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A QUARK MODEL FOR NUCLEAR MATTER AND THE COULOMB SUM RULE

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ABSTRACT

The quark correlation function, $g(r)$, of nuclear matter is calculated in a simple model with many of the essential composite features of nucleons. From g the Coulomb sum rule is calculated and found not to go to unity because there is substantial overlap of momentum scales describing NN correlations and nucleon substructure.

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In this paper we examine a simple model in which calculations for the many-quark system are tractable. This model has the essential composite features of nuclear matter: that all quarks are confined into hadrons yet the wavefunction is explicitly antisymmetric even for the exchange of quarks "from" different hadrons.

This allows one to avoid the traditional "package model" assumptions, that quarks are confined into different packages such as the Nucleon, Delta or six quark bag. The overlap or interference of different clusters, as they interact in the nucleus, is neglected. Package models do not allow any "tunneling" of quarks between hadrons and deal with probabilities rather than amplitudes.

One very common package model is the impulse approximation for the electromagnetic current. Here the free nucleon current operator is used even in the medium. For our model, we construct the current operator from the quark coordinates directly and compare to an impulse approximation.

What changes takes place in the structure of a nucleon when placed in the medium? Does the length scale for quark confinement depend on the nuclear density? Indeed, deep inelastic scattering data [1] have been interpreted by [2] as showing the "length scale for quark confinement" is greater in Fe than in a free proton.

One would like to examine this within a dynamical quark model. Unfortunately, the bag model with its imposed boundary conditions does not provide any insight into the dynamics of this partial deconfinement. At best, all that can be calculated is a simple geometric overlap [3].

Using Monte Carlo techniques, we have calculated some of the ground state properties of uniform nuclear matter by solving variationally a Hamiltonian in the quark coordinates. This nonrelativistic Hamiltonian is described below. The energy, wavefunction and single quark momentum distribution have been examined as a function of density both in one [4] and three space dimensions [5]. In addition, the model has been used to describe the scattering of two hadrons [6].

The quark correlation function or two-body density, $g(r)$ has also been calculated [5]. This function provides considerable information on the dynamics of quark confinement in the medium. Closely related to $g(r)$ is the structure function or Coulomb sum rule, $S(q)$. The sum rule is the integral over all energy transfers, ω , of the longitudinal response function $R(q, \omega)$ measured in electron scattering for momentum transfer q . It is commonly believed that $S(q)$ per nucleon should go to one at high momentum transfer. This would be the case if incoherent scattering from each proton occurred simultaneously with coherent scattering from all the quarks in a given nucleon. However, it is known that one must go to relatively high momentum transfer, $q \approx 500 \text{ MeV}/c$, to get above $N - N$ correlations. But already by this q the square of the elastic nucleon form factor is less than $\frac{1}{2}$, and thus the nucleon substructure is starting to play a role. Is one caught between NN correlations and nucleon substructure, with no range of q where a sum rule over nucleons is appropriate? We test this nuclear Coulomb sum rule below.

First we describe our quark potential model and recent three dimensional nuclear matter results [5] which we use to construct the Coulomb sum rule. Finally, we discuss general features of corrections to the Coulomb sum rule from nucleon

substructure which may have validity beyond that of our simple model.

Nonrelativistic quark models have proven remarkably useful in describing hadron spectroscopy [7]. Therefore, for simplicity, this paper is entirely nonrelativistic (this is expected to at least give insight into the qualitative features of hadron substructure.) Furthermore, we focus on the valence quarks and choose to eliminate gluon or other nonquark degrees of freedom in favor of an effective potential V acting in the quark coordinates. The neglect of q-qbar pairs or mesons may be a limitation of the model when one is interested in fine details of the NN force at large distances. Here, however, we examine the qualitative features of quarks in nuclei which may depend on the short range valence quark core.

In order to minimize the number of degrees of freedom needed in Monte Carlo simulations we use model two-quark "baryons" and omit internal degrees of freedom such as color and spin. Work is underway to extend the calculations to more detailed three quark nucleons, but the interesting composite features already exist for these two quark clusters.

We confine two quarks into an isolated hadron by using a harmonic osc. potential,

$$v(r_{12}) = \frac{1}{2} \vec{r}_{12}^2. \quad (1)$$

Other radial forms are expected to give similar results.

There are a number of ways to generalize equation (1) for the many-quark system. If a two-body confining potential is used $V = \sum_{i < j} v(r_i - r_j) L_i \cdot L_j$ (where, L_i , are the $SU(3)$ color matrices) then long range Van der Waals forces will be introduced [7]. Such forces which only fall off as a power law of the hadron separation may significantly distort the properties of infinite quark matter.

