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ON THE SHORT-DISTANCE BEHAVIOUR OF
THE CALLAN-RUBAKOV CONDENSATE

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A B S T R A C T

The implications of the finite size of the 't Hooft-Polyakov monopole for the Callan-Rubakov effect are studied. The chirality violating condensate is derived without imposing a boundary condition on the fermion field at the origin. Finite distance corrections to the condensate are worked out.

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1. - INTRODUCTION

The Callan-Rubakov (CR) effect¹⁾ is an outstanding example which displays to what dramatic extent quantum effects may modify classical "tree-level" physics. The interactions of fermions with an electric field, which is quantized in the presence of a classical non-Abelian monopole, change the classical dyon spectrum completely and lead to the formation of a chirality breaking vacuum around the monopole core, which induces the celebrated catalysis of baryon decay.

Unfortunately the original derivation heavily relied on several simplifying assumptions. This fact has challenged several authors to re-examine it. In particular the effects of finite fermion masses^{2),3)}, confinement⁴⁾ and additional gauge fields⁵⁾⁻⁷⁾ have been examined. Though the proper inclusion of these effects modifies the details, so far there is no conclusive evidence which necessitates a complete revision of the picture given in Ref. 1).

Here we will focus our attention on one of the simplifications of the original derivation and demonstrate how it can be circumvented, leading to a refinement of the original picture. As is well known, the 't Hooft-Polyakov monopole is an object of finite spatial extent. If the mass of the Higgs field associated with the monopole is denoted by m_W , the monopole extends typically over a region of size $r \sim 1/m_W$. Therefore, from infinity, it looks like a pointlike object. In this respect, there is no difference between a non-Abelian monopole and a Dirac monopole. The original analysis of CR has been performed in the limit of shrinking the size of the monopole to zero. Therefore, effectively only a Dirac monopole, embedded into a non-Abelian gauge theory is at work. The only trace which is left over from the core structure of the non-Abelian monopole is a boundary condition which has to be imposed on solutions of the Dirac equation at the origin. Therefore, the results of Ref. 1) are trustworthy only at spatial infinity.

The purpose of the present paper is to study the implications of keeping the monopole core finite. The finite size of the monopole core manifests itself in a modification of both the fermion field as well as the electric field. In the original analysis, only part of the finite core effects on the fermion field was included by requiring the boundary condition. Here we shall take into account the core effects on both fields. In contrast to the original derivation we shall not impose any boundary condition on the fermion field. For simplicity, we shall work with massless fermions.

One of the remarkable things discovered by CR is that the chirality violating condensate is neither suppressed by inverse powers of m_W , nor by factors of $e^{-\text{const}/\alpha}$, a phenomenon common to instanton effects⁸⁾. However, due to the inherent limitations of the original analysis, this can only be taken for granted at large enough distances. On general grounds, one expects the condensate to vanish near the core. The detailed form of the suppression factor, however, is not known. We shall show that the form of the condensate given in Ref. 1) is indeed the leading term of a large distance expansion. The next-to-leading corrections are worked out in the so-called Bogomolny-Prasad-Sommerfield (BPS)⁹⁾ limit. They show that the condensate is exponentially suppressed near the core, the suppression setting in at a scale of the order of $1/m_W$.

The paper is organized as follows. In the next section we shall set up the necessary framework for the calculation of the condensate, including in all the equations the full monopole structure. We shall show how the condensate emerges in the pointlike limit. Sections 3 and 4 are devoted to the study of the fermion field, respectively the electric field in the background of an extended monopole. Our interest focuses mainly on the consequences for the final shape of the condensate. We conclude this paper with a discussion of our results and possible implications.

2. - THE CALLAN-RUBAKOV MODEL

For later reference, let us first recapitulate the essentials of the CR model. The underlying theory is a SU(2) gauge theory with one isodoublet of massless Dirac fermions, and a Higgs field ϕ in the adjoint representation, which breaks SU(2) into U(1). The charge associated with the unbroken U(1) generator will be referred to as electric charge.

The classical field equations of this theory admit a static solution with non-vanishing magnetic charge, the 't Hooft-Polyakov magnetic monopole¹⁰⁾. In the temporal gauge, the monopole is given by

$$\vec{A}_0^a(x) = 0 \tag{2.1a}$$

$$A_i^a(x) = \epsilon_{aij} \hat{r}_j (1 - K(r)) / r \quad (2.1b)$$

$$\phi_a(x) = \hat{r}_a H(r) / r \quad (2.2)$$

The index a refers to the adjoint representation of the gauge group and $\hat{r}_i$ is the i^{th} component of a unit vector pointing radially outwards. The functions $K(r)$ and $H(r)$ satisfy some complicated coupled differential equations. It can be shown that they smoothly interpolate between their values at the origin and at infinity

$$\begin{aligned} K(0) &= 1 & \lim_{r \rightarrow \infty} K(r) &= 0 \\ H(0) &= 0 & \lim_{r \rightarrow \infty} H(r) &= m_w r \end{aligned} \quad (2.3)$$

m_w is the mass of the heavy gauge bosons generated by the Higgs mechanism. Both functions, $K(r)$ and $H(r)$ approach their value at infinity exponentially.

