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MAGNITUDE OF $O(\alpha^2)$ CORRECTIONS AND SEARCH
FOR NEW EFFECTS IN THE ELECTROWEAK INTERACTIONS

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ABSTRACT

Using renormalization scheme independence of physical quantities, the magnitude of $O(\alpha^2)$ radiative corrections in the electroweak theory is estimated in terms of the $O(\alpha)$ corrections which are known for various processes. $O(\alpha^2)$ corrections to such quantities like vector boson masses or neutral current cross-sections turn out to be of the order of a tenth of a per cent. They are comparable to the one-loop corrections coming from such new effects like heavy fermions or supersymmetric contributions.

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1. - INTRODUCTION

Testing $O(\alpha)$ corrections in the electroweak theory does not lie in a very far future. It seems that new experiments at the $\bar{p}p$ collider and at LEP will be able to achieve the necessary accuracy¹⁾. One would then like to know whether such high precision data could also, eventually, give some evidence for new effects going beyond the minimal standard model.

Many modifications of the minimal model are conceivable and have been discussed in the recent literature [see, e.g., Ref. 2) and references therein]. Some can affect even the tree level effective Fermi Lagrangian, other manifest themselves in higher order corrections to it, beginning at the one-loop level. Simple qualitative arguments suggest that such new one-loop effects may be rather small, in fact comparable to the $O(\alpha^2)$ corrections in the minimal model, and therefore hard to detect.

Indeed, let us first focus on the one-loop ($O(\alpha G_F)$) corrections in the minimal model: explicit calculations show that $O(\alpha G_F) \sim O(1/10) \times O(G_F)$ rather than $O(1/100) \times O(G_F)$ as one could naively expect. This is due to large logarithms $\ln(m/M_W)$, where m is a light fermion mass. Consequently, one can also expect that the two-loop corrections in the minimal model satisfy the following order of magnitude equation:

$$O(\alpha^2 G_F) \sim O\left(\frac{1}{10}\right) \times O(\alpha G_F) \sim O\left(\frac{1}{100}\right) \times O(G_F)$$

On the other hand, many of the new effects considered in the literature involve new particles with masses $m \sim O(M_W)$. No large logs are then present in the one-loop contributions of such particles and therefore, one may expect these contributions to be close to the naive expectation $O(1/100) \times O(G_F)$ and thus comparable to the $O(\alpha^2 G_F)$ corrections in the minimal model.

In the present paper, using renormalization scheme independence of physical quantities, we express some combinations of the $O(\alpha^2)$ corrections in terms of the $O(\alpha)$ corrections known for various processes. These exact relations provide us with a numerical estimate of the magnitude of the $O(\alpha^2)$ corrections.

Next, we collect a sample of results on two extensions beyond the minimal model: heavy fermion contribution and supersymmetric extension [for a review, see Ref. 3)]. Explicit calculation of the one-loop $O(\alpha_{\text{strong}})$ supersymmetric

contribution to the ratio $R_\nu = \sigma_{\nu N}^{\text{NC}} / \sigma_{\nu N}^{\text{CC}}$ is included in the last section.

A comparison of the one-loop new effects with the estimated magnitude of the $O(\alpha^2)$ corrections in the minimal model confirms the qualitative expectations summarized in the beginning of this introduction. This is an obvious theoretical difficulty in attempts to discriminate between the minimal model and certain of its non-minimal extensions in high precision standard measurements.

2. - ESTIMATE OF $O(\alpha^2)$ CORRECTIONS IN THE MINIMAL ELECTROWEAK THEORY

Apart from the fermion mass matrix and the Higgs sector parameters, the Glashow-Salam-Weinberg (GSW) model is a three parameter theory. Therefore, any physical quantity can be calculated in terms of three input quantities taken from experiment. The electric charge and the Fermi constant are known with high accuracy and it is customary to use them always as input quantities in the theoretical calculations. This will be implicitly understood in our considerations.

Let us consider three physical quantities x , y and z (neither of them is e or G_F). Order by order in perturbation expansion in α , the theory predicts, say, quantities x and z in terms of y :

$$x = f(y) = f^{(n)}(y) + \Delta f^{(n)}(y) \quad (1a)$$

$$z = g(y) = g^{(n)}(y) + \Delta g^{(n)}(y) \quad (1b)$$

(y scheme)

where $x^{(n)} = f^{(n)}(y)$ is called the result in the n^{th} order and $\Delta f^{(n)}(y)$ is called the sum over all radiative corrections of order higher than n .

A specification of the experimental input of the theory belongs to a choice of a definite renormalization scheme. Therefore, we can equally well choose the z scheme in which the quantity z is our input quantity and calculate the quantities x and y in terms of it. Analogously to Eqs. (1), we have now

$$z = \bar{z} \quad (2a)$$

$$x = h(z) = h^{(n)}(z) + \Delta h^{(n)}(z) \quad (z \text{ scheme}) \quad (2b)$$

$$y = G(z) = G^{(n)}(z) + \Delta G^{(n)}(z) \quad (2c)$$

where $x^{(n)} = h^{(n)}(z)$, $\Delta h^{(n)}(z)$ is the sum over all radiative corrections of order higher than n and similarly for $G^{(n)}(z)$ and $\Delta G^{(n)}(z)$. Finally, we can consider the x scheme in which

$$x = \bar{x} \quad (3a)$$

$$y = F(x) = F^{(n)}(x) + \Delta F^{(n)}(x) \quad (x \text{ scheme}) \quad (3b)$$

$$z = H(x) = H^{(n)}(x) + \Delta H^{(n)}(x) \quad (3c)$$

From the renormalization scheme independence of the physical quantities, x , y , z it follows that, e.g.:

