_3^2(0)$ at M_x , radiative corrections cannot generate a μ_3^2 large enough to get $\sin 2\theta \approx 1$ at the M_W scale. One needs to complicate appropriately the model in order to obtain the appropriate $SU(2) \times U(1)$ breaking. Moreover, if one starts from $m = 0$ at the GUT scale, the SUSY spectrum will in general be very heavy. This is because experimentally we know that $m_{\chi_R}^2 \gtrsim (20 \text{ GeV})^2$. Using Eq. (31e), one then gets that $M \gtrsim 50 \text{ GeV}$, implying $m_{\tilde{g}} \gtrsim 150 \text{ GeV}$ and $m_{\tilde{q}} \gtrsim 140 \text{ GeV}$.

Another point to remark concerns the parameter A . As we stated above, in the case of minimal kinetic terms one has $B = A-1$. Sometimes this equation is assumed although one is not considering minimal kinetic terms. One must be careful in any case because once one assumes $B = A-1$ a wide range of A values $-1 \lesssim A \lesssim 3$ (including $A = 0$!) is forbidden if we want to be consistent with $m_t \approx 40 \text{ GeV}$.

*) Notice that Figs. 2 and 3 correspond to the approximation $|\sin 2\theta(M_W)| \approx 1$. In general, relaxing this approximation gives rise to some "bands" instead of the lines drawn for different A 's. However, these bands are extremely narrow for the regions of m and M of interest. We also remark that in fact in the unphysical $m \rightarrow \infty$ limit, the boundary lines corresponding to $|\sin 2\theta(M_W)| < 1$ do not close and one may have arbitrarily large A 's, x 's and y 's.

5. - THE LOW ENERGY SUPERSYMMETRIC SPECTRUM

We discuss in some detail in this section the SUSY spectra consistent with the appropriate radiative $SU(2) \times U(1)$ breaking and a t-quark with $m_t \approx 40$ GeV. The free parameters in an arbitrary $N = 1$ supergravity (grand unified) model are

$$m, M, \mu_0, A, B, m_t \quad (56)$$

If we fix $m_t \approx 40$ GeV and impose $SU(2) \times U(1)$ breaking, one still has four free parameters. If one knew, e.g., the squark and gluino masses $m_{\tilde{q}}$ and $M_{\tilde{g}}$ one would infer from them the values of m and M . We would be left then with two free parameters, e.g., A and B , which would parametrize the rest of the supersymmetric spectrum. For the sake of simplicity we are going to further assume that $B = A-1$ as in the "canonical" kinetic term case. In fact this assumption has only a certain influence on the Higgs and Higgsino sector of the spectrum and it is reasonable to expect that the consideration of more general situations will essentially lead to very similar spectra. For fixed $m_{\tilde{q}}$ and $M_{\tilde{g}}$ values we will then parametrize the spectra by one single parameter A . On the other hand, A is, as we discussed above, severely restricted by $|A| \lesssim 3$ (for fixed x and y) in the general case (furthermore the region $-1 \lesssim A \lesssim 3$ is forbidden if $B = A-1$) so that not many different spectra are possible within the above hypothesis.

In order to obtain a one-parameter (A) family of spectra, we thus need to fix $m_{\tilde{q}}$ and $M_{\tilde{g}}$. We will do that by considering values of these parameters which could be of interest for giving a supersymmetric explanation to the jets + missing p_T events recently observed in the $p\bar{p}$ collider at CERN⁷⁾. The most outstanding of these events are the "mono-jet" events in which a single (narrow) jet recoils against a large amount of invisible (~ 40 GeV) p_T . In a SUSY explanation of these events the photino(s) is the natural candidate to carry the missing p_T . However, several (essentially three) different SUSY mechanisms have been proposed in order to interpret the data. We will call them the $1-\tilde{q}$ (one squark), $2-\tilde{q}$ (two squarks) and $2-\tilde{g}$ (two gluinos) interpretations. The $1-\tilde{q}$ mechanism^{15),16)} was in fact proposed¹⁵⁾ well before the monojet events were found. It assumes very light gluinos (say $M_{\tilde{g}} \approx 3-7$ GeV) and heavy squarks ($m_{\tilde{q}} \approx 80-140$ GeV). In this candidate-explanation, a quark from the proton (anti-p) fuses with a gluino from the "sea" of the antiproton (proton) to form a heavy squark which decays practically at rest. The spectator gluino does not carry p_T enough to be detected. The heavy $\tilde{q}$ so created decays $\sim 2\%$ of the times into $q + \tilde{\gamma}$ giving rise to a jet + missing p_T signature. The dominant decay into $q + \tilde{g}$ is difficult to

disentangle from the usual QCD background. In the 2- $\tilde{q}$ mechanism¹⁷⁾, one assumes squark masses $m_{\tilde{q}} \approx 40$ GeV and heavier gluinos (typically slightly heavier than the $\tilde{q}$'s) so that the dominant $\tilde{q}$ decay mode is $\tilde{q} \rightarrow q + \tilde{\gamma}$. The squarks are pair-produced in the collider and then decay into a couple of quarks and a couple of photinos. One would naively expect two-jet + missing p_T events from this type of mechanism but the experimental p_T cuts and jet criteria make that many of these would-be two-jet events look like monojets. Finally in the 2- $\tilde{g}$ mechanism¹⁸⁾ gluinos with $m_{\tilde{g}} \approx 40$ GeV are assumed and $m_{\tilde{q}} \gg m_{\tilde{g}}$. Again, the gluinos are pair-produced in the collider and decay into four quarks and two photinos. As in the 2- $\tilde{q}$ mechanism the experimental procedure means that much of the times one observes monojets. We discuss the SUSY spectra corresponding to these three scenarios in turn.