In general, no analytic expressions for $K(r)$ and $H(r)$ are known. However, if we let the scalar potential vanish, keeping the vacuum expectation value of the Higgs field fixed, i.e., in the so-called BPS limit, $H(r)$ and $K(r)$ are given by⁹⁾

$$H(r) = m_w r \coth m_w r - 1 \quad (2.4a)$$

$$K(r) = m_w r / \sinh m_w r \quad (2.4b)$$

We shall make use of the solutions (2.4) later repeatedly.

In the CR model, the monopole configuration (2.1), (2.2) serves as a classical background field. The degree of freedom of the long-ranged field associated with the unbroken generator, the so-called "charge rotator" is then quantized about that background. In the chosen gauge, this field is spherically symmetric. Therefore we have

$$\begin{aligned}
 H_0^a(\sigma, t) &= \hat{\tau}_a a_0(\sigma, t) \\
 H_i^a(\sigma, t) &= \hat{\tau}_a \hat{\tau}_i a_i(\sigma, t) + H_{i,CL}^a(\sigma, t) \\
 \phi^a(\sigma, t) &= \phi_{CL}^a(\sigma, t)
 \end{aligned} \tag{2.5}$$

with

$$a_0(\sigma, \pm\infty) = a_i(\sigma, \pm\infty) = 0 \tag{2.6}$$

where $A_{i,CL}^a(r,t)$ represents the monopole (2.1b) and the $a_i(r,t)$ $i = 0,1$, are the quantum fields.

The potential (2.5) induces the electric field

$$E_i^a = \hat{\tau}_a \hat{\tau}_i E_L + (\delta_{ia} - \hat{\tau}_i \hat{\tau}_a) E_{T,1} + \epsilon_{aij} \hat{\tau}_j E_{T,2} \tag{2.7a}$$

with

$$E_L = \dot{a}_1 - \dot{a}_0, \quad E_{T,1} = a_0 K / \sigma, \quad E_{T,2} = 0 \tag{2.7b}$$

and the magnetic field

$$B_i^a = \hat{\tau}_a \hat{\tau}_i B_L + (\delta_{ia} - \hat{\tau}_i \hat{\tau}_a) B_{T,1} + \epsilon_{aij} \hat{\tau}_j B_{T,2} \tag{2.8a}$$

$$B_L = -(1 - K^2) / \sigma^2, \quad B_{T,1} = K' / \sigma, \quad B_{T,2} = -a_1 K / \sigma \tag{2.8b}$$

where a dot (dash) denotes differentiation with respect to $t(r)$.

The residual gauge freedom left over, after symmetry breaking with the Higgs mechanism is parametrized by

$$U(\sigma, t) = \exp [i \lambda(\sigma, t) \hat{\tau} / 2] \tag{2.9}$$

$\lambda(r,t)$ is the gauge function, and the τ_i are the SU(2) generators. Equation (2.9) induces the following transformation of the quantum fields

$$a_0^U = a_0 + \dot{\lambda} \quad a_1^U = a_1 + \dot{\lambda} \quad (2.10)$$

leaving E_L and B_L invariant. On the transversal fields, $E_{T,i}$, $i = 1, 2$, respectively $B_{T,i}$, $U(r, t)$ acts like a rotation with an angle $\lambda(r, t)$.

Inserting (2.7) and (2.8) into the action of the gauge fields

$$S_A = - \frac{1}{2e^2} \int d^4x \, \text{Tr} \, \bar{F}_{\mu\nu} \bar{F}^{\mu\nu} \quad (2.11)$$

and performing the integration over the angular variables, one obtains the two-dimensional system

$$S_A = \frac{2\pi}{e^2} \int dr r^2 dt \left\{ (\dot{a}_0 - \dot{a}_1)^2 + 2(a_0^2 + a_1^2) K^2 / r \right\} \quad (2.12)$$

In going from (2.11) to (2.12), the static energy of the monopole has been dropped.

The topological density

$$\gamma = 2 E_i^a B_i^a \quad (2.13)$$

expressed in terms of the decomposition (2.7a) and (2.8a) reads

$$\gamma = 4 \left\{ \beta_{T,1} E_{T,1} + \beta_{T,2} E_{T,2} + \frac{1}{2} \beta_L E_L \right\} \quad (2.14a)$$

$$= \left\{ \partial_r [a_0 (K^2 - 1)] - \partial_t [a_1 (K^2 - 1)] \right\} / r^2 \quad (2.14b)$$

It is invariant under U for arbitrary λ if $r \neq 0$. The point $r = 0$ requires special care, because U is singular at $r = 0$ unless $\lambda(0, t) = 0$. Though the fields E and B themselves do not pick up additional singular pieces under a singular gauge transformation, γ does, due to the behaviour of E_T like $1/r$. Therefore in order not to change the topological charge we must have

$$\lambda(0, t) = 0 \quad (2.15)$$

We shall come back to this point later on.