$$\begin{aligned} x &= f(F(x)) \\ y &= G(g(y)) \\ z &= g(G(z)) \\ h(z) &= f(G(z)) \\ f(y) &= h(g(y)) \end{aligned} \quad (4)$$

etc., for any value of the respective variable. We also observe that in the zeroth order approximation, $O(G_F)$ similar relations hold by definition. For example,

$$\begin{aligned} x &= f^{(0)}(F^{(0)}(x)) \\ h^{(0)}(z) &= f^{(0)}(G^{(0)}(z)) \end{aligned} \quad (5)$$

etc., for any value of x and z , respectively. The latter relations follow simply from the fact that a convolution of two zeroth order relations gives again a zeroth order relation. However, at any order $n \neq 0$ of perturbation theory, the renormalization scheme independence, as expressed by relations (4), is equivalent to the following relations:

$$x = f^{(n)}(F^{(n)}(x)) + O(\alpha^{n+1}) \quad (6)$$

$$h^{(n)}(z) = f^{(n)}(G^{(n)}(z)) + O(\alpha^{n+1}) \quad (7)$$

etc., again for any value x and z , respectively.

We can make use of relations (4) or (6) and (7) to estimate order α^2 corrections in terms of order α contributions. Let us specify the relation (6) as follows:

$$f^{(1)}(F^{(1)}(x)) = x + O(\alpha^2) \quad (8)$$

Calling Δf_1 and ΔF_1 the order α contributions [distinguish them from $\Delta f^{(1)}$ and $\Delta F^{(1)}$ which we call sums over contributions of order α^2 and higher] and expanding, we get

$$\begin{aligned} f^{(1)}(F^{(1)}(x)) &= f^{(0)}(F^{(0)}(x) + \Delta F_1(x)) + \Delta f_1(F^{(0)}(x) + \Delta F_1(x)) = \\ &= x + E(\Delta f_1(x), \Delta F_1(x)) \end{aligned} \quad (9)$$

where

$$\begin{aligned} E(\Delta f_1(x), \Delta F_1(x)) &= \sum_{k=1}^{\infty} \frac{d f^{(0)}(y)}{d y^k} \Big|_{y=F^{(0)}(x)} \cdot \frac{[\Delta F_1(x)]^k}{k!} + \\ &+ \Delta f_1(F^{(0)}(x)) + \sum_{k=1}^{\infty} \frac{d \Delta f_1}{d y} \Big|_{y=F^{(0)}(x)} \cdot \frac{[\Delta F_1(x)]^k}{k!} \end{aligned} \quad (10)$$

We conclude, by comparing with (8), that a certain non-linear function $E[\Delta f_1(x), \Delta F_1(x)]$ of first order corrections is $O(\alpha^2)$. Thus, Eq. (10) implies

$$\left. \frac{df^{(0)}}{dy} \right|_{y=F^{(0)}(x)} \cdot \Delta F_1(x) + \Delta f_1(F^{(0)}(x)) = 0 \quad (11)$$

Now, we can repeat our procedure starting with

$$f^{(2)}(F^{(2)}(x)) = x + O(\alpha^3) \quad (12)$$

Calling Δf_2 and ΔF_2 the order α^2 contributions, we get

$$E(\Delta f_1(x) + \Delta f_2(x), \Delta F_1(x) + \Delta F_2(x)) = O(\alpha^3) \quad (13)$$

Thus, in this particular combination, corrections of order α and α^2 must cancel up to terms $O(\alpha^3)$. Repeating our argument for $n \rightarrow \infty$, we get

$$E(\Delta f^{(0)}(x), \Delta F^{(0)}(x)) = 0 \quad (14)$$

which we can also obtain by inserting $f = f^{(0)} + \Delta f^{(0)}$ and $F = F^{(0)} + \Delta F^{(0)}$ directly into (4).

Neglecting the $O(\alpha^3)$ terms in Eqs. (13) or (14) and using (11), one has

$$\begin{aligned} & \left. \frac{df^{(0)}(y)}{dy} \right|_{y=F^{(0)}(x)} \cdot \Delta F_2(x) + \Delta f_2(F^{(0)}(x)) = \\ & = - \left[\left. \frac{d\Delta f_1}{dy} \right|_{y=F^{(0)}(x)} \cdot \Delta F_1(x) + \frac{1}{2} \left. \frac{d^2 f^{(0)}(y)}{dy^2} \right|_{y=F^{(0)}(x)} \cdot (\Delta F_1(x))^2 \right] \end{aligned} \quad (15)$$

which is the result we are looking for: it gives a combination of the order α^2 corrections ΔF_2 and Δf_2 in terms of order α corrections.

Quite analogously, using relation (7) we can derive the following:

$$\Delta h_1(z) = E(\Delta f_1(z), \Delta G_1(z)) + O(\alpha^2) \quad (16)$$

$$\Delta h_1(z) + \Delta h_2(z) = E(\Delta f_1(z) + \Delta f_2(z), \Delta G_1(z) + \Delta G_2(z)) + O(\alpha^3) \quad (17)$$

etc., or

$$\Delta h^{(0)}(z) = E(\Delta f^{(0)}(z), \Delta G^{(0)}(z)) \quad (18)$$

Neglecting terms $O(\alpha^3)$ we get now

$$\begin{aligned} \frac{df^{(0)}(y)}{dy} \Big|_{y=G^{(0)}(z)} \cdot \Delta G_2(z) + \Delta f_2(G^{(0)}(z)) - \Delta h_2(z) = \\ = - \left[\frac{d\Delta f_1}{dy} \Big|_{y=G^{(0)}(z)} \cdot \Delta G_1(z) + \frac{1}{2} \frac{df^{(0)}(y)}{dy} \Big|_{y=G^{(0)}(z)} \cdot (\Delta G_1(z))^2 \right] \end{aligned} \quad (19)$$

We are again able to express certain linear combinations of order α^2 corrections in terms of order α corrections.