a) The 1-squark mechanism

In this case^{15),16)}, one assumes a heavy squark and a very light gluino. We show spectra for various allowed A values in Table 2a ($m_{\tilde{q}} \approx 80$, $M_{\tilde{g}} \approx 3$ GeV) and Table 2b ($m_{\tilde{q}} \approx 120$, $M_{\tilde{g}} \approx 3$ GeV). One can see that in this case the possible A values are confined to $-2.9 \lesssim A \lesssim -1$ and $A \approx +3$. We show results for $A = -2.5, -2, -1, 2.95$ and 3 . One has $|M| \approx 1$ GeV and due to the smallness of M its sign is usually irrelevant. When it is not, we show separately both spectra [if allowed by the $SU(2) \times U(1)$ breaking and stability constraints]. We see that in both Table 2a and 2b, the slepton mass is practically equal to the squark mass (it is slightly higher because of the terms proportional to $\cos 2\theta$ which are in this case more important than the ones due to a non-vanishing M ; in the tables, the $\tilde{q}$ and $\tilde{\ell}$ masses correspond to $\tilde{U}_R$ and $\tilde{e}_L$). There is one lighter ($\tilde{t}_l$) and one heavier ($\tilde{t}_h$) top-squarks most of the times far above present experimental bounds. However, in the $m = 80$ GeV case and for $|A| \approx 2.5-3$, one may have very light t -squarks as in the example provided for $A = -2.5$ and $A = 2.95$. The photino is $m_{\tilde{\gamma}} \approx 0.5$ GeV and the rest of the neutralinos are in general much heavier. However, for $A = -2.5$, $A = -2$, there is a neutralino (mostly a Higgsino) with mass in the range 20-40 GeV which can be pair produced in Z^0 decay. There is also a lighter chargino ($\tilde{W}_l$) with mass in the range 30-65 GeV which can be produced in W and Z^0 decays. The heavy chargino ($\tilde{W}_h$) lies in the range 90-180 GeV. All the Higgs scalars are very heavy except for the neutral one H_b with mass in the range 2-9 GeV. As we remarked in Section 3, there is always such a light neutral Higgs in any supergravity scheme with $m_t \approx 40$ GeV. This Higgs could be present in the decay products of the T (through $T \rightarrow H_b + \gamma$) or toponium states and couples to fermions

$\sqrt{2}$ times more strongly than the standard Higgs. Quite often there is no phase space for H_b appearing in charmonium decays. As a general comment, we should mention that in order to get this scenario with very heavy $\tilde{q}$'s and very light $\tilde{g}$'s no special teasing of the supergravity parameters is required, it is a scenario which appears naturally for a wide range of values of these parameters.

b) The 2-squark mechanism

In this scenario¹⁷⁾, one takes $\tilde{q}$ masses $m_{\tilde{q}} \approx 40$ GeV and heavier gluinos so that the dominant $\tilde{q}$ decay mode is $\tilde{q} \rightarrow q + \tilde{\gamma}$. However, it is not easy to get gluinos which are heavier than squarks in an $N = 1$ supergravity scheme^{*}). This is easily seen¹⁹⁾ using Eqs. (31) and (42) which yield for the average squark mass in terms of gluino mass

$$m_{\tilde{q}}^2 = m^2 + 0.72 M_{\tilde{g}}^2 \quad (57)$$

so that large $M_{\tilde{g}}$ implies large $m_{\tilde{q}}$. One can try to lower the value of $m_{\tilde{q}}$ by diminishing m but the experimental constraint $m_{\tilde{\chi}_R} \gtrsim 18$ GeV implies from Eq. (31)

$$m_{\tilde{\chi}_R}^2 = m^2 + 0.015 M_{\tilde{g}}^2 \gtrsim (18 \text{ GeV})^2 \quad (58)$$

so that if we set, e.g., $m_{\tilde{q}} \approx 45$ GeV, one obtains from (57) and (58)

$$M_{\tilde{g}} \lesssim 49 \text{ GeV} \quad (59)$$

The gluino can only be slightly heavier than the squark in a low energy supergravity model¹⁹⁾. This is further illustrated in Fig. 4 in which we plot $m_{\tilde{q}}$ versus $M_{\tilde{g}}$ for $m \approx 18$ GeV. We observe that the gluino can only be heavier than the squark for $m_{\tilde{q}} \gtrsim 35$ GeV or so. For higher m (which might be required if the limits on selectron masses are improved) one needs even heavier squarks. The range of supergravity parameters for this scenario to work is extremely restricted, one needs $m \approx 20$ GeV, $M \approx 15$ GeV and small variations of these parameters spoil the scenario because either $m_{\tilde{g}} \lesssim m_{\tilde{q}}$ or $m_{\tilde{\chi}_R} \lesssim 18$ GeV. On the other hand, this possibility implies a light supersymmetric spectrum which could be observed in the near future as was remarked by Ellis and Sher¹⁹⁾. One finds for $m_{\tilde{q}} \approx 45$ GeV, $m_{\tilde{g}} \approx 45$

^{*}) This was already remarked by U. Ellwanger in Ref. 22).

GeV ($m = 20$ GeV, $|M| = 15$ GeV) the spectra shown in Table 3. We show results for $A = -3, -1.8, -1, 3$ and $M = \pm 15$ GeV which cover the range of allowed values for $B = A-1$. Values $A \approx -3 \sim -2$ are forbidden either due to the stability of the potential or due to having $\langle \tilde{t} \rangle \neq 0$. One of the most prominent features of the spectra is the lightness of the slepton masses, $m_{\tilde{\chi}} \approx 23-24$ GeV which could be tested in e^+e^- experiments (through, e.g., $e^+e^- \rightarrow \tilde{\chi}\tilde{\chi}$ and $e^+e^- \rightarrow \tilde{e}e\tilde{\gamma}$) as well as in the weak gauge boson decays $Z^0 \rightarrow \tilde{\chi}^+\tilde{\chi}^-$ and $W \rightarrow \tilde{\chi}\tilde{\nu}$ at the CERN collider¹⁹⁾. Concerning the top-squark masses, in this scheme, a general tendency exists for obtaining too light t-squarks. As we commented above, for $A = -3 \sim -2$ and $M = -15$ GeV, one gets $m_{\tilde{t}_\ell}^2 < 0$ and even for $A = 3$ and $A \sim -1.8$ the obtained $\tilde{t}_\ell$ mass is experimentally excluded. Only for $A = -3$ and $A \approx -1$, t-squark masses may be safe. We will comment later on about the possible parameters leading to light stops in the three scenarios. The heavy t-squark has a mass $m_{\tilde{t}_h} \approx 70-80$ GeV. For the photino, one has $M_{\tilde{\gamma}} \approx 7.5$ GeV and concerning the rest of the neutralinos, two of them are usually quite heavy (85-105 GeV and correspond to zeeno-Higgsino states. The other neutralino (mostly a Higgsino) can be rather light, 7-40 GeV, and in fact for the $A = -3, M = +15$ GeV example it is the lightest supersymmetric particle. The lightest chargino is usually quite heavy (68-78 GeV) so that there is not much phase space for the decay $W \rightarrow \tilde{W}_\ell + \tilde{\gamma}$ nor $Z^0 \rightarrow \tilde{W} + \tilde{W}$. The heavy chargino weighs 78-95 GeV. As in the $l-\tilde{q}$ scenario, the Higgs scalars are usually heavy except for the neutral one H_b which in this case may be even lighter than in the $l-\tilde{q}$ scenario, $m_{H_b} \approx 0.3-0.5$ GeV. This Higgs could be present in charmonium decays. Let us also mention that there is another neutral scalar (H_c) which may be relatively light (as light as ~ 30 GeV) and be produced in toponium decays.