Let us now turn to the fermion fields. As is well known, in the presence of a monopole field, the usual total angular momentum operator has to be modified. It picks up an additional piece, which is proportional to the isospin $\vec{T}$, i.e.,

$$\vec{J} = \vec{L} + \vec{S} + \vec{T} \quad (2.16)$$

Therefore, angular momentum eigenstates couple spin and isospin. As $\vec{J}$ commutes with the Dirac operator, it is convenient to decompose the Dirac field into eigenstates of definite angular momentum. As has been observed in Ref. 11), the $J = 0$ state is afflicted by a peculiar behaviour. The Dirac Hamiltonian in the presence of a pointlike monopole is self-adjoint only after imposing a particular boundary condition on the zero angular momentum wave function at the origin. The CR effect is strongly related to this peculiarity. In particular, as has been shown by a careful analysis in Ref. 1), the $J \neq 0$ states are not essential for the formation of the condensate, which finally gives rise to the CR effect. Therefore we shall simply neglect all the $J \neq 0$ states right from the beginning. We shall decompose the Dirac spinor ψ into states of definite chirality

$$\psi = \frac{1}{\sqrt{2}} (\psi^+ + \psi^-) \quad , \quad \gamma_5 \psi^\pm = \pm \psi^\pm \quad (2.17)$$

A convenient parametrization of the $J = 0$ partial wave has been proposed by Jackiw and Rebbi¹²⁾

$$X_{\alpha A}^\pm = \frac{-i}{\sqrt{8\pi}} \frac{1}{r} \left[q_\pm \epsilon_{\alpha A} + p_\pm \hat{r}^a \sigma_{\alpha\beta}^a \epsilon_{\beta A} \right] \quad (2.18)$$

where

$$\psi^\pm = \begin{pmatrix} X^\pm \\ \pm X^\pm \end{pmatrix} \quad (2.19)$$

In (2.18), α is a Dirac index, while A is an isospin index. Inserting (2.17)-(2.19) into the fermionic action

$$S_F = \int d^4x \bar{\psi} \gamma^\mu (i\partial_\mu + A_\mu) \psi \quad (2.20)$$

it becomes

$$S_F = \int dr dt \left\{ \bar{\chi}^+ \left[i \not{\partial} - \frac{1}{2} \not{A} \bar{\gamma}^5 + i \frac{K}{r} \bar{\gamma}^5 \right] \chi^+ + \bar{\chi}^- \left[i \not{\partial} + \frac{1}{2} \not{A} \bar{\gamma}^5 + i \frac{K}{r} \bar{\gamma}^5 \right] \chi^- \right\} \quad (2.21)$$

where

$$\chi^\pm = \begin{pmatrix} q^\pm \\ \pm i \rho^\pm \end{pmatrix} \quad (2.22)$$

In (2.21), we have introduced the two-dimensional Dirac algebra defined by

$$\bar{\gamma}^0 = \tau_3, \quad \bar{\gamma}^1 = i \tau_1, \quad \bar{\gamma}^5 = \bar{\gamma}^0 \bar{\gamma}^1 \quad (2.23)$$

and the notation

$$\not{\partial} = \bar{\gamma}^0 \partial_0 + \bar{\gamma}^1 \partial_1 \quad (2.24)$$

$$\not{A} = \bar{\gamma}^0 a_0 - \bar{\gamma}^1 a_1 \quad (2.25)$$

The gauge transformation U acts on χ like a chiral transformation

$$\chi^\pm_\sigma = \exp \left\{ \pm \frac{1}{2} i \lambda(r, t) \bar{\gamma}^5 \right\} \chi^\pm \quad (2.26)$$

(2.21) is invariant under (2.9) and (2.26). (2.12) and (2.21) will be the starting point of our analysis in the next section. In the remainder of this section we shall show how the condensate emerges in the pointlike limit.

For large r , $K(r)$ vanishes exponentially. Therefore restricting our attention to large distances, we may set $K(r)$ in (2.12) and (2.21) to zero. We are then essentially dealing with the massless Schwinger model, which is known to be soluble. However, in contrast to the ordinary Schwinger model, this model is defined only on the half line. This requires that (2.12) and (2.21) have to be supplemented by a boundary condition at $r = 0$. A suitable boundary condition is obtained from analyzing the $r \sim 0$ behaviour of the solutions of the two-dimensional Dirac equation before dropping the K function

$$\left(i \not{\partial} + \frac{1}{2} \not{A} \bar{\gamma}^5 + i \bar{\gamma}^5 K(r)/r \right) \chi^\pm(r) = 0 \quad (2.27)$$

The irregular solution of (2.27) is characterized by $(1-\bar{\gamma}_0)\chi^\pm \sim r^{-1}$. Therefore in order to ensure a regular behaviour we must have

$$(1-\bar{\gamma}^0)\chi^\pm \sim r \quad (2.28)$$

This is the boundary condition adopted in Ref. 1). The $(1-\bar{\gamma}^0)\chi^\pm$ component of the regular solution of (2.27) vanishes in fact with the second power of r . However, the corresponding boundary condition cannot be imposed on solutions of (2.27) with $K \equiv 0$.