3. - EXAMPLES

We apply now the results of the previous section to study the $O(\alpha^2)$ corrections to $M_W = M_W(M_Z)$ and $M_Z = M_Z(M_W)$ relations. However, instead of M_W and M_Z , it will be more convenient for us to use the dimensionless parameters w and z defined as follows:

$$w = \frac{4A}{M_W^2}, \quad z = \frac{4A}{M_Z^2} \quad (20)$$

where $A = \pi\alpha/\sqrt{2} G_F = (37.281 \text{ GeV})^2$. We also adopt a notation in which $w = w^{(n)}(z) + \Delta w^{(n)}(z)$, etc. Now (with x replaced by w and y by z), Eq. (15) reads:

$$\Delta W_2(z^{(0)}(w)) + \left. \frac{dW^{(0)}}{dz} \right|_{z=z^{(0)}(w)} \cdot \Delta z_2(w) =$$

$$= - \left\{ \left. \frac{d\Delta W_1}{dz} \right|_{z=z^{(0)}(w)} \cdot \Delta z_1(w) + \frac{1}{2} \left. \frac{dW^{(0)}}{dz} \right|_{z=z^{(0)}(w)} \cdot [\Delta z_1(w)]^2 \right\} \quad (21)$$

With ⁴⁾

$$W^{(1)}(z) = W^{(0)}(z) + \Delta W_1(z) = 2(1 - \sqrt{1-z}) + \frac{(1 - \sqrt{1-z})^2}{\sqrt{1-z}} \cdot \mathcal{J}_\mu(z) \quad (22)$$

$$z^{(1)}(w) = z^{(0)}(w) + \Delta z_1(w) = w - \frac{1}{4} w^2 - \frac{1}{4} w^2 \mathcal{J}_\mu(w) \quad (23)$$

[δ_μ is the radiative correction to the muon decay rate; numerically $\delta_\mu \approx 0.07^{(4)-7)}$], we obtain

$$\Delta W_2(z^{(0)}(w)) + \frac{2}{2-w} \cdot \Delta z_2(w) =$$

$$= \frac{w^3}{(2-w)^3} \left(1 - \frac{3}{8} w\right) \mathcal{J}_\mu^2 - \frac{1}{8} \frac{w^4}{2-w} \mathcal{J}_\mu \frac{d\mathcal{J}_\mu}{dz} \quad (24)$$

For the actual experimental value of $M_W \approx 82$ GeV ($w \approx 0.83$) and for $d\delta_\mu/dz = -0.036$ [the last number can be extracted from Refs. 4), 5) and 8)]

$$\Delta W_2(z^{(0)}(w)) + 1.70 \Delta z_2(w) \approx 0.0013 \quad *) \quad (25)$$

Of course, the right-hand side of (24) [and thus that of (25)] depends on the value of w (M_W). We have illustrated this dependence in Fig. 1.

Equation (25) should be compared with the first order corrections: $\Delta w_1(z) \approx 0.019$ and $\Delta z_1(w) \approx -0.012$. One should also keep in mind that our result does not

*) Equation (25) can be easily translated into the relation for the corrections to M_W and M_Z using $\Delta w_2 = -2w \Delta M_{W2}/M_W + 3w(\Delta M_{W1})^2/M_W^2 = -2w \Delta M_{W2}/M_W + 3/4w \cdot (\Delta W_1)^2$ and analogous formula for Δz_2 .

give any constraint from above on the $O(\alpha^2)$ corrections ^{*}).

Similarly, we can consider for instance, $\sigma_{\nu \mu e} = \sigma_{\nu \mu e}^{(M_Z)}$ and $\sigma_{\nu \mu e} = \sigma_{\nu \mu e}^{(M_W(M_Z))}$. Once again, it is convenient to use a dimensionless quantity

$$X = \frac{12 A^2}{\pi \alpha^2 m_e} \cdot \frac{\sigma_{\nu e}}{E_{\nu \mu}} \quad (26)$$

We have ⁷⁾

$$\begin{aligned} X^{(4)}(w) &= X^{(0)}(w) + \Delta X_1(w) = \\ &= w^2 - 3w + 3 + w^2 \mathcal{J}_\nu(w) - 3w [\mathcal{J}_\nu(w) - \mathcal{J}_\mu(w)] + \\ &+ 3 [\mathcal{J}_\nu(w) - 2\mathcal{J}_\mu(w)] = \\ &= 1.20 + (-0.10) \end{aligned} \quad (27)$$

$$\begin{aligned} X^{(4)}(z) &= X^{(0)}(z) + \Delta X_1(z) = 4(1 - \sqrt{1-z})^2 - 6(1 - \sqrt{1-z}) + 3 + \\ &+ [4(1 - \sqrt{1-z})^2 - 6(1 - \sqrt{1-z}) + 3] \cdot \mathcal{J}_\nu(z) + \\ &+ [4 \frac{(1 - \sqrt{1-z})^3}{\sqrt{1-z}} - 3 \frac{(1 - \sqrt{1-z})^2}{\sqrt{1-z}} - 6\sqrt{1-z}] \mathcal{J}_\mu(z) = \\ &= 1.23 + (-0.13) \end{aligned} \quad (28)$$

where the numerical values have been obtained with $M_W = 82$ GeV, $M_Z = 93$ GeV and $\delta_\nu = 0.121$ ⁷⁾. Since $d\delta_\nu/dw \approx 0$ ⁷⁾, we get from Eq. (19) (in which we put w instead of y)

$$\Delta X_2(w^{(0)}(z)) - \Delta X_2(z) - 1.4 \Delta w_2(z) \approx -0.004 \quad (29)$$

This result should be compared with $\Delta x_1(w) = -0.10$, $\Delta x_1(z) = -0.13$ and $\Delta w_1(z) = 0.019$.