c) The 2-gluino mechanism

In this case¹⁸⁾, one considers gluino masses $M_{\tilde{g}} \approx 40$ GeV and the missing momentum comes from the two photinos appearing in gluino decay. The squarks are assumed to be heavier than the gluino. This scenario, as the l -squark one, may appear with a wide range of supergravity parameters. As shown in Fig. 4, it is very easy to get $m_{\tilde{q}} \gg m_{\tilde{g}}$. As an example, we show in Table 4 the spectrum for $m_{\tilde{q}} \approx 70$ GeV, $m_{\tilde{g}} \approx 40$ GeV. The allowed values for A are $-3 \lesssim A \lesssim -1$ and $A \approx 3$. The sleptons are a few GeV lighter than the squarks and both stop eigenstates are relatively heavy except for $A \approx 3, M \approx +13$ in which there may be too light a stop. The photino weights ~ 6.5 GeV and the qualitative structure of the rest of

the neutralino spectrum is similar to that in the $2\tilde{q}$ case, with a relatively light Higgsino. The mass of the lightest chargino is $\approx 50\sim 68$ GeV and thus it can be produced in W decays (it may be lighter if $m_{\tilde{q}}$ is increased). The heavier chargino is typically $m_{\tilde{W}_h} \approx 90\sim 130$ GeV. All Higgses are very heavy except for H_b with $m_{H_b} \approx 1\sim 4$ GeV. This spectrum is a sort of interpolation between those of the $1\tilde{q}$ and $2\tilde{q}$ schemes.

The three types of spectra described above give a general flavour of the possibilities allowed by the appropriate $SU(2)\times U(1)$ breaking from broken $N = 1$ supergravity. There are, however, some features which are general and can be discussed without committing oneself to a particular spectrum. For example, it is interesting to know for what ranges of parameters one can expect charginos or stops light enough so that one could produce them in present accelerators. Figure 5 shows the mass of the lightest chargino versus the μ_0 parameter for various values of M in the $\cos 2\theta \approx 0$ approximation. It is clearly seen that relatively light charginos may only be obtained for large M or μ_0 . Thus, e.g., if one has $m_{\tilde{q}} \approx 100\sim 120$ GeV and $M_g \approx 0$, one may have $\mu_0 \sim 100$ GeV leading to a chargino weighing ~ 35 GeV. On the contrary, for $m_{\tilde{q}} \approx m_g \approx 40$ GeV, the lightest chargino should weigh ~ 60 GeV at least. This explains why the lightest chargino is usually lighter in the $1\tilde{q}$ scheme than in the other two, μ_0 may be larger in the former case [recall the bounds in Eq. (54)].

The case of the lightest t -squark is more complicated since its mass depends on several parameters (A, m, μ_0, M). To simplify matters, we assume $B = A^{-1}$ and consider the cases of special interest $M = 0, \pm 15$ GeV. We show in Fig. 6 the mass of the lightest stop versus m for several (allowed) A values. For $A = 3$ and $A = -1$, two possible lines (corresponding to two possible μ_0 's) may occur. Notice that the $\tilde{t}_\lambda$ may be lighter than $m_t \approx 40$ GeV for certain values of A in all the three $M = 0, \pm 15$ GeV cases. For $M = +15$ GeV and $A = 3$, one always gets too light t -squarks unless the sfermions are heavy enough. Only for $A \approx -2, -2.5$, one may have m as low as ~ 30 GeV leading to $m_{\tilde{q}} \sim 50$ GeV, $M_g \sim 45$ GeV. For $M = 0$ and light squarks, one always gets too light t -squarks except for $|A| \lesssim 2$. For large enough m , one is safe. Finally, in the case $M = -15$ GeV and light squarks, only for $A \approx -1$, one is safe but one may improve the situation by increasing m . As a general conclusion, the safer way not to have too light t -squarks is by having a large enough m parameter. Especially dangerous are large $|A|$ values. This is obvious from Eq. (28).

It is also of some interest to study the possible values of the lightest

Higgs field H_b mass. As in the $\tilde{t}_\ell$ case, the mass of H_b depends of various parameters so that we will assume $B = A-1$ and take the interesting values $M = 0, \pm 15$ GeV. We show in Fig. 7 the mass of the H_b Higgs versus m for different (allowed) A values. The general features of the three cases considered ($M = 0, \pm 15$ GeV) are the same¹⁰⁾: the smaller m (and $|A|$) the smaller m_{H_b} is. For $m \approx 20$ (as in the $2-\tilde{q}$ scheme) this Higgs is a very light $m_{H_b} \sim 0.5$ GeV and should be produced in charmonium decays. Notice, however, that for such a small H_b mass, the one-loop corrections to this mass may be relevant.

Let us make a couple of final comments about the spectra shown above. First, in the given numbers, we have assumed $h_t(0) = 0.08$ ($m_t \approx 37-41$ GeV) but the spectra will practically remain unchanged in the interval $30 \lesssim m_t \lesssim 50$ GeV. Second, we have shown the parameters renormalized at the M_W scale. As we commented in Section 4, in the case of $\tilde{q}$'s or $\tilde{g}$'s lighter than M_W , one needs to further evolve $m_{\tilde{q}}$ and $m_{\tilde{g}}$ down to the appropriate scale. However, numerically it makes very little difference.