We can now solve the CR model in exactly the same way as the original Schwinger model was solved. A combined gauge and chiral transformation decouples the gauge fields and fermions in (2.27) with $K(r) \equiv 0$ completely

$$\chi^\pm = \exp(\pm i \bar{\Phi}) \chi_0^\pm \quad (2.29)$$

where

$$\not{\partial} \bar{\Phi} = -\frac{1}{2} \not{\partial} \bar{\gamma}^5 \quad (2.30)$$

and $\chi_0^\pm$ is a solution of the free two-dimensional Dirac equation which satisfies (2.28). Due to the particular properties of the two-dimensional Dirac algebra $\bar{\Phi}$ can be decomposed into u and v

$$\bar{\Phi} = U \bar{\gamma}^5 + V \quad (2.31)$$

As u corresponds to a gauge transformation U given by (2.9) it has to satisfy

$$U(\sigma=0, t) = 0 \quad (2.32)$$

In the $a_1 = 0$ gauge, (2.30) implies

$$\square U = \frac{1}{2} \dot{a}_0, \quad \square V = -\frac{1}{2} \dot{a}_0, \quad \dot{V} = -\dot{U} \quad (2.33)$$

where

$$\square = \partial_t^2 - \partial_r^2 \quad (2.34)$$

The Green's function $G_{\pm}(r_1, t_1; r_2, t_2)$ of the Dirac operator

$$D = \not{\partial} \pm \frac{1}{2} i \not{\gamma}^5 \quad (2.35)$$

$$i D G_{\pm}(r_1, t_1; r_2, t_2) = \delta(r_1 - r_2) \delta(t_1 - t_2) \quad (2.36)$$

is given by

$$G_{\pm}(r_1, t_1; r_2, t_2) = \exp \left[\pm i (u(r_1, t_1) \bar{\gamma}^5 + v(r_1, t_1)) \right] \cdot G_0(r_1, t_1; r_2, t_2) \exp \left[\pm i (u(r_2, t_2) \bar{\gamma}^5 - v(r_2, t_2)) \right] \quad (2.37)$$

where

$$G_0(r_1, t_1; r_2, t_2) = S(r_1 - r_2, t_1 - t_2) + S(r_1 + r_2, t_1 - t_2) \bar{\gamma}^0 \quad (2.38)$$

and

$$S(r, t) = \frac{1}{2\pi i} \frac{\bar{\gamma}^0 t - \bar{\gamma}^1 r}{t^2 - r^2 - i\epsilon} \quad (2.39)$$

$G_0(r_1, t_1; r_2, t_2)$ satisfies the boundary condition

$$(1 - \bar{\gamma}^0) G_0 \big|_{r_1=0} = G_0 (1 - \bar{\gamma}^0) \big|_{r_2=0} = 0 \quad (2.40)$$

Having the fermion fields and the gauge fields decoupled, the fermion fields can be integrated out, leaving over the effective action

$$S_{eff} = S_A - i \log \text{Det} [i D] \quad (2.41)$$

The fermionic determinant can be calculated in the standard way, e.g., by point splitting. Going into the $a_r = 0$ gauge, we obtain

$$\log \text{Det} [i D] = \frac{-i}{2\pi} \int dr dt \dot{v} a_0 \quad (2.42)$$

Note that the fermionic determinant is not affected by the boundary condition (2.28).

Now we are ready to calculate the chirality breaking condensate. To this end we first calculate the two-point function of the composite chirality carrying operator $\bar{\psi}_+\psi_-$, and then apply clustering arguments to the $|t_2-t_1| \rightarrow \infty$ limit of that Green's function. Neglecting $J \neq 0$ partial waves we have

$$\bar{\psi}_+ \psi_- = \frac{1}{2\pi r^2} \bar{\chi}_+ \chi_- \quad (2.43)$$

As the effective action S_{eff} is quadratic in v , the corresponding functional integral can be carried out easily. Going to Euclidean space-time, we get

$$\begin{aligned} \langle \bar{\chi}_+ \chi_- (\tau_1, t_1) \bar{\chi}_- \chi_+ (\tau_2, t_2) \rangle &= T \tau G_0(\tau_1, t_1; \tau_2, t_2) G_0(\tau_2, t_2; \tau_1, t_1) \\ &\cdot \exp \left[2L^{-1}(\tau_1, t_1; \tau_1, t_1) + 2L^{-1}(\tau_2, t_2; \tau_2, t_2) - 4L^{-1}(\tau_1, t_1; \tau_2, t_2) \right] \end{aligned} \quad (2.44)$$

where L^{-1} is the v field Green's function, defined by

$$\left(\frac{16\pi}{e^2} \square(\tau^2 \square) - \frac{2}{\pi} \square \right) L^{-1}(\tau, t; \tau', t') = \delta(\tau - \tau') \delta(t - t') \quad (2.45)$$

Due to (2.32) and (2.33) the derivative of L^{-1} must vanish at $r = 0$. This determines L^{-1} uniquely to be

$$\begin{aligned} L^{-1}(\tau_1, t_1; \tau_2, t_2) &= -\frac{1}{2} \pi \left\{ \square^{-1}(\tau_1 - \tau_2; t_1 - t_2) + \square^{-1}(\tau_1 + \tau_2; t_1 - t_2) \right. \\ &\quad \left. - M(\tau_1, t_1; \tau_2, t_2) \right\} \end{aligned} \quad (2.46)$$

where

$$\square^{-1}(\tau, t) = \frac{1}{4\pi} \log(\mu^2(\tau^2 + t^2)) \quad (2.47)$$

$$\left(\square_1 - \frac{e^2}{8\pi^2 \tau_1^2} \right) M(\tau_1, t_1; \tau_2, t_2) = \delta(\tau_1 - \tau_2) \delta(t_1 - t_2) \quad (2.48)$$

and

$$\partial_{\tau_1} M(\tau_1, t_1; \tau_2, t_2) \Big|_{\tau_1=0} = 0 \quad (2.49)$$