Our prescription to define the potential energy operator for a system of A quarks is to (1) Group the quarks into $A/2$ pairs; (2) Assume the confining potential equation (1) acts only between the two quarks in a given pair. Sum this potential over the $A/2$ pairs; (3) Repeat (2) for all $(A-1)!$ possible ways to group A quarks into $A/2$ nucleons and take that pairing which gives the lowest potential energy [4].

$$V = \min_{\{P\}} \sum_{i=1}^{A/2} v(\vec{r}_{p_{i1}} - \vec{r}_{p_{i2}}) \quad (2)$$

Here, the minimum is taken over all possible pairings $\{P\}$ which confine quarks p_{i1} and p_{i2} into the i^{th} cluster. This could represent an adiabatic approximation for the gluon flux tube dynamics and assures the potential will be symmetric in the quark coordinates. Equation (2) is an A body operator: changing the position of one quark could change all of the pairings.

For a system of four quarks the potential is the minimum of three possible pairings, $V = \frac{1}{2} \min\{(\vec{r}_1 - \vec{r}_2)^2 + (\vec{r}_3 - \vec{r}_4)^2, (\vec{r}_1 - \vec{r}_3)^2 + (\vec{r}_2 - \vec{r}_4)^2 + (\vec{r}_1 - \vec{r}_4)^2 + (\vec{r}_2 - \vec{r}_3)^2\}$. This can describe, for example, a nucleon made of quarks one and two scattering from a nucleon made of three and four. At large nucleon separation, the potential would be given by $(\vec{r}_1 - \vec{r}_2)^2 + (\vec{r}_3 - \vec{r}_4)^2$. But as the nucleons overlap or if a quark exchange takes place, the potential could be given by a different pairing.

The Hamiltonian for the system is obtained by adding to Equation (2) a nonrelativistic kinetic energy

$$H = \sum_i -\nabla_i^2 + V(r_1 \dots r_A) \quad (3)$$

We use units such that twice the quark mass is one. A simple variational wavefunction for the ground state, $|\Psi_0\rangle$, of uniform nuclear/quark matter is

$$|\Psi_0\rangle = e^{-\lambda V} \Phi_{Fg} \quad (4)$$

Here, Φ_{Fg} is a free Fermi gas, V is the full many-body potential of Equation (2) and λ is a variational parameter. At low density $\lambda = \frac{1}{2}$ reproduces the osc. wavefunction for isolated nucleons while at high density $\lambda \rightarrow 0$ reproduces the correct free Fermi gas. Thus, $|\Psi_0\rangle$ is exact both in the high and low density limits. Also, at intermediate densities this wavefunction is very good in one dimension [4].

The amount of quark clustering in the ground state is determined by λ with $\lambda^{-\frac{1}{2}}$ providing a "length scale of quark confinement" (at high density the momentum scale in the quark wavefunction is set by k_F in Φ_{Fg}).

Three dimensional variational calculations have been carried out for Equations (3) and (4), see reference [5] for details. The value of λ which minimizes the energy is found to decrease with density. If one chooses the osc. length to reproduce the charge radius of a proton, *, then normal nuclear density ($.16 \text{ fm}^{-3}$) corresponds to $\rho_0 = .0076$ in our units. At this density, λ is about 15% smaller than at zero density. Near $2\rho_0$, λ rapidly diminishes. This represents a transition from nuclear to quark matter.

Reference [2] has fit deep inelastic lepton scattering data with a similar change in the confinement scale in the medium. However, they provide no dynamical information as to how this scale change comes about. We find that the overlap of the quark wavefunctions and exchange symmetry is all that is needed.

The quark correlation function, $g(r)$, or two-body density gives the probability of finding another quark a distance r from a given quark (and is normalized to one at large r).

$$g(r) = \frac{1}{A\rho} \sum_{i \neq j} \delta^3(\vec{r} - (r_i - r_j)) \quad (5)$$

Figure (1) shows $g(r)$ for a density just below ρ_0 . A peak in $g(r)$ is seen for small r . This peak describes the other quark bound into each nucleon and is similar to $g(r)$ for a free nucleon. However, there is a shift (although small) of this peak to larger r . This shows the quark confinement scale is increasing with density. At intermediate distances $g(r)$ shows a dip (compared to a Fermi gas) indicating the anticorrelation of nucleons (indeed the model NN phase shifts show a hard core like repulsion [4]).

* If the substructure is to represent only the core of a proton charge distribution, then one could identify ρ_0 with a lower density. But one must still account for the full proton charge distribution when calculating $S(q)$.

As the density is increased, the peak in g starts to differ substantially from that of a free nucleon. At still higher density g relaxes to that of a Fermi gas.

Inclusive lepton scattering with four momentum transfer, q, ω can measure the longitudinal response function, $R(q, \omega)$.

$$R(q, \omega) = \sum_n |\langle \Psi_0 | \hat{\rho}(-q) | \Psi_n \rangle|^2 \delta(\omega - E_n + E_0) \quad (6)$$

Here, $|\Psi_n\rangle$ are target states of energy E_n and $\hat{\rho}(q)$ is the charge operator

$$\hat{\rho}(q) = \int d^3x e^{-iqx} \sum_i e_i \delta(\vec{x} - \vec{x}_i) \quad (7)$$

$$e_i = \begin{cases} 1 & \text{Proton} \\ \frac{1}{2} & \text{Quark} \end{cases} \quad (8)$$

We arbitrarily assign each quark in our "proton" a charge $e_i = \frac{1}{2}$.