6. - A SHORT DIGRESSION ON SOME NON-STANDARD SCENARIOS

One of the less attractive features of the supergravity class of models considered in the previous sections is the fact we already discussed that one necessarily has $\mu_3^2(0) \sim m^2$ for $m_t \approx 40$ GeV. These two parameters have nothing to do with each other and thus it is rather puzzling that they have to be almost equal in order to get a consistent $SU(2) \times U(1)$ breaking. There are several ways in which one can relax relation (50). One trivial way is to consider the existence of a fourth quark-lepton generation. In this case, the Yukawa coupling of this fourth family will be large enough to get μ_2^2 (or μ_1^2) negative, leading to $SU(2) \times U(1)$ breaking even if $\mu_3^2(0) \ll m^2$. Another way is to consider models in which the low energy chiral sector is not minimal but has extra light multiplets which transform like (adjoint + $2(1,1,+2)+2(1,1,-2)$) under $SU(3) \times SU(2) \times U(1)$. This type of low energy matter does appear in some $N = 1$ supersymmetric GUTs²⁰⁾. One can obtain in this case $SU(2) \times U(1)$ breaking with $\mu_3^2(0) \ll m^2$ for $m_t \gtrsim 43$ GeV, still compatible with experiment. However, one needs to use extreme supergravity parameters: for example, $A = 3$, $m = 50$, $M = 216$ GeV are consistent with $m_t \approx 47$ GeV but one can get $m_t \approx 43$ GeV only for $A = 3$, $m, M \rightarrow \infty$. Thus in this type of models, all the sparticles would be very heavy.

A third interesting possibility is disposing of the universality of the scalar masses at the GUT scale and assuming a Higgs mass at the GUT scale m_H smaller than the common $\tilde{q}, \tilde{\ell}$ masses m . This is not as artificial as it sounds since the Higgses may have couplings to other (superheavy) fields which may render $m_H < m$ at the GUT scale even if they were equal at the Planck mass²¹⁾. One can easily check that for $m_H \neq m$ the formulae (37) and (38) get modified to

$$\mu_2^2 = (m^2(h - \kappa A^2) + e M^2 + f m M A) + (m_H^2 - m^2) \frac{(1 + 3\gamma_b F)}{D(t)} \quad (60)$$

$$\mu_1^2 = m_H^2 + g M^2 \quad (61)$$

in the $\mu \rightarrow 0$ limit. In this case, one may break $SU(2) \times U(1)$ with a low mass t -quark and $\mu_3^2(0) \ll m^2$. Having $m_H \approx \frac{1}{2}m$ at the GUT scale may be sufficient to radiatively break $SU(2) \times U(1)$ with $m_t \approx 40$ GeV. Some possible values of A, m, m_H, m_t and M in this scenario are shown in Table 5. One usually needs large values for A and m but it is still possible to get relatively light gaugino masses (e.g., $A = 3, m = 150, m_H = 37.5, m_t = 39, M = 21$ GeV). However, the general tendency is to get heavy spectra. We hope that the "non-minimal" scenarios briefly discussed here give an idea of what one can expect from more sophisticated models.

7. - FINAL REMARKS AND CONCLUSIONS

We have presented in this paper a detailed study of the constraints imposed on low energy $N = 1$ supergravity models by the existence of a low mass ($m_t \lesssim 60$ GeV) t -quark. We give analytical formulae for the scale evolution of masses and couplings in terms of the parameters m, M, μ_0, A and B (for any h_t). We find numerical constraints for these parameters when we fix $m_t \approx 40$ GeV. In particular, we find that $\mu_3^2 \approx (0.6-1.0)(m^2 + \mu_0^2)$ at the GUT scale. Since, by definition, $B = -\mu_3^2/m\mu_0$, we conclude that models with $B(M_x) = 0$ are inconsistent with $m_t \approx 40$ GeV. This includes the simplest versions of the recently proposed models based on a Kahler manifold $SU(n,1)/SU(n) \times U(1)$ in which the observable sector is globally supersymmetric (except for gaugino terms) at the GUT scale. One also finds restrictions for the bare Higgsino mass parameter μ_0 . Depending on the value of gaugino masses at the GUT scale (M) one gets bounds on μ_0 . Thus, for example, one gets (for $\mu_3^2 > 0$) $\mu_0 \lesssim 1.2 m$ ($M = 0$), $\mu_0 \lesssim 0.8 m$ ($M/m \approx 0.5$) and

$\mu_0 < 1.5 m$ ($M/m = -0.5$). All these results are general and apply also to models with non-minimal kinetic terms. If one further assumes the relation $B = A-1$, one obtains stronger constraints. In particular (for the A convention defined in Section 2), the range of values $-1 \lesssim A \lesssim 2.8$ is forbidden. Sometimes in the literature the relationship $B = A-1$ is assumed even though the scalar metric is non-minimal. One should be careful in any case since, e.g., values like $A = 0.0$ are inconsistent with radiative $SU(2) \times U(1)$ breaking. The restrictions on μ_0 and M get also more stringent if one assumes $B = A-1$. Thus, for example, one gets (for $\mu_3^2 > 0$) $0.15 m \lesssim \mu_0 \lesssim m$ ($M/m = 0$); $0.17 m \lesssim \mu_0 \lesssim 1.2 m$ ($M/m = -\frac{1}{2}$); $0.22 m \lesssim \mu_0 \lesssim 0.5 m$ ($M/m = +\frac{1}{2}$), etc. Similar bounds (more stringent) are obtained in the $\mu_3^2 < 0$ case. In any case, one always has $\mu_0 \gtrsim 0.15 m$. Another interesting feature for $B = A-1$ is that the gauginos cannot be massless for $X = \mu_0/m > 1$. Also one may have large values for X , Y and A but only if we let them all increase simultaneously. Thus the usual stability assumption $|A| \lesssim 3$ only applies for relatively small X and Y .