Finally, let us apply our method to $R(M_W)$ and $M_W(R)$, where R is the ratio of neutral and charged current induced cross-sections in the deep inelastic $\nu_\mu N$ scattering, $R = \sigma_{\nu N}^{cc}/\sigma_{\nu N}^{nn}$. Within the valence parton model approximation for the nucleon structure function we have ⁹⁾

^{*}) Recent explicit calculations of a part of $O(\alpha^2)$ corrections ⁸⁾ to $M_W(M_Z)$ give $\Delta w_2 = 0.0014$.

$$\begin{aligned}
 R^{(1)}(w) &= R^{(0)}(w) + \Delta R_1(w) = \\
 &= \frac{1}{2} \left(1 - \frac{1}{2} w + \frac{5}{54} w^2 \right) + \frac{1}{2} \left\{ \mathcal{J}_R(w) - \frac{1}{2} w [\mathcal{J}_\mu(w) + \mathcal{J}_R(w)] + \right. \\
 &\quad \left. + \frac{5}{54} w^2 [2\mathcal{J}_\mu(w) + \mathcal{J}_R(w)] \right\} = \\
 &= 0.325 - 0.019
 \end{aligned} \tag{30}$$

with $\delta_R = -0.03$ ^{9),10)}. In writing (30), we have neglected the small difference between $\sin^2\theta_W = 1 - M_W^2/M_Z^2$ and $\sin^2\theta_W$ defined in the $\overline{MS}$ renormalization scheme at the scale $\mu = M_W$ ¹¹⁾. Inverting (30) we get^{*})

$$\Delta w_1(R) = -0.14 \tag{31}$$

[for the CDHS¹²⁾ value of $R = 0.307$]. Since using the formulae of Ref. 9), we get

$$\frac{d\mathcal{J}_R}{dw} = 0.024, \tag{32}$$

the second order corrections have to fulfil the relation

$$\Delta R_2(w^{(0)}(R)) - 0.16 \Delta w_2(R) = 0.0006 \tag{33}$$

We see that in this case our estimate of $O(\alpha^2)$ corrections gives the result much below the theoretical uncertainty caused by strong interaction effects¹³⁾.

Our next point is to compare the estimated magnitude of the $O(\alpha^2)$ corrections with some extensions of the minimal model discussed in the literature. First, we recall the results of Refs. 4) and 14) for new heavy fermion generation contributions to $w(z)$ [or $M_W(M_Z)$]. They, of course, depend on the actual values of the masses in the new doublet (and on the mass splitting in particular) but unless the ratio $m_{t'}/m_{b'}$, (where t' and b' are called the up- and down-type member of the new family, respectively) is very large, the order of magnitude of such a contribution to $\Delta w_1(z)$ is¹⁴⁾ $\sim 0.001 \pm 0.002$. This is also true if, for instance,

^{*})The small difference between (31) and a similar relation obtained from Ref. 9), is due to QCD corrections to the parton model which we have neglected.

$m_t, \sim m_b, \gg M_W$ and $m_t/m_b, \sim 1$. Even for $m_t/m_b, \approx 20$ the contribution to $\Delta w_1(z)$ is ⁴⁾ -0.004 and becomes comparable with the standard one-loop effect only for $m_t/m_b, \approx 50$ (that is, for $m_t, > 1$ TeV).

In the supersymmetric extension of the standard model, one-loop corrections to $M_W(M_Z)$ have been studied by Lynn ¹⁵⁾. For $m_t \approx 40$ GeV, the supersymmetric $O(\alpha)$ contribution to $\Delta w_1(z)$ is at most ~ 0.002 .

It is also of some interest to consider supersymmetric contributions to the ratio R . At the one-loop level, there are two types of supersymmetric corrections to R : $O(\alpha_{\text{strong}})$ and $O(\alpha)$. At first sight, one might expect the first one to be more important (they are given by the diagrams in Fig. 2). However, if we neglect the external quark masses, the divergent parts of the self-energy and the vertex corrections cancel each other and, in addition, in the limit of the degenerate squark masses, the finite parts also cancel. This is expected on the basis of the Ward identity ($Z_1 = Z_2$) and it is indeed confirmed by the explicit calculation in the next section. Therefore, the $O(\alpha_s)$ corrections to R are proportional to the difference of squares of squark masses.

The results of the next section show that the supersymmetric $O(\alpha_s)$ corrections to R change δ_R defined below Eq. (30) by less than 0.002. Consequently, the contribution to $\Delta w_1(R)$ is also less than 0.002.

Since $O(\alpha_s)$ corrections to R are small, those of $O(\alpha)$ may be equally or more important. They have been recently calculated too ¹⁶⁾. Numerical results exist for only one supergravity model with extremely light gaugino masses $0.1 \times m_W$. Even in this case, the total supersymmetric contribution to $\Delta w_1(R)$ is ¹⁶⁾ 0.008.

4. - $O(\alpha_s)$ SUPERSYMMETRIC CORRECTIONS TO THE DEEP INELASTIC νN SCATTERING

In this section, we present the results of the calculation of $O(\alpha_s)$ supersymmetric corrections to the deep inelastic νN scattering [see also Ref. 16)]. To start with, we need to formulate the Feynman rules. They are discussed extensively in the literature, for instance in Ref. 3). For completeness and also to settle our notation, we collected the necessary rules for us in Fig. 3.

