LOCALIZATION OF THE DYON CHARGE

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ABSTRACT

We consider three possible expressions for the dyon charge in the presence of fermions. We find that although none of them fully admit an interpretation as a localized charge, one expression may be identified as the core charge under suitable conditions. Comparison with Callan's expressions are also made.

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Recently¹, it has been shown that the charge $Q_{rot} = I\dot{\phi}$ associated with the dyon (rotor) degree of freedom² cannot be interpreted as a localized charge in the presence of fermions. The question therefore arises whether there exists any other expression which does admit such an interpretation⁹. We find that the answer is best characterized as yes and no. The purpose of this note is to explain the subtleties involved in this answer, and to compare the results with those of Callan⁷.

Note. Most of the material in this note is already contained in Ref. 1 and Ref. 9. However, we hope that a joint paper on the points we have reached agreement will serve to clarify the issues involved.

Let us first briefly recall why Q_{rot} cannot be interpreted as a localized charge. It is convenient to work in the gauge where the vector potential has the form

$$eA_0(\vec{x}) = (\hat{x} \cdot \vec{r}) \frac{J(r)}{2r} \quad (1)$$

$$e\vec{A}(\vec{x}) = (\hat{x} \times \vec{r}) \frac{K(r) - 1}{2r} \quad (2)$$

with the asymptotic behavior

$$\frac{J(r)}{r} = O(r) \quad (r \rightarrow 0) \quad \frac{J(r)}{r} \sim -\dot{\phi} + \frac{e^2}{4\pi r} Q_{tot} \quad (r \rightarrow \infty) \quad (3)$$

$$K(r) = 1 + O(r^2) \quad (r \rightarrow 0) \quad K(r) = O(e^{-m_W r}) \quad (r \rightarrow \infty) \quad (4)$$

The electric field is then given by

$$E_i^a(x) = -(\delta_{ia} - \hat{x}_i \hat{x}_a) \frac{J(r)K(r)}{er^2} - \hat{x}_i \hat{x}_a \frac{d}{dr} \frac{J(r)}{er} \quad (5)$$

and Gauss' law reads

$$r^2 J''(r) - 2J(r)K^2(r) = -\frac{e^2}{4\pi} r \rho(r) \quad (6)$$

where $\rho(r)$ is the (one-dimensional) charge density of the fermions, including contributions from vacuum polarization. Integrating with the boundary condition (3), we find³

$$\frac{J(r)}{r} = -J(r)\dot{\phi} + \frac{e^2}{4\pi} \int_0^\infty dr' \mathcal{G}(r, r') \rho(r') \quad (7)$$

$$\mathcal{G}(r, r') = J(r)\theta(r' - r)J(r') + J(r)\theta(r - r')J(r') \quad (8)$$

where $J(r)$ and $J(r)$ are the solutions of the homogeneous equation

$$r \frac{d^2}{dr^2} r J(r) - 2J(r)K^2(r) = 0 \quad (9)$$

which behave as

$$J(r) = O(r) \quad (r \rightarrow 0) \quad J(r) = 1 - \frac{Ie^2}{4\pi r} + O(e^{-m_W r}) \quad (r \rightarrow \infty) \quad (10)$$

$$J(r) = O\left(\frac{1}{r^2}\right) \quad (r \rightarrow 0) \quad J(r) = \frac{1}{r} + O(e^{-m_W r}) \quad (r \rightarrow \infty) \quad (11)$$

The scale is set by the core radius $r_0 = m_W^{-1}$; in particular, I is a positive constant of order r_0/e^2 .

Eqs. (7)-(11) lead to the expressions for the total charge

$$Q_{tot} = Q_{rot} + \int_0^\infty dr J(r) \rho(r) \quad (12)$$

the radial electric field

$$E_r = \hat{x}_i \hat{x}_a E_i^a \sim \frac{e}{4\pi r^2} \left(Q_{tot} - \int_r^\infty dr' \rho(r') \right) \quad (13)$$

and the electrostatic energy

$$\mathcal{E} = \frac{1}{2} \int dV \sum_{i,a} (E_i^a)^2 = \frac{1}{2I} Q_{rot}^2 + \frac{e^2}{8\pi} \int_0^\infty \frac{dr}{r^2} \left(\frac{1}{J(r)} \int_0^r dr' J(r') \rho(r') \right)^2 \quad (14)$$

Since there is no cross term between Q_{rot} and $\rho(r)$ in (14)⁴, we find that Q_{rot} cannot be interpreted as a charge localized at the core.

However, Eq. (13) suggests another possibility, if r_0 is small compared to other scales. Taking

$$Q_{tot}(r) = Q_{tot} - \int_r^\infty dr' \rho(r') \quad (15)$$

as the total charge within the radius r , we may introduce

$$Q_{core} = Q_{tot} - \int_0^\infty dr \rho(r) = Q_{rot} - \int_0^\infty dr (1 - J(r)) \rho(r) \quad (16)$$

Since

$$Q_{tot}(r) = Q_{core} + \int_0^r dr' \rho(r') \quad (17)$$

we may regard Q_{core} as the charge located at the core. Furthermore, rewriting (14) as

$$\mathcal{E} = \frac{1}{2I} Q_{core}^2 + \frac{Q_{core}}{I} \int_0^\infty dr (1 - J(r)) \rho(r) + \frac{e^2}{8\pi} \int_0^\infty \frac{dr}{r^2} \left(\int_0^r dr' \rho(r') \right)^2 + O(r_0) \quad (18)$$

we may identify respectively the self-energy of the core charge, the interaction energy between the core charge and fermions, and the self-energy of the fermions.