We also discussed the low energy supersymmetric spectra consistent with the above restrictions. Of the five free parameters μ_0 , M , m , A and B , one is related to the others through the $SU(2) \times U(1)$ breaking condition. We fix M and m in order to obtain squark and gluino masses which could be interesting in trying to give a supersymmetric explanation to the CERN missing p_T events. In particular, we consider three cases: a) $1-\tilde{q}$ mechanism, with $M_{\tilde{g}} \approx 3$ GeV, $m_{\tilde{q}} \approx 80-120$ GeV; b) $2-\tilde{q}$ mechanism, with $m_{\tilde{q}} \approx M_{\tilde{g}} \approx 40$ GeV, and c) $2-\tilde{g}$ mechanism where $M_{\tilde{g}} \approx 40$ GeV and $m_{\tilde{q}} \gg M_{\tilde{g}}$. Each of the three mechanisms lead to characteristic spectra which are described in detail. By fixing M and m , we are left only with two free parameters A and B . To simplify matters, we show the results in the case $B = A-1$ in which a single parameter A parametrizes the possible spectra (for fixed M, m). We argued, however, that the assumption $B = A-1$ only affects essentially the spectra of Higgses and Higgsinos and that the results should be similar for more general situations. There are some general features of the masses of the sparticles in the three cases considered. In general, the spectrum in the $2-\tilde{q}$ case is lighter and some SUSY signatures should be soon available¹⁹⁾ in present accelerators (e.g., $Z^0 \rightarrow \tilde{\chi}^+ \tilde{\chi}^-$, $W \rightarrow \tilde{\chi} \tilde{\nu}$, $e^+ e^- \rightarrow \tilde{\gamma} \tilde{\gamma}$). In the other two cases, the sparticles are in general quite heavy. In the $1-\tilde{q}$ and $2-\tilde{g}$ schemes one "chargino" may be relatively light (~ 30 GeV) and could be produced in W and Z^0 decay through $W \rightarrow \tilde{W}_{\chi} \tilde{\gamma}$ and $Z^0 \rightarrow \tilde{W} \tilde{W}$. In the other ($2-\tilde{q}$) case, these processes have in general little phase space. The photino is practically always the lightest supersymmetric particle but there is also a neutralino (mostly a Higgsino) which can be relatively light (even the lightest SUSY particle) and contribute to Z^0 and W

decay (along with a chargino). The lightest top-squark is usually heavier than its spartner in the $1\text{-}\tilde{q}$ and $2\text{-}\tilde{g}$ schemes, but in some restricted ranges of parameters $m_{\tilde{t}_\lambda}^2$ may be very (too) small (even negative!) so that one must be careful and check if one's parameters are safe. This is especially the case in the $2\text{-}\tilde{q}$ mechanism where most of the times $m_{\tilde{t}_\lambda}^2$ is too small. All the Higgs scalars are usually heavier than M_W except for the neutral H_b which is very light (of the order of a few GeV) especially in the $2\text{-}\tilde{q}$ scheme in which it may be as light as ~ 0.5 GeV and be present in charmonium decay. In the $2\text{-}\tilde{q}$ case there is another neutral Higgs which may be as light as ~ 30 GeV.

Of the three mechanisms proposed to explain the CERN missing p_T events, the $1\text{-}\tilde{q}$ and $2\text{-}\tilde{g}$ schemes can be obtained from an $N=1$ supergravity model very easily. In the $2\text{-}\tilde{q}$ case the range of supergravity parameters leading to $M_{\tilde{g}} > m_{\tilde{q}}$ is very small but on the other hand the predicted spectra lead to quite dramatic signatures ($\tilde{\chi}\tilde{\chi}$ production) which should be seen soon. In the $1\text{-}\tilde{q}$ and $2\text{-}\tilde{g}$ schemes, one may, on the other hand, have relatively light charginos and neutralinos which could be observed in W and Z^0 decays. Moreover, a bump in the multijet cross-section¹⁵⁾ should be observed around the squark mass in the case of the $1\text{-}\tilde{q}$ mechanism. Thus forthcoming data may soon decide about the possibilities of these mechanisms. We should remark, however, that the spectra discussed above are just possible (but not the only) supersymmetric spectra. If no trace of supersymmetric particles are found in present accelerators, it could just mean that the range of energies explored is still too small. Let us hope that we will not have to wait for the supercolliders to settle the question.

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APPENDIX I

The only relevant renormalization group equations with a non-trivial h_t dependence are:

$$\frac{d\mu_1^2}{dt} = (3\tilde{\alpha}_2 M_2^2 + \tilde{\alpha}_1 M_1^2) + (3\tilde{\alpha}_2 + \tilde{\alpha}_1) \mu^2 - 3\gamma_t \mu^2 \quad (I.1)$$

$$\frac{d\mu_2^2}{dt} = (3\tilde{\alpha}_2 M_2^2 + \tilde{\alpha}_1 M_1^2) + (3\tilde{\alpha}_2 + \tilde{\alpha}_1) \mu^2 - 3\gamma_t (m_a^2 + m_u^2 + \mu_2^2 + A_t^2 m^2) \quad (I.2)$$

$$\frac{d\mu_3^2}{dt} = \left(\frac{3}{2}\tilde{\alpha}_2 + \frac{1}{2}\tilde{\alpha}_1 - \frac{3}{2}\gamma_t\right) \mu_3^2 + 3\mu m \gamma_t A_t - \mu (3\tilde{\alpha}_2 M_2 + \tilde{\alpha}_1 M_1) \quad (I.3)$$

$$\frac{dm_a^2}{dt} = \left(\frac{16}{3}\tilde{\alpha}_3 M_3^2 + 3\tilde{\alpha}_2 M_2^2 + \frac{1}{9}\tilde{\alpha}_1 M_1^2\right) - \gamma_t (m_a^2 + m_u^2 + \mu_2^2 + A_t^2 m^2 - \mu^2) \quad (I.4)$$

$$\frac{dm_u^2}{dt} = \left(\frac{16}{3}\tilde{\alpha}_3 M_3^2 + \frac{16}{9}\tilde{\alpha}_1 M_1^2\right) - 2\gamma_t (m_a^2 + m_u^2 + \mu_2^2 + A_t^2 m^2 - \mu^2) \quad (I.5)$$

$$\frac{d\mu^2}{dt} = (3\tilde{\alpha}_2 + \tilde{\alpha}_1 - 3\gamma_t) \mu^2 \quad (I.6)$$

$$\frac{dA_t}{dt} = \left(\frac{16}{3}\tilde{\alpha}_3 \frac{M_3}{m} + 3\tilde{\alpha}_2 \frac{M_2}{m} + \frac{13}{9}\tilde{\alpha}_1 \frac{M_1}{m}\right) - 6\gamma_t A_t \quad (I.7)$$

$$\frac{dB}{dt} = (3\tilde{\alpha}_2 \frac{M_2}{m} + \tilde{\alpha}_1 \frac{M_1}{m}) - 3\gamma_t A_t \quad (I.8)$$

$$\frac{dY_t}{dt} = Y_t \left(\frac{16}{3} \tilde{\alpha}_3 + 3 \tilde{\alpha}_2 + \frac{13}{9} \tilde{\alpha}_1 \right) - 6 Y_t^2 \quad (I.9)$$

where

$$Y_t = \frac{h_t^2}{(4\pi)^2} \quad ; \quad \tilde{\alpha}_i = \frac{\alpha_i}{(4\pi)} \quad ; \quad t = 2 \log (M_x/a) \quad (I.10)$$

and the conventions for the couplings are given in Chapter 2. Here we have considered a non-negligible μ which was not included in Ref. 3).