The coefficients Γ , M and N , which are present in the vertices, are defined as follows: let $u_{iL,R}^0$ ($d_{iL,R}^0$), $i = 1, 2, \dots, n$, be called the up- (down-)

type quark fields which are weak eigenstates and $u_{iL,R}$, $d_{iL,R}$ - fields with well-defined mass (i.e., we consider n generations of fermions and allow them to mix). Then, we define matrices $U_{L,R}$ and $V_{L,R}$ in the flavour space

$$u_{jL}^0 = (U_L)_{ji} u_{iL}, \quad u_{jR}^0 = (U_R)_{ji} u_{iR} \quad (34)$$

$$d_{jL}^0 = (V_L)_{ji} d_{iL}, \quad d_{jR}^0 = (V_R)_{ji} d_{iR} \quad (35)$$

(summation over the repeated indices is assumed). Similarly, scalar fields $\tilde{u}_i^{0\alpha}$ ($\tilde{d}_i^{0\alpha}$), $\alpha = L, R$, associated with quarks u_i^0 (d_i^0) of appropriate helicity can be expressed in terms of the mass matrix eigenstates $\tilde{u}_i^\beta$, $\tilde{d}_i^\beta$ ($\beta = 1, 2$)

$$\tilde{u}_{jL}^{0\alpha} = ({}^u A)_{ji}^{\alpha\beta} \tilde{u}_i^\beta, \quad \tilde{d}_{jR}^{0\alpha} = ({}^d A)_{ji}^{\alpha\beta} \tilde{d}_i^\beta \quad (36)$$

Since the original coupling is of the form

$$\begin{aligned} \mathcal{L} = & \dots - g_s \sqrt{2} \cdot \sum_{q=u,d} \left[\bar{q}_{iL}^0 \frac{\lambda^a}{2} \tilde{q}_{iL}^{0L} \psi_{\tilde{g}}^a + \right. \\ & \left. - \bar{q}_{iR}^0 \frac{\lambda^a}{2} \tilde{q}_{iR}^{0R} \psi_{\tilde{g}}^a + h.c. \right] + \\ & + \frac{g}{\cos \theta_W} \cdot \sum_{q=u,d} \left[(I_3 - Q \sin^2 \theta_W) \tilde{q}_{jL}^{0L\dagger} i \overleftrightarrow{\partial}_\mu \tilde{q}_{jL}^{0L} + \right. \\ & \left. - Q \sin^2 \theta_W \tilde{q}_{jR}^{0R\dagger} i \overleftrightarrow{\partial}_\mu \tilde{q}_{jR}^{0R} \right] Z^\mu + \\ & + \frac{g}{\sqrt{2}} \left[\tilde{u}_{jL}^{0L} i \overleftrightarrow{\partial}_\mu \tilde{d}_{jL}^{0L} W^\mu + h.c. \right] + \dots \end{aligned} \quad (37)$$

(g_s is the strong and g the SU(2) the coupling constant, λ^a ($a = 1, 2, \dots, 8$) are Gell-Mann matrices, $\psi_{\tilde{g}}^a$ denotes the gluino, θ_W is the Weinberg angle, I_3 and Q are the third component of the weak isospin and electric charge, respectively), we get

$$({}^u \Gamma_L)_{ik}^\beta = - ({}^u A^*)_{ji}^{L\beta} \cdot (U_L)_{jk} \quad (38)$$

$$({}^u\Gamma_R)_{ik}^\beta = ({}^uA^*)_{ji}^{R\beta} \cdot (U_R)_{jk} \quad (39)$$

$$({}^uN)_{ji}^{\delta\beta} = ({}^uA^*)_{ji}^{L\delta} \cdot ({}^uA)_{ki}^{L\beta} \quad (40)$$

$$M_{ji}^{\delta\beta} = ({}^uA^*)_{kj}^{L\delta} \cdot ({}^dA)_{ki}^{L\beta} \quad (41)$$

and three formulae analogous to (38)-(40) for down squarks, with $U_{L,R}$ replaced by $V_{L,R}$ (and the superscript u replaced by d).

The one-loop corrections to the νN scattering arising from the supersymmetric version of QCD are given by the sum of the corrections to the $q\bar{q}$ -weak boson three-point Green's functions. For the case of the Z-boson vertex, they are depicted in Fig. 2. To calculate them, we neglect all the external momenta and quark masses in comparison with the gluino and squark masses. Since one expects that the gluino and squark masses cannot be much smaller than M_W , our result applies to the deep inelastic scattering in the range of the momentum transfer $\lesssim 0(100 \text{ GeV}^2)$. For the ingoing u_i and outgoing u_j quarks, we get

$$\begin{aligned} \Delta \quad u_i \bar{u}_j Z \Gamma_\mu = & \frac{i}{4\pi} \alpha_s \frac{g}{\cos \vartheta_W} \cdot 2 I_3 \cdot \\ & \cdot \left\{ \left[S(u_{X_k}^\beta, u_{X_m}^\delta) \cdot ({}^uN)_{mk}^{\delta\beta} ({}^u\Gamma_L^*)_{mj}^\delta ({}^u\Gamma_L)_{ki}^\beta + \right. \right. \\ & \left. \left. + C(u_{X_k}^\beta) \cdot ({}^u\Gamma_L^*)_{kj}^\beta ({}^u\Gamma_L)_{ki}^\beta \right] \delta_\mu L + \right. \\ & \left. + S(u_{X_k}^\beta, u_{X_m}^\delta) \cdot ({}^uN)_{mk}^{\delta\beta} ({}^u\Gamma_R^*)_{mj}^\delta ({}^u\Gamma_R)_{ki}^\beta \delta_\mu R \right\} \quad (42) \end{aligned}$$