Unfortunately, Monte Carlo calculations only give ground state (or low lying excited state) information so $R(q, \omega)$ can't be directly calculated. Therefore, we consider the structure function $S(q)$, instead.

$$S(q) = \int_0^\infty d\omega R(q, \omega) \quad (9)$$

This is simply related to $g(r)$.

$$S(q) = A e_i^2 \{1 + \rho \int d^3r e^{iqr} (g(r) - 1)\} \quad (10)$$

Figure (2) shows $S(q)$ (per proton) for a density just below ρ_0 .

A large enhancement is seen in $S(q)$ over that of a Fermi gas. This is due to the correlations between quarks in a given nucleon. Thus, there is some tendency to scatter coherently from both quarks in a nucleon. However, this coherence is incomplete. The naive expectations for a nuclear Coulomb sum rule would have $S(q)$ go to unity for a range of intermediate momentum transfer. This does not happen since $S(q)$ is always significantly less than one. Therefore, in the model, the NN correlation momentum scale $2k_F$ (which describes the rising slope of $S(q)$ at small q) and the internal quark excitation scale have substantial overlap.

One way to try and deal with this overlap is to hope the contributions from the nucleon substructure and nuclear structure "factor" and can be dealt with separately.

Data is often analyzed within the impulse approximation for the electromagnetic current operator. Here, the free nucleon current operator is used even in the medium. This suggests replacing the proton charge in Equation (8) with the elastic form factor, $F_e(q)$. If this is a good approximation, then the quantity,

$$\begin{aligned} S_{IA}(q) &\equiv S(q) / |F_e(q)|^2 \\ F_e(q) &= \left(1 - \frac{q^2}{3}\right) e^{-\frac{q^2}{3}} \end{aligned} \quad (11)$$

should be nearly one over a range of intermediate q values of "nuclear" interest. Clearly, S_{IA} must significantly exceed one at very large q because of internal excitations of the nucleon. To emphasize that there is no general theory for Equation (11), we note that F_e has a node in our model (and S_{IA} a pole). Even below this node there is no level region with S_{IA} near one (see Figure (2)). Therefore, even with the arbitrary ansatz of an impulse approximation there is no clear separation between momenta describing NN correlations and internal quark excitations.

This could have a number of implications:

- It is not enough for experiments to examine the Coulomb sum rule at one q . Instead, one should look to see over how broad a q region is S_{IA} near one. As experiments are pushed to higher q substantial excess strength may be found and the sum rule will become ill defined.
- The impulse approx. (Equation (11)) may not be the most appropriate way to analyze experimental data if a substantial amount of strength is going through virtual (or real) excitations of the nucleon in the medium. We emphasize, the impulse approx. is only a model. Experimentalists must publish first the raw response functions without any nucleon form factors divided out.
- The NN correlation function may become badly defined due to the quark substructure at relatively low q . I.e., as the nucleons start to overlap, the distance between their centers becomes ill defined (given the quark antisymmetry). Alternatively, one could define a partial sum rule by only integrating $R(q, \omega)$ up to some cut off in ω . However.
- It is in general much harder to calculate this.
- One still needs to develop a quantitative theory for a partial sum rule. (If the nuclear and nucleon energy scales overlap by an amount X then S_{IA} will differ from unity by an amount Y .)
- Why should the nuclear and quark excitations be so much better separated in energy than in momentum transfer?
- Even below a threshold energy there can be substantial virtual nucleon excitations.

A many-body potential model has been used which confines quarks into hadrons and avoids Van der Waals forces. Variational calculations were presented of the quark correlation function for the ground state of uniform nuclear matter. This function shows the change in nucleon substructure with density without having to make "package model" assumptions which neglect interference terms. The structure function shows no clear separation in q for scattering from nucleons or quarks.

Figure 1

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FIGURE CAPTIONS

Figure 1

Quark correlation function, $g(r)$, vs. r for a density, $\rho = .0059$, slightly below normal nuclear density (solid curve). See Equation (5). Also shown is the correlation function of a quark Fermi gas (dotted) and that of an isolated nucleon (dashed). A "length scale for quark confinement" $\lambda^{-1/2}$, is indicated on the r axis. This variationally determined value is slightly greater than that of an isolated nucleon, $\lambda^{-1/2} = \sqrt{2}$.

Figure 2

Coulomb sum rule per proton, Equations (9, 10) vs. momentum transfer at $\rho = .0059$ (solid curve). Also shown is the sum rule for a quark Fermi gas (dotted) and the impulse approximation results, Equation (11), (dashed) obtained by dividing the solid curve by the square of our proton elastic form factor. The exp. proton charge radius gives $q = 1$ in our units = $570 \text{ MeV}/c$.

Figure 2