APPENDIX II

We collect here all the functions H_1 , F_1 and $G_{1,2}$ not defined in the text. These only depend on the running couplings $\alpha_i(t)$ and not on h_t or any other parameter. We prefer to give here the general expressions for these functions and not just their numerical values in the minimal model because in this way it is easy to see the modifications needed if one considers non-minimal scenarios like, e.g., adding a fourth generation or some extra light Higgses. One has

$$E(t) = (1 + \beta_3 t)^{16/(3h_3)} (1 + \beta_2 t)^{3/h_2} (1 + \beta_1 t)^{13/(9h_1)} \quad (\text{II.1})$$

$$F(t) = \int_0^t E(t') dt' \quad (\text{II.2})$$

$$H_1(t) = \tilde{\alpha}(0) \left(\frac{16}{3} \frac{1}{(1 + \beta_3 t)^2} + \frac{3}{(1 + \beta_2 t)^2} + \frac{13}{9} \frac{3/5}{(1 + \beta_1 t)^2} \right) \quad (\text{II.3})$$

$$H_2(t) = \int_0^t H_1(t') dt' = \tilde{\alpha}(0) \left(\frac{16}{3} h_3 + 3 h_2 + \frac{13}{15} h_1 \right) \quad (\text{II.4})$$

$$H_3(t) = \int_0^t dt' E(t') H_2(t') = t E(t) - F(t) \quad (\text{II.5})$$

$$H_4(t) = F(t) H_2(t) - H_3(t) \quad (\text{II.6})$$

$$H_5(t) = \tilde{\alpha}(0) \left(-\frac{16}{3} f_3 + 6 f_2 - \frac{22}{15} f_1 \right) \quad (\text{II.7})$$

$$H_6(t) = \int_0^t H_2(t')^2 E(t') dt' \quad (\text{II.8})$$

$$H_7(t) = 3 \tilde{\alpha}_2(0) h_2(t) + \tilde{\alpha}_1(0) h_1(t) \quad (\text{II.9})$$

$$H_8(t) = \tilde{\alpha}(0) \left(-\frac{8}{3} f_3 + f_2 - \frac{1}{3} f_1 \right) \quad (\text{II.10})$$

$$F_2(t) = \tilde{\alpha}(0) \left(\frac{8}{3} f_3 + \frac{8}{15} f_1 \right) \quad (\text{II.11})$$

$$F_3(t) = F(t) F_2(t) - \int_0^t dt' E(t') F_2(t') \quad (\text{II.12})$$

$$F_4(t) = \int_0^t E(t') H_5(t') dt' \quad (\text{II.13})$$

$$G_1 = F_2 - \frac{1}{3} H_2^2 \quad (\text{II.14})$$

$$G_2 = 6 F_3 - F_4 - 4 H_2 H_4 + 2 F H_2^2 - 2 H_6 \quad (\text{II.15})$$

where the functions $f_i(t)$ and $h_i(t)$ were already defined in (II.1):

$$f_i(t) = \frac{1}{\beta_i} \left(1 - \frac{1}{(1 + \beta_i t)^2} \right) ; h_i(t) = \frac{t}{(1 + \beta_i t)} \quad (\text{II.16})$$

and $\beta_i = \tilde{\alpha}(0) b_i = \alpha_i(0) b_i / (4\pi)$. Whenever an α_i appears in formulae (II.1-15), it refers to its value at the GUT scale. $\tilde{\alpha}(0) = \alpha(0)/4\pi$ corresponds to the GUT

coupling and the hypercharge normalization is such that $\alpha_1(0) = 3/5\alpha(0)$. The numerical values of the functions $q, \ell, h, k, f, e, g, r$ and s defined in the text are given in Table 1 in the case of the minimal low energy content of the theory. The numerical values for f_i and h_i at the M_W scale are

$$\begin{aligned} f_3 &= 803 & ; & & f_2 &= 100 & ; & & f_1 &= 38 \\ h_3 &= 201 & ; & & h_2 &= 55 & ; & & h_1 &= 27 \end{aligned} \quad (\text{II.17})$$

in the case of minimal content. For the functions F and E relevant for the renormalization of the Y_t Yukawa coupling, one has

$$F \approx 290 \quad ; \quad E \approx 14 \quad (\text{II.18})$$

TABLE CAPTIONS

Table 1 - Numerical values of the coefficients defined in the text in the case of minimal low energy content.

Table 2 - Supersymmetric particle spectra corresponding to the $1-\tilde{q}$ scheme. The tables correspond to $M_{\tilde{g}} = 3$ GeV, $m_{\tilde{q}} = 80$ GeV (a), and $M_{\tilde{g}} = 3$ GeV, $m_{\tilde{q}} = 120$ GeV (b). The values of m and M are essentially fixed by the $M_{\tilde{g}}$ and $m_{\tilde{q}}$ values. The different spectra are then parametrized by a single parameter A , although for some A values (e.g., $A = -1$) there are two different spectra allowed. Only results for allowed A values are shown. The sign of M is usually (in this case) numerically irrelevant. When it is not we show the spectra corresponding to both signs separately (if allowed by the stability bounds). All the masses are in GeV.

Table 3 - Supersymmetric spectra corresponding to the $2-\tilde{q}$ scheme. In this case, $m \approx 20$, $M = \pm 15$ GeV, leading to $m_{\tilde{q}} \approx M_{\tilde{g}} \approx 45$ GeV. Different signs for M give rise to different spectra although sometimes a given sign is not allowed. The A values correspond to typical allowed values and sometimes there are two spectra for a given A .