where $\alpha_s = g_s^2/4\pi$ and the strong coupling constant g_s is taken at the scale of the order of the gluino mass, $u_{X_k}^\beta = \frac{m_{\tilde{g}}^2}{u_k^\beta m_g^2}$, $L, R = (1 \mp \gamma_5)/2$ and the functions C and S are given by the formulae:

$$G(x) = -\frac{2}{3} \frac{x}{1-x} + \frac{2}{3} \frac{x^2-2x}{(1-x)^2} \ln x \quad (43)$$

$$S(x, y) = \frac{2}{3} \frac{1}{1-y} \left[\frac{x}{1-x} \ln x + \frac{y}{x-y} (x \ln x - y \ln y) \right] \quad (44)$$

Of course, a similar result, with u replaced by d , holds for down quarks.

The $O(\alpha_s)$ supersymmetric corrections to the $q\bar{q}W$ Green's function are given by diagrams analogous to those of Fig. 2. The result for the sum of them reads as follows (to be specific, we choose d_i as an ingoing and u_j as an outgoing quark):

$$\begin{aligned} \Delta u_i \bar{d}_j W_\mu &= \frac{i}{4\pi} \alpha_s \frac{g}{\sqrt{2}} \cdot \\ &\cdot \left\{ \left[2S(d_{X_k}^\beta, u_{X_m}^\sigma) \cdot M_{mk}^{\sigma\beta} (u\Gamma_L^*)_{mj}^\sigma (d\Gamma_L)_{ki}^\beta + \right. \right. \\ &+ C(u_{X_m}^\sigma) \cdot (u\Gamma_L^*)_{mj}^\sigma (u\Gamma_L)_{mk}^\sigma K_{ki} + C(d_{X_k}^\beta) \cdot (d\Gamma_L^*)_{km}^\beta (d\Gamma_L)_{ki}^\beta K_{jm} \Big] \delta_\mu \mathbb{L} \\ &+ \left. \left[2S(d_{X_k}^\beta, u_{X_m}^\sigma) \cdot M_{mk}^{\sigma\beta} (u\Gamma_R^*)_{mj}^\sigma (d\Gamma_R)_{ki}^\beta \right] \delta_\mu \mathbb{R} \right\} \quad (45) \end{aligned}$$

with K_{ji} denoting the quark mixing matrix,

$$K_{ji} = (U_L^*)_{rj} (V_L)_{ni}$$

Now, using the relation $S(x, x) = -C(x)$ and the definitions (38)-(41) of the coefficients Γ , M and N , it is easy to see that both corrections to the $q\bar{q}$ -weak boson Green's functions vanish in the limit of the degeneration of all squark masses.

However, in general, one can expect some mass splitting between various squarks and, consequently, non-zero values for $\Delta\Gamma_\mu$'s.

If non-zero, Eqs. (42) and (45) can be used to calculate the corrections to the directly measurable quantities. We investigate in more detail the parameter $R = \sigma_{\nu N}^{NC}/\sigma_{\nu N}^{CC}$ which has been studied experimentally and which we have discussed in the previous section. For simplicity, we still use the valence parton model for nucleon and thus neglect the contribution due to antiquark and strange sea, scaling violation, etc. Those effects introduce an uncertainty of the order of 10% into our results for the supersymmetric $O(\alpha_s)$ correction to R . Within the above approximation, only the quarks of the first family contribute to R . Thus, with $i = j = 1$ in Eqs. (42) and (45) and keeping the terms $O(\alpha_s)$, we get

$$R = R^{(0)} (1 + \tilde{\delta}_R) \quad (46)$$

where

$$R^{(0)} = \frac{1}{2} (1 - 2 \sin^2 \theta_W + \frac{40}{27} \sin^4 \theta_W) \quad (47)$$

$[\cos^2 \theta_c = 1$ in Eq. (47) but we keep non-zero quark mixing in $\tilde{\delta}_R$] and

$$\begin{aligned} \frac{\tilde{\delta}_R}{\alpha_s/2\pi} = \frac{1}{2 R^{(0)}} \left\{ (1 - \frac{4}{3} \sin^2 \theta_W) \cdot \right. \\ \cdot \left[(u_L^*)^\beta_{k1} (u_L)^\beta_{k1} C(u_{X_k}^\beta) + \right. \\ \left. + (u_N)^{\delta\beta}_{mk} (u_L^*)^\delta_{m1} (u_L)^\beta_{k1} \cdot S(u_{X_k}^\beta, u_{X_m}^\delta) \right] + \\ \left. + (1 - \frac{2}{3} \sin^2 \theta_W) \cdot [u \rightarrow d] \right\} + \end{aligned} \quad (48)$$

$$\begin{aligned}
& -\frac{2}{9} \frac{\sin^2 \theta_W}{2R^{(0)}} \cdot \left[2 (u_N)^{\delta\beta}_{mk} (u\Gamma_R^*)^{\delta}_{m1} (u\Gamma_R)^{\beta}_{k1} \cdot S(uX_k^{\beta}, uX_m^{\delta}) + \right. \\
& \quad \left. + (d_N)^{\delta\beta}_{mk} (d\Gamma_R^*)^{\delta}_{m1} (d\Gamma_R)^{\beta}_{k1} \cdot S(dX_k^{\beta}, dX_m^{\delta}) \right] + \\
& \quad - (u\Gamma_L^*)^{\delta}_{m1} (u\Gamma_L)^{\delta}_{mk} K_{k1} \cdot C(uX_m^{\delta}) + \\
& \quad - (d\Gamma_L^*)^{\beta}_{km} (d\Gamma_L)^{\beta}_{k1} K_{1m} \cdot C(dX_k^{\beta}) + \\
& \quad - 2M^{\delta\beta}_{mk} (u\Gamma_L^*)^{\delta}_{m1} (d\Gamma_L)^{\beta}_{k1} \cdot S(dX_k^{\beta}, dX_m^{\delta})
\end{aligned} \tag{48}$$

cont.