(2.48) and (2.49) are solved by the Legendre function Q_ν

$$M(\tau_1, t_1; \tau_2, t_2) = -\frac{1}{2\pi} Q_\nu \left(\frac{\tau_1^2 + \tau_2^2 + (t_1 - t_2)^2}{2\tau_1\tau_2} \right) \quad (2.50)$$

with

$$\nu = \left(\frac{1}{4} + \frac{e^2}{8\pi^2} \right)^{1/2} - \frac{1}{2} \quad (2.51)$$

Inserting (2.46)-(2.51) into (2.44) and letting $|t_1 - t_2|$ go to infinity, one discovers that the right-hand side of (2.44) does not vanish. Therefore we must have

$$\langle \bar{\chi}_+ \chi_- (\tau, t) \rangle = \frac{1}{2\pi\tau} (1 + \mathcal{O}(e^2)) \quad (2.52)$$

The reason for the formation of the condensate (2.52) is well known from the Schwinger model¹³⁾. The functional integral representing the left-hand side of (2.44) is saturated by a gauge field configuration, which is a superposition of a vortex and an antivortex located at r_1, t_1 respectively r_2, t_2 . These vortex configurations are responsible for the non-vanishing of (2.52). Note that the functional integral representing the left-hand side of (2.44) is defined only for topological trivial gauge field configurations. Therefore we cannot tolerate singular gauge transformations, i.e., we must have (2.32).

3. - THE FERMION PROPAGATOR IN THE EXTENDED MONOPOLE BACKGROUND

In the last section we have shown how the CR condensate is calculated in the limit of a pointlike monopole, i.e., for large distances. This amounted essentially to setting the function $K(r)$ everywhere to zero, and imposing a suitable boundary condition on the fermion fields at $r = 0$. In order to take the core structure of the magnetic monopole fully into account, we must keep the function $K(r)$ in (2.12), (2.21) and (2.27). In this section we shall focus our attention on the fermion action with $K(r)$ non-vanishing. The next section is devoted to the effect of keeping $K(r)$ different from zero in the gauge field action.

The presence of the $\bar{\gamma}^5 K(r)/r$ term in (2.27) has far reaching consequences concerning the explicit solvability. It acts essentially like a mass term. Therefore, the associated Schwinger model is no longer solvable exactly.

In principle, a solution can be constructed with the help of a transformation (2.29), where, however, Φ does not have the particular form (2.30), (2.31). The matrix Φ of such a transformation has to be constructed in such a way that it eliminates both the a_μ and the K fields from (2.27). This may be achieved, e.g., by representing Φ as an infinite series, where the single terms are determined recursively. However, all but the lowest order terms of such an expansion seem to be too complex to be determined explicitly. Therefore, here we shall choose an alternative route.

Suppose we start again with the transformation (2.29)-(2.31). Then $\chi_0^\pm$ appears as a solution of

$$(i\not{\partial} + i\bar{\gamma}^5 K(r)/r e^{\pm 2i\sigma\bar{\gamma}^5}) \chi_0^\pm = 0 \quad (3.1)$$

Though now we do not have to impose (2.28) on the field $\chi_0^\pm$, we still must have (2.32) for $u(r,t)$. As the function $K(r)$ is non-vanishing only in a small neighbourhood of $r = 0$, it seems to be a good approximation to substitute $u(0,t)$ for $u(r,t)$ in (3.1). Notice in particular that the corrections are non-singular at $r = 0$. In this way, we obtain the Dirac equation in the presence of a non-Abelian monopole, however, without the additional gauge field a_μ .

This equation has been solved in the BPS limit by Marciano and Muzinich¹⁴⁾. In Ref. 15), these solutions have been used to construct the corresponding Green's function. However, here we are not interested in the full Green's function. Rather we only require its limiting form for large time separations. This can be obtained without knowing $K(r)$ explicitly.