Table 4 - As in Tables 2 and 3 but for the $2-\tilde{g}$ scheme and $M_{\tilde{g}} \approx 40$, $m_{\tilde{q}} \approx 70$ GeV.

Table 5 - Some allowed values of the supergravity parameters in the scenario discussed in Section 6 with a Higgs mass m_H smaller than the common scalar mass m at the GUT scale. This assumes $\mu_0 = 0 = \mu_2^2(0)$ and $\omega \rightarrow \infty$.

$q = 1.38$	$\lambda = 1.9$	$h = 0.9$
$K = 0.031$	$f = -0.15$	$e = 0.08$
$g = 0.53$	$r = -0.72$	$s = 0.046$

- Table 1 -

m	80	80	80	80	80	80	80	80
M	+1	-1	± 1	± 1	+1	-1	± 1	± 1
A	-2.5	-2.5	-2	-1	-1	2.95	2.95	+3
μ_0	15	15	20	47	66	42	67	74
m_t	39	39	38	38	38	39	38	38
<hr/>								
$\tilde{q}$	79	79	79	80	80	80	80	80
$\tilde{t}_\ell$	21	9	49	84	82	14	35	38
$\tilde{t}_h$	114	116	108	87	90	113	109	108
$\tilde{l}$	81	81	81	80	80	81	81	81
$\tilde{g}$	3	3	3	3	3	3	3	3
$\tilde{\gamma}$	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
N° 's	100	99	103	127	145	122	146	153
	78	79	75	61	53	65	54	51
	22	21	27	66	91	56	93	102
$\tilde{W}_\ell$	66	67	64	51	44	53	44	43
$\tilde{W}_\tau$	90	89	93	117	136	112	137	144
H^-	134	134	138	163	186	152	184	193
H_a	140	140	144	168	191	158	189	198
H_b	6	7	5	2	2	7	5	5
H_c	109	109	114	143	169	131	167	177

- Table 2a -

m	120	120	120	120	120	120
M	± 1	± 1	± 1	± 1	± 1	± 1
A	-2.5	-2	-1	-1	2.95	2.95
μ_0	23	30	70	100	56	101
m_t	39	39	38	38	39	39
$\tilde{q}$	119	120	120	120	119	120
$\tilde{t}_\ell$	62	84	119	117	52	71
$\tilde{t}_h$	153	146	124	126	153	145
$\tilde{\chi}$	121	121	120	120	121	121
$\tilde{g}$	3	3	3	3	3	3
$\tilde{\gamma}$	0.5	0.5	0.5	0.5	0.5	0.5
N^0 's	106	111	150	181	139	182
	73	70	52	43	56	43
	32	41	98	138	84	139
$\tilde{W}_\ell$	62	59	42	34	47	35
$\tilde{W}_h$	96	101	141	173	129	174
$H^\pm$	181	187	228	266	208	261
H_a	185	192	232	270	212	265
H_b	9	7	3	2	10	6
H_c	163	170	214	255	193	249

- Table 2b -

m	20	20	20	20	20
M	+15	-15	-15	-15	+15
A	-3	-1.8	-1	-1	3
μ_0	5	5	9	25	27
m_t	37	38	38	37	38
$\tilde{q}$	45	45	45	45	45
$\tilde{t}_\ell$	54	2	28	41	13
$\tilde{t}_h$	62	81	77	71	79
$\tilde{\chi}$	23	24	24	24	24
$\tilde{g}$	45	45	45	45	45
$\tilde{\gamma}$	7.5	7.5	7.5	7.5	7.5
$N^0's$	97	87	90	103	105
	79	91	89	79	79
	7	7.5	12	35	37
$\tilde{W}_\ell$	68	75	78	70	69
$\tilde{W}_h$	87	81	79	93	94
$H^\pm$	84	83	85	97	97
H_a	94	93	95	105	106
H_b	0.4	0.8	0.7	0.9	1.5
H_c	32	30	34	58	58

- Table 3 -

m	60	60	60	60	60	60
M	+13	+13	+13	-13	-13	+13
A	-3	-2.5	-2	-1	-1	3
μ_0	11	13	17	28	62	65
m_t	38	38	38	38	38	38
$\tilde{q}$	69	69	69	69	69	69
$\tilde{t}_\ell$	41	55	67	56	70	10
$\tilde{t}_h$	98	92	85	92	83	102
$\tilde{\chi}$	62	62	62	61	61	62
$\tilde{g}$	39	39	39	39	39	39
$\tilde{\gamma}$	6.5	6.5	6.5	6.5	6.5	6.5
N° 's	100	102	105	107	138	142
	76	75	72	77	62	61
	15	18	23	39	86	90
$\tilde{W}_\ell$	64	63	61	67	54	53
$\tilde{W}_h$	91	92	95	96	123	132
$H^\pm$	113	115	117	126	165	167
H_a	121	122	125	133	171	172
H_b	4.5	3	3	2.5	2	4.5
H_c	82	85	88	99	146	147

- Table 4 -

A	m	m_H	m_t	M_1	M_2
3	100	10	45	140	51
3	100	10	51	524	13
3	150	75	45	193	95
3	150	37.5	39	124	21
3	200	50	33	88	19
3	200	20	39	251	-57
3	1000	500	45	1983	-66
1	200	50	51	224	134
1	200	20	51	343	15
1	1000	250	45	724	-85
0	1000	100	39	302	-302
0	1000	100	33	195	-195