In order to estimate the magnitude of δ_R , we have to specify the squark mass matrix. Its general form is suggested by supergravity models [Ref. 17) and references therein]

$$M_{\tilde{q}}^2 = \begin{pmatrix} \mu_L^2 \mathbb{1} + M_d M_d^\dagger + c M_u M_u^\dagger & A^* \mu M_d \\ A \mu M_d^\dagger & \mu_R^2 \mathbb{1} + M_d^\dagger M_d \end{pmatrix} \tag{49}$$

[in the basis of $SU(2) \times U(1)$ eigenstates] and analogously for up-type squarks. $M_{d(u)}$ denotes the down (up) quark mass matrix, μ_L , μ_R and μ are all of order of the global supersymmetry breaking scale [which itself is expected to be $O(M_W)$], A is a $O(1)$ complex parameter depending on the details of the original supergravity theory. The parameter c characterizes the flavour changing radiative corrections

to the tree level mass matrix. It is negative and can be of order $O(1)^{18)}$ [for a more detailed discussion of the c parameter in the minimal supersymmetric extension of the standard model, see Ref. 19)].

One of the features of the matrix (49) is its almost flavour-diagonal form. This can be seen most easily after rotating it to the basis in which the d-quark mass matrix is diagonal. With ${}^d A = {}^d A_1 \cdot {}^d A_2$ and

$${}^d A_1 = \begin{pmatrix} e^{-i\phi_A} V_L & 0 \\ 0 & e^{i\phi_A} V_R \end{pmatrix} \quad (50)$$

where $\phi_A = -\frac{1}{2} \arg A$, we have

$${}^d A_1^\dagger M_{\tilde{d}}^2 {}^d A_1 = \begin{pmatrix} \mu_L^2 11 + \hat{M}_d^2 + c K^\dagger \hat{M}_u K & |A| \mu \hat{M}_d \\ |A| \mu \hat{M}_d & \mu_R^2 11 + \hat{M}_d^2 \end{pmatrix} \quad (51)$$

Assuming three generations of quarks, the mass of the top quark ~ 40 GeV, as indicated by the results from the CERN collider²⁰⁾, and using the experimental values of the Kobayashi-Maskawa matrix K , we are able to calculate flavour non-diagonal elements in (51). For example, with the Wolfenstein parametrization of K ²¹⁾

$$K^\dagger \hat{M}_u^2 K = \begin{pmatrix} 0.24 & 1.2 & 16 \\ 1.2 & 5.8 & 85 \\ 16 & 85 & 1600 \end{pmatrix} \text{GeV}^2 \quad (52)$$

(only absolute values are given). Although the actual values of the mixing matrix elements can be slightly different from those used to obtain Eq. (52), the

general picture is clear: with μ 's of order of several tens of GeV, non-diagonal elements of (52) represent at most 10% corrections to the leading, flavour conserving part of M_d^2 . Thus, within the employed approximation for R, they can be safely neglected.

Now, it is easy to see that d_{A_2} is flavour diagonal and, consequently,

$$\begin{aligned} (d_L^\dagger)_{ik}^\beta &= \delta_{ik} (d_L^\dagger)_{ii}^\beta \\ (d_R^\dagger)_{ik}^\beta &= \delta_{ik} (d_R^\dagger)_{ii}^\beta \end{aligned} \quad (53)$$

with

$$\begin{aligned} (d_L^\dagger)_{ii}^\beta &= \begin{pmatrix} \cos \mathcal{J}_{d_i} \\ -\sin \mathcal{J}_{d_i} \end{pmatrix} \\ (d_R^\dagger)_{ii}^\beta &= \begin{pmatrix} \sin \mathcal{J}_{d_i} \\ \cos \mathcal{J}_{d_i} \end{pmatrix} \end{aligned} \quad (54)$$

where

$$\tan 2\mathcal{J}_{d_i} = \frac{-2|A|\mu m_{d_i}}{\mu_L^2 - \mu_R^2 + b_i} \quad (55)$$

$[b_i = m_{d_i}^2 + \text{diag}_{(i)}(cK + M_U^2 K)]$. The masses of the squarks are given by

$$m_{\tilde{d}_i^{1,2}}^2 = m_{d_i}^2 + \frac{1}{2} \left[\mu_L^2 + \mu_R^2 \pm \sqrt{(\mu_L^2 + b_i - \mu_R^2)^2 + 4|A|^2 \mu^2 m_{d_i}^2} \right] \quad (56)$$

The above formulae can also be used (with the obvious modifications) in the case of up squarks. In fact, they are almost exact in this case, since the down quarks are lighter than up ones and the flavour non-diagonal elements of M_u^2 - much smaller than those of M_d^2 .