Nevertheless, the identification of Q_{core} as a localized charge is not without restriction. For simplicity, let us assume that fermions are absent, so that $Q_{tot} = Q_{core} = Q_{rot}$. Then we find that the dyon charge is associated with a homogeneous solution of Gauss' law⁵, so there is no natural way to introduce its density. In other words, as far as the physics outside the core is concerned, one may simply adopt the view that the dyon charge is distributed in some manner inside the core; however, it is not possible to probe inside the core and determine its actual distribution, since electromagnetism is not uniquely defined there.

On the other hand, in accordance with the fact that the energy comes purely from the surface term at infinity

$$\begin{aligned} \mathcal{E} &= \frac{1}{2e^2} \int dV \left(\vec{\nabla} \frac{J(r)}{r} \right)^2 + \frac{1}{e^2} \int dV \left(\frac{J(r)K(r)}{r^2} \right)^2 \\ &= \frac{1}{2e^2} \int d\vec{S} \cdot \left(\vec{\nabla} \frac{J(r)}{r} \right) \frac{J(r)}{r} - \frac{1}{2e^2} \int dV \frac{J(r)}{r^4} (r^2 J''(r) - 2J(r)K^2(r)) \end{aligned} \quad (19)$$

one may also think of the dyon as a capacitor with a capacitance I , charge Q_{rot} , and voltage $\dot{\varphi}$,⁶ regardless of whether r_0 is small or large.

Finally, how do our results compare with Callan's⁷? The answer is, close, but not quite the same. Dropping terms which formally vanish as $r_0 \rightarrow 0$, we may write (18) as

$$\mathcal{E} = \frac{e^2}{8\pi} \int_{r_C}^\infty \frac{dr}{r^2} Q_{tot}^2(r) \quad (20)$$

with $r_C = Ie^2/4\pi$. On the other hand, Callan gives

$$E_r = -\frac{e}{8\pi^{3/2}r^2} \left(\phi_+ - \phi_- + \frac{\theta}{\sqrt{\pi}} \right) \quad (21)$$

$$\mathcal{E} = \frac{e^2}{32\pi^2} \int_{r_0}^{\infty} \frac{dr}{r^2} \left(\phi_+ - \phi_- + \frac{\theta}{\sqrt{\pi}} \right)^2 \quad (22)$$

Therefore, apart from changing r_0 to r_C , it would seem that (15)(20) and (21)(22) are the same, provided we identify $Q_{tot}(r)$ with

$$\frac{-1}{2\sqrt{\pi}} \left(\phi_+ - \phi_- + \frac{\theta}{\sqrt{\pi}} \right) \quad (23)$$

There is however a problem with this identification. After bosonization, $\rho(r)$ is proportional to the derivative of the scalar fields, so our expressions (12)-(14) are invariant under

$$E_r(r) \rightarrow E_r(r) \quad \phi_{\pm}(r) \rightarrow \phi_{\pm}(r) + \alpha_{\pm} \quad (24)$$

whereas (21)(22) are not. The problem is well known in the Schwinger model⁸, where dropping surface terms leads to an answer for the Coulomb energy

$$\frac{g^2}{2\pi} \int dz \phi^2(z) \quad (25)$$

which also breaks $\phi \rightarrow \phi + \alpha$.

To circumvent this problem, we may evaluate (20) directly in terms of

$$2\sqrt{\pi}\rho(r) = -\phi'_+(r) + \phi'_-(r) \quad (26)$$

so that

$$\mathcal{E} = \frac{e^2}{32\pi^2} \int_{r_C}^{\infty} \frac{dr}{r^2} \left(2\sqrt{\pi}Q_{tot} - \phi_+(r) + \phi_-(r) + \phi_+(\infty) - \phi_-(\infty) \right)^2 \quad (27)$$

If the fermions are massless, we may exploit the invariance of (27) to choose a 'gauge'

$$2\sqrt{\pi}Q_{tot} + \phi_+(\infty) - \phi_-(\infty) = -\theta/\sqrt{\pi} \quad (28)$$

so that (15)(20) and (21)(22) are effectively equivalent. Mass terms however, are not invariant under the translation (24), and in general, (15)(20) and (21)(22) need not give the same results for massive fermions⁵.

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REFERENCES

1. H. Yamagishi, MIT-CTP preprints 1139 and 1154.
2. B. Julia and A. Zee, Phys. Rev. D **11**, 2227 (1975); R. Jackiw and J. Goldstone, as reported in *Gauge Theories and Modern Field Theory*, edited by N. Arnowitt and P. Nath (MIT Press, 1976).
3. See also H. Sonoda, Nucl. Phys. **B238**, 259 (1984); W. Goldstein, SLAC-PUB-3272 (1983).
4. See also A. Abouelsaood, Nucl. Phys. **B226**, 309 (1983).
5. See also A. Niemi, M. Paranjape, and G. Semenoff, Phys. Rev. Lett. **53**, 515 (1984).
6. See also Y. Srivastava and A. Widom, Lett. Nuovo Cim. **37**, 77 (1983).
7. C. G. Callan, Phys. Rev. D **26**, 2058 (1982); J. Harvey, Phys. Lett. **131B**, 104 (1983).
8. J. Kogut and L. Susskind, Phys. Rev. D **11**, 3594 (1975).
9. A. Sen, Fermilab-Pub-84/61-T.