- Table 5 -

REFERENCES

- 1) Some review articles are:
H.P. Nilles - Physics Reports 110 (1984) 1;
J. Ellis - CERN Preprint TH.3718 (1983);
P. Nath, R. Arnowitt and A.H. Chamseddine - Northeastern Preprint NUB(2613) (1983);
L.E. Ibáñez - Madrid Preprint FTUAM 84-7 (1984);
H. Haber and G. Kane - Michigan Preprint UM-HE 83-17 (1984) to appear in Physics Reports.
- 2) L.E. Ibáñez - Nucl.Phys. B218 (1983) 514;
L.E. Ibáñez and C. Lopez - Phys.Lett. 126B (1983) 54;
L. Alvarez-Gaumé, J. Polchinski and M. Wise - Nucl.Phys. B221 (1983) 495;
J. Ellis, D.V. Nanopoulos and K. Tamvakis - Phys.Lett. 121B (1983) 123.
- 3) L.E. Ibáñez and C. Lopez - Nucl.Phys B233 (1984) 511.
- 4) S. Jones and G.G. Ross - Phys.Lett. 155B (1984) 69;
K. Inoue, A. Kakuto and S. Takeshita - Kyushu Preprint 83-HE-6 (1983).
- 5) C. Kounnas, A. Lahanas, D.V. Nanopoulos and M. Quirós - Nucl.Phys. B236 (1984) 438.
- 6) G. Arnison et al. - CERN-EP/84-134 (1984).
- 7) G. Arnison et al. - Phys.Lett. 139B (1984) 115;
P. Bagnaia et al. - Phys.Lett. 139B (1984) 105.
- 8) K. Inoue et al. - Prog.Theor.Phys. 67 (1982) 1859.
- 9) K. Inoue et al. - Prog.Theor.Phys. 68 (1982) 927; Errata, Kyushu Preprint 83-HE-5 (1983).
- 10) H.P. Nilles and M. Nusbaumer - Geneva Preprint UGVA-1984/05-431 (1984);
S. Li and M. Sher - Irvine Preprint UCL-TR 84-7 (1984);
P. Majumdar and P. Roy - Tata Institute Preprint TIFR/TH/84-19 (1984).
- 11) J. Ellis and S. Rudaz - Phys.Lett. 128B (1983) 248.
- 12) M. Claudson, L. Hall and I. Hinchliffe - Nucl.Phys. B228 (1983) 501.
- 13) L. Hall, J. Lykken and S. Weinberg - Phys.Rev. D27 (1983) 2359.
- 14) E. Cremmer et al. - Phys.Lett. 133B (1983) 61;
J. Ellis, C. Kounnas and D.V. Nanopoulos - CERN Preprint TH.3824 (1984), and CERN Preprint TH.3848 (1984);
U. Ellwanger, N. Dragon and M. Schmidt - CERN Preprint TH.3794 (1984).
- 15) M.J. Herrero, L.E. Ibáñez, C. López and F.J. Yndurain - Phys.Lett. 132B (1983) 199; Phys.Lett. 145B (1984) 430.
- 16) V. Barger, K. Hagiwara, W. Keung and J. Woodside - Phys.Rev.Lett. 53 (1984) 641; Madison Preprint MAD/PH/197 (1984);
A. De Rújula and R. Petronzio - CERN Preprint TH.4070 (1984).

- 17) J. Ellis and H. Kowalski - DESY Preprint 84-045 (1984);
A. Allan, E. Glover and A. Martin - Durham Preprint DTRP/84/20 (1984);
V. Barger, K. Hagiwara and W. Keung - Madison Preprint MADPH/183 (1984).
- 18) E. Reya and D.P. Roy - Phys.Lett. 141B (1984) 442; Phys.Rev.Lett. 52 (1984) 881;
J. Ellis and H. Kowalski - CERN Preprint TH.3843 (1984).
- 19) J. Ellis and M. Sher - CERN Preprint TH.3968 (1984).
- 20) L.E. Ibáñez - Phys.Lett. 126B (1983); Phys.Lett. 130B (1983) 463;
L.E. Ibáñez and G.G. Ross - Phys.Lett. 131B (1983) 335.
- 21) K. Inoue, A. Kakuto and S. Takeshita - Kyushu Preprint 83-HE-6 (1983);
H. Komatsu, Tokyo preprint INS-Rep.-469 (1983);
B. Gato, J. Leon, J. Perez-Mercader and M. Quirós - Madrid Preprint IEM.TH 84-2-1 (1984);
P. Moxhay and K. Yamamoto - Chapel Hill Preprint, IFP-215-UNC (1984).
- 22) U. Ellwanger - Phys.Lett. 141B (1984) 435.

FIGURE CAPTIONS

Figure 1 Values of A , μ_0 , M and $\sin 2\theta(M_X)$ consistent with radiative $SU(2) \times U(1)$ breaking as explained in the text. a) $M/m = -\frac{1}{2}$, b) $M = 0$, c) $M/m = \frac{1}{2}$. These results are general and do not assume minimal kinetic terms.

Figure 2 Values of A , $x = \mu_0/m$ and $y = M/m$ consistent with radiative $SU(2) \times U(1)$ breaking assuming $B = A-1$. The shaded regions correspond to $|\sin 2\theta(M_X)| > 1$. Results are shown for $M/m = 0$ (continuous line), $M/m = -1$ (dashed line) and $M/m = +1$ (dotted line).

Figure 3 Allowed values of the parameters $y = \mu_0/m$, $y = M/m$, A in the assumption $B = A-1$. The hatched areas are forbidden due to the violation of the $|\sin 2\theta(t)| < 1$ or stability constraints for a) $\sin 2\theta(M_W) < 0$, b) $\sin 2\theta(M_W) > 0$.

Figure 4 $m_{\tilde{q}}$ versus $M_{\tilde{g}}$ in a general low energy supergravity model. The shaded area corresponds to $M_{\tilde{g}} > m_{\tilde{q}}$ and $m_{\tilde{\chi}} \gtrsim 18$ GeV. Only for $m_{\tilde{q}} \gtrsim 35$ GeV this situation is possible.

Figure 5 The mass of the lightest chargino versus the μ_0 parameter for various M values in a general SUSY theory (in the $\cos 2\theta \approx 0$ approximation).

Figure 6 The mass of the lightest top-squark versus m for several (allowed) A values. The equality $B = A-1$ is assumed for simplicity. Results are shown for a) $M = 15$; b) $M = 0$; c) $M = -15$ GeV.

Figure 7 Mass of the lightest neutral Higgs H_b versus m for several (allowed) A values. The equality $B = A-1$ is assumed. Results are shown for a) $M = 0$; b) $M = 15$ GeV; c) $M = -15$ GeV. Interrupted curves indicate that beyond that point, charge and colour are broken through $\langle \tilde{t}_L \rangle \neq 0$.

- Figure 1 -

- Figure 2 -

- Figure 3 -

- Figure 4 -

- Figure 5 -

- Figure 6 -

- Figure 7 -