Neglecting flavour non-conserving couplings, we get for $\tilde{\delta}_R$

$$\begin{aligned}
 \frac{\tilde{\delta}_R}{\alpha_s/2\pi} &= \frac{\frac{1}{4} - \frac{2}{9} \sin^2 \theta_u}{2R^{(0)}} \cdot \sin^2 2\theta_u \cdot [G(x_u^1) + G(x_d^2) + 2S(x_u^1, x_u^2)] + \\
 &+ \frac{\frac{1}{4} - \frac{1}{9} \sin^2 \theta_d}{2R^{(0)}} \cdot \sin^2 2\theta_d \cdot [G(x_d^1) + G(x_u^2) + 2S(x_d^1, x_d^2)] + \\
 &- \cos^2 \theta_u \cdot G(x_u^1) - \cos^2 \theta_d \cdot G(x_d^1) - 2\cos^2 \theta_u \cos^2 \theta_d \cdot S(x_d^1, x_u^1) \\
 &- \sin^2 \theta_u \cdot G(x_u^2) - \sin^2 \theta_d \cdot G(x_d^2) - 2\sin^2 \theta_u \sin^2 \theta_d \cdot S(x_d^2, x_u^2) \\
 &- 2\sin^2 \theta_u \cos^2 \theta_d \cdot S(x_d^1, x_u^2) + \\
 &- 2\cos^2 \theta_u \sin^2 \theta_d \cdot S(x_d^2, x_u^1)
 \end{aligned} \tag{57}$$

$[x_u^1 = x_u^1, x_d^1 = x_d^1, \text{ etc.}; \text{ we used the identities } S(x, x) = -C(x), S(x, y) = S(y, x)]$.

Let us observe now that with the typical value of the difference $\mu_L^2 - \mu_R^2$ of order of several GeV^2 , e.g. ²²⁾, Eq. (55) gives approximately zero for θ_u and θ_d (no left-right mixing within the first family). Thus, the expression for $\tilde{\delta}_R$ simplifies significantly:

$$\tilde{\delta}_R \simeq - \frac{\alpha_s}{2\pi} [G(x_u^1) + G(x_d^1) + 2S(x_d^1, x_u^1)] \tag{58}$$

Furthermore, within the same accuracy

$$|m_{\tilde{d}_1}^2 - m_{\tilde{u}_1}^2| \simeq \frac{1}{2} |c \cdot (b_1 - t_1)| \simeq \frac{1}{2} |c| \cdot t_1 \tag{59}$$

$[t_1 = m_{u_1}^2 + \text{diag}_{(1)}(c\hat{K}_d^2 K^+)]$, since μ_L is the same for both up and down squarks.

Using Eq. (52) and $c = -1$ we get

$$|m_{\tilde{d}_1}^2 - m_{\tilde{u}_1}^2| \simeq 0.1 \text{ GeV}^2$$

However, with the maximal allowed by experiment value of the K_{td} element of the mixing matrix²³⁾, $|K_{td}| = 0.07$, we cannot exclude that

$$|m_{\tilde{d}_1}^2 - m_{\tilde{u}_1}^2| \simeq 4 \text{ GeV}^2$$

Anyway, with the gluino mass of order of several tens of GeV, we see that $\Delta_1 = x_u^1 - x_d^1 \ll 1$. For example, with $m_{\tilde{g}} = 38 \text{ GeV}$, i.e., the lowest value allowed by the analysis of the flavour changing neutral interactions¹⁸⁾, we get

$$\Delta_1 < 0.003$$

Thus, recalling the relation $S(x, x) = -C(x)$, we can rewrite Eq. (58) in the form

$$\tilde{\mathcal{O}}_R \simeq - \frac{\alpha_s}{2\pi} \cdot \Delta_1 \cdot P(x^1) \quad (60)$$

where $x^1 = \frac{1}{2}(x_u^1 + x_d^1)$ and

$$P(x) = 2 + \frac{2}{3} \frac{x-3}{(1-x)^2} + \frac{4}{3} \frac{1-2x}{(1-x)^2} \ln x \quad (61)$$

Even on the more general ground, without assuming the squark mass matrices induced by supergravity models, one can obtain the upper bound for Δ_1 from the analysis of the nuclear parity violation²⁴⁾. Although this analysis suffers from several theoretical uncertainties²⁵⁾, it seems safe to deduce

$$\Delta_1 \leq 0.1$$

With $\alpha_s \simeq 0.1$, Eq. (57) gives us

$$|\tilde{\mathcal{O}}_R| \leq 0.002 \quad (62)$$

However, in supergravity models the value of $\tilde{\mathcal{O}}_R$ is much smaller

$$|\tilde{\mathcal{O}}_R| \leq 0.0001 \quad (63)$$

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FIGURE CAPTIONS

- Fig. 1 : The right-hand side of Eq. (24) as a function of the parameter $w = (74.562 \text{ GeV}/M_W)^2$. This function gives an order of magnitude estimate of second order radiative corrections to $M_W(M_Z)$ and $M_Z(M_W)$.
- Fig. 2 : The one-loop $O(\alpha_s)$ diagrams contributing to the quark-antiquark-Z boson Green's function in supersymmetric extensions of the standard model ($\tilde{g}$ and $\tilde{q}_k^\beta$ are called the gluino and the squark fields, respectively).
- Fig. 3 : The Feynman rules for the squark-quark-gluino and squark-squark-weak boson vertices in supersymmetric extensions of the standard model. All fields are the mass matrix eigenstates.

Fig. 1

Fig. 2

$$\tilde{g}^\alpha = ig\sqrt{2} \frac{\lambda^\alpha}{2} [\Gamma_L \cdot L + \Gamma_R \cdot R]_{ji}^\beta$$

$$Z_\mu = i \frac{g}{\cos \vartheta_W} [-Q \sin^2 \vartheta_W \delta_{ji} \delta^{\gamma\beta} + I_3^q N_{ji}^{\gamma\beta}] \cdot (p_i + p_j)_\mu$$

$$W_\mu = i \frac{g}{\sqrt{2}} M_{ji}^{\gamma\beta} (p_i + p_j)_\mu$$

Fig. 3