Suppose we have found a complete set of solutions $\tilde{\chi}_E(r)$ of the equation

$$(\bar{\gamma}^0 E + i\bar{\gamma}^1 \partial_\tau + i\bar{\gamma}^5 K(r)/r) \tilde{\chi}_E(r) = 0 \quad (3.2)$$

then the Green's function $G_0(r_1, t_1; r_2, t_2)$ is given by

$$G_0(\tau_1, t_1; \tau_2, t_2) = \int_0^\infty \frac{dE}{2\pi} e^{-iE(t_1-t_2)} \tilde{G}(E; \tau_1, \tau_2) \quad (3.3)$$

where

$$\tilde{G}(E; \tau_1, \tau_2) = \tilde{\chi}_E(\tau_1) \tilde{\chi}_E^*(\tau_2)$$

In the limit of large time separation, $|t_1 - t_2| \rightarrow \infty$, we can approximate $\tilde{G}(E; r_1, r_2)$ by $\tilde{G}(0; r_1, r_2)$. Therefore we only need to know the solution of (3.2) for $E = 0$. It is given by

$$\tilde{\chi}_0 = \exp(\bar{y}^0 \log \xi(\tau)) \chi_c \quad (3.4)$$

where χ_c is a constant vector, and

$$\log \xi(\tau) = \int_{\tau_0}^{\tau} d\tau' \frac{K(\tau')}{\tau'} \quad (3.5)$$

(3.4) can be rewritten as

$$\tilde{\chi}_0 = \frac{1}{2} \left\{ \xi(\tau) (1 + \bar{y}^0) + \xi(\tau)^{-1} (1 - \bar{y}^0) \right\} \chi_c \quad (3.6)$$

As ξ vanishes like r for small r , χ_c must be annihilated by $(1 - \bar{y}^0)$ in order for $\tilde{\chi}_0$ to be normalizable. Therefore we finally obtain

$$\tilde{\chi}_0 = C \xi(\tau) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (3.7)$$

with C a normalization constant. It is now a simple task to show that the presence of the function $\xi(r)$ has the effect of multiplying the condensate in the form given by (2.51) by a factor

$$\frac{1}{2} C^2 \xi^2(r) \quad (3.8)$$

Let us briefly discuss the qualitative behaviour of $\xi(r)$. As already pointed out, for small values of r , $\xi(r)$ vanishes like r . Therefore the inclusion of the core effects leads to a suppression of the condensate at small distances. For large values of r , $\xi(r)$ goes into a constant, leaving the behaviour of the condensate as obtained in the pointlike limit unchanged, as had to be expected. In the BPS limit, the function $\xi(r)$ can be calculated explicitly. It turns out to be

$$\xi(\tau) = \hbar \frac{1}{2} m_w \tau \quad (3.9)$$

4. - THE ν FIELD PROPAGATOR IN THE EXTENDED MONOPOLE BACKGROUND

The previous section has been devoted to the study of the consequences of keeping the K function in the fermionic action (2.21). In this section we shall study the consequences of keeping the full monopole core structure in the gauge field action (2.12).

However, before doing so, let us take again a look at the pointlike limit. It is instructive to analyze in greater detail how the condensate and the propagator L^{-1} defined by (2.45) are related. As can be seen from (2.44), the calculation of the condensate does not necessarily require the knowledge of L^{-1} for all possible values of its arguments. What has to be known is: (i) $\lim_{|t_1-t_2| \rightarrow \infty} L(r_1, t_1; r_2, t_2)$ and (ii) $L(r_1, t_1; r_1, t_1)$. The first quantity has to behave exactly like $-\frac{1}{2} \log(t_1-t_2)^2$ plus terms which are independent of $|t_1-t_2|$, in order to cancel the fall-off of $G_0(r_1, t_1; r_2, t_2)$ in (2.44). Otherwise the condensate does not exist. The function (ii) determines the shape of the condensate. Comparing the large $|t_1-t_2|$ behaviour of the three pieces L^{-1} consists of, [compare (2.46)] one discovers that $\square^{-1}(r_1-r_2, t_1-t_2)$ and $\square^{-1}(r_1+r_2, t_1-t_2)$ supply equal contributions to (i), whereas M is negligible. Notice that the appearance of $\square^{-1}(r_1+r_2, t_1-t_2)$ is intimately related to the boundary condition (2.32). For (ii) the situation is almost reversed. Here $\square^{-1}(r_1-r_2, t_1-t_2)$ and $M(r_1, t_1; r_2, t_2)$ become singular individually, however $L^{-1}(r_1, t_1; r_2, t_2)$ is finite. $\square^{-1}(r_1-r_2, t_1-t_2)$ does not contribute to the final form of the condensate.

At first sight, it might be puzzling that the condensate, as given by (2.52), does not depend on the electric charge, whereas (2.45) and (2.48) obviously do. The short-distance structure of $M(r_1, t_1; r_2, t_2)$ is determined by the $\square$ piece of the operator on the left-hand side of (2.48). Therefore it does not depend on e^2 . However, this does not mean that we can simply set $e^2 = 0$ from the beginning if we are only interested in the short-distance behaviour. The crucial point is the boundary condition (2.49). We have $\partial_r Q_\nu \sim r^\nu$ for small r . Therefore setting $e^2 = 0$ in (2.48) runs into conflict with (2.49).

Let us now turn to the effects induced by keeping the full $K(r)$ in the gauge action (2.12). Expressing a_0 in terms of the v field, the action becomes non-local. The corresponding equation for $L^{-1}(r_1, t_1; r_2, t_2)$ is given by

$$\left(\frac{16\tilde{\eta}}{e^2} \square_1 (\tau_1^2 \square_1) - \frac{2}{\tilde{\eta}} \square_1 \right) L^{-1}(\tau_1, t_1; \tau_2, t_2) + \frac{32\tilde{\eta}}{e^2} \square_1 \int_0^{\tau_1} d\tau' K^2(\tau') \int_{\tau'}^{\infty} d\tau_3 \square_3 L^{-1}(\tau_3, t_1; \tau_2, t_2) = \delta(\tau_1 - \tau_2) \delta(t_1 - t_2) \quad (4.1)$$

The time derivative in $\square_3$ acts on t_1 . We have chosen the boundaries of the $d\tau'$ and $d\tau_3$ integration in such a way that potential boundary terms possibly contributing to (4.1) are automatically eliminated. We start now again with the ansatz (2.46). Inserting (2.46) into (4.1) leads to

$$\frac{1}{2} \tau_1^2 [\delta(\tau_1 - \tau_2) \delta(t_1 - t_2) - \square_1 M(\tau_1, t_1; \tau_2, t_2)] + \frac{e^2}{16\tilde{\eta}^2} M(\tau_1, t_1; \tau_2, t_2) + \int_0^{\tau_1} d\tau' K^2(\tau') \int_{\tau'}^{\infty} d\tau_3 [\delta(\tau_3 - \tau_2) \delta(t_1 - t_2) - \square_3 M(\tau_3, t_1; \tau_2, t_2)] = 0 \quad (4.2)$$

In contrast to the situation encountered in Section 2, the presence of the K function prevents the reduction of (4.2) into a differential equation for $M(r_1, t_1; r_2, t_2)$ simply by extracting a factor of $r_1^2/2$. Nevertheless it is possible to bring (4.2) into a form similar to (2.48).

Suppose we know the auxiliary Green's function $H(r_1, r_2)$ defined by

$$\left[\frac{1}{2} \partial_{\tau_1} (\tau_1^2 \partial_{\tau_1}) - K^2(\tau_1) \right] H(\tau_1, \tau_2) = \delta(\tau_1 - \tau_2) \quad (4.3)$$

Then we can eliminate the inhomogeneous term in (4.2) by acting with

$$\partial_{\tau_3} \int_0^{\infty} d\tau_1 H(\tau_1, \tau_2) \partial_{\tau_1} \quad (4.4)$$

from the left on (4.2), and performing a partial integration, leading to

$$\left\{ \square_1 M(\tau_1, t_1; \tau_2, t_2) + \frac{e^2}{16\pi^2} \partial_{\tau_1} \int_0^\infty d\tau \partial_\tau H(\tau, \tau_1) M(\tau, \tau_2) \right\} = \delta(\tau_1 - \tau_2) \delta(t_1 - t_2) + B.T. \quad (4.5)$$

where B.T. stands for

$$\frac{1}{2} \partial_{\tau_1} \left\{ -H(\tau, \tau_1) \tau^2 \left[\square M(\tau, \tau_2) - \frac{e^2}{8\pi^2 \tau^2} M(\tau, \tau_2) - \delta(\tau - \tau_2) \right] \right\} \Bigg|_{\tau=0}^{\tau=\infty} \quad (4.6)$$

(4.5) is the generalization of (2.48).

We shall now show how the auxiliary Green's function $H(r_1, r_2)$ can be determined explicitly in the BPS limit. With $K(r)$ given by (2.4b) the second order differential equation defined by (4.3) has two solutions

$$U_1(\tau) = \frac{1}{\tau m_w} \cosh \tau m_w \quad \text{and} \quad U_2(\tau) = \cosh \tau m_w - \frac{1}{\tau m_w} \quad (4.7)$$

These solutions form a fundamental system, i.e., their Wronskian

$$W(\tau) \equiv U_1 U_2' - U_2 U_1' = (\tau^2 m_w)^{-1} \quad (4.8)$$

is non-vanishing. Therefore they can be used to construct $H(r_1, r_2)$. In order to determine $H(r_1, r_2)$ uniquely, we must impose boundary conditions. We shall require that for large r (4.5) goes over into (2.48). This requirement can be met by letting $H(r_1, r_2)$ vanish for all values of the arguments which satisfy $r_1 < r_2$. With this boundary condition, the auxiliary Green's function becomes

$$H(\tau_1, \tau_2) = 2m_w \Theta(\tau_1 - \tau_2) [U_1(\tau_2) U_2(\tau_1) - U_1(\tau_1) U_2(\tau_2)] \quad (4.9)$$

Inserting (4.9) back into (4.5), we finally obtain

$$\left\{ \left[\square_1 - \frac{e^2}{8\pi^2 \tau_1^2} \right] M(\tau_1, t_1; \tau_2, t_2) + \frac{e^2 m_w}{8\pi^2} \int_{\tau_1}^\infty d\tau [U_1'(\tau_1) U_2'(\tau) - U_1'(\tau) U_2'(\tau_1)] M(\tau, t_1; \tau_2, t_2) \right\} = \delta(\tau_1 - \tau_2) \delta(t_1 - t_2) + B.T. \quad (4.10)$$

where the $e^2/8\pi^2 \tau_1^2$ term is a consequence of (4.8).

Comparing (4.10) with (2.47) one discovers that the K function contributes just the integral part to (4.10). For large r we have

$$U_1'(\tau_1) U_2'(\tau) - U_1'(\tau) U_2'(\tau_1) = 4 \left[e^{-2\tau_1 m_w} / \tau_2^2 - e^{-2\tau_2 m_w} / \tau_1^2 \right] (1 + O(1/\tau)) \quad (4.11)$$

This confirms that (2.48) is indeed the large distance limit of (4.10). As has been discussed at the beginning of this section, the condensate is determined by (i) $\lim_{(t_1-t_2) \rightarrow \infty} M$ and by (ii) $M(r, t; r, t)$. As the logarithmic growth of $\square^{-1}(r_1-r_2; t_1-t_2)$ and $\square^{-1}(r_1+r_2, t_1-t_2)$ for large time separations is a consequence of the infra-red singularities associated with the massless Schwinger model, whereas in (4.10) there appears an effective mass term, we expect M to vanish for large time separations, even after having included the K function. Let us therefore turn to $M(r, t; r, t)$.

In order to gain some insight into the corrections caused by the additional term, we have evaluated M by using the approximation (4.11) and iterating (4.10) once, i.e.,

$$M(\tau_1, t_1; \tau_2, t_2) = M_0(\tau_1, t_1; \tau_2, t_2) + \Delta(\tau_1, t_1; \tau_2, t_2) \quad (4.12)$$

where

$$\Delta(\tau_1, t_1; \tau_2, t_2) = - \frac{e^2 m_w}{8\pi^2} \int_{-\infty}^{+\infty} dt \int_0^{\infty} d\sigma' M_0(\tau_1, t_1; \sigma', t') \cdot \int_{\sigma'}^{\infty} d\sigma'' [U_1'(\sigma') U_2'(\sigma'') - U_1'(\sigma'') U_2'(\sigma')] M_0(\sigma'', t''; \tau_2, t_2) \quad (4.13)$$

and $M_0(r_1, t_1; r_2, t_2)$ is the exact solution in the absence of the K function given by (2.50). Evaluating (4.13) in the limit of large $r_1 = r_2 \equiv r$ at $t_1 = t_2 \equiv t$ we obtain

$$\Delta(\tau, t; \tau, t) = - \frac{1}{2\pi^2} \frac{1}{m_w \tau} (1 + O(e^2, 1/(m_w \tau))) \quad (4.14)$$

Note that despite the appearance of one power of e^2 in front of the definition of $\Delta(r, t; r, t)$ (4.13), $\Delta(r, t; r, t)$ is not suppressed by any power of e^2 . The inverse power of e^2 , which cancels the factor e^2 explicitly appearing in (4.13)

results from the dr' integral over $M_0(r_1, t_1; r', t')$ which has a singularity at $r' = 0$ if we set $e^2 = 0$ in the integrand. Furthermore, though the correction in (4.10) which is due to the K function is exponentially suppressed for large r , $\Delta(r, t; r, t)$ is only power-law suppressed.

5. - DISCUSSION

Let us first combine the results of the two preceding sections. We have shown that in the BPS limit the chirality violating condensate behaves like

$$\langle \bar{\Psi} \Psi \rangle = \frac{1}{(2\pi)^2} \frac{1}{r^3} \text{th} \frac{m_W r}{2} e^{-1/(2\pi m_W r)} (1 + O(e^2, (m_W r)^{-1})) \quad (5.1)$$

The factor $\text{th}(m_W r/2)$ is a consequence of the fact that the fermions pick up an effective, r -dependent mass, whereas the exponential is a consequence of the photon becoming effectively massive in the vicinity of a non-Abelian monopole. Therefore we infer that at small distances the second effect is the dominant one.

The result indicates that the condensate vanishes at the origin. However, one has to be careful with such an interpretation. (5.1) is strictly valid only for large r . Therefore it can only be considered to give an indication of the scale at which the finite size effects become relevant. In general, as the CR model contains only those degrees of freedom which are important at large distances, it does not allow any conclusion about the dynamics inside the monopole core.

We have kept the fermions massless throughout. A realistic estimate of the qualitative behaviour of the condensate at large distances¹⁶⁾ has to include fermion masses. On general grounds^{2), 3)}, it has to be expected that the condensate is exponentially suppressed at distances large compared to the Compton wavelength of the fermion. We have therefore two competing suppression mechanisms.

As (5.1) shows, the scale at which the finite monopole size effects become negligible is typically given by m_W . In contrast to this result, in Ref. 17), it

has been argued that finite size effects die out for distances larger than $1/(m_W e^2)$. We believe that this discrepancy has to be attributed to the fact that in 17) the core effects were effectively taken into account by shifting both, the boundary condition (2.28) as well as (2.32) to $r = r_0 \sim 1/m_W$. This approximation has been used by several authors¹⁸⁾⁻²⁰⁾. However, we believe that this approximation is only justified if it is supplemented by the limit $r_0 \rightarrow 0$. Therefore it does not allow any conclusion about finite size effects. Though for fermions imposing a boundary condition at $r = r_0$ mainly amounts to approximate the K function by a step function, and therefore is perfectly legitimate, the requirement of the gauge transformation associated with v being non-singular, forces us to fix u at $r = 0$.

In this paper we have only considered the chirality violating condensate of 1). It would obviously be of much interest to apply our approach as well to the charge violating condensate considered in 18)-21).

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