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THRESHOLD BEHAVIOUR OF TOP PRODUCTION

IN e^+e^- ANNIHILATION^{*}

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A B S T R A C T

Implications of electroweak interactions on the threshold behaviour of top production are analyzed. Weak decays of densely spaced high radial excitations of toponium lead to a form of the cross-section below threshold that is difficult to distinguish from continuum production. We study the contribution of the vector and axial-vector current to the continuum and calculate the rate for Z decays into top particles as a function of the top quark mass, including QCD corrections.

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A main task of e^+e^- colliders operating in the high energy region will be the isolation of toponium states which supposedly should be found in the mass region around 80 GeV /1/. Even if the mass of top mesons should be known by then to an accuracy of ± 2 GeV, some effort will be necessary to scan a region of 5 - 10 GeV to find the narrow resonances. One of the strategies already proposed for PETRA, was to operate at the highest possible energy (in our case perhaps at the Z^0) and to establish the existence of a new flavor through its marked signature: multijet events of high thrust and acoplanarity from nonleptonic decays and, in addition, energetic isolated leptons from semileptonic decays. Having localized the threshold for this new channel, a fine scan can be performed to find the lowest lying narrow resonances. Although this strategy can be employed again in the 80 GeV region, the threshold properties will be quite different from the familiar behavior below 50 GeV due to the increasing importance of electroweak interactions, as will be shown in this note. Three effects are of relevance in this context:

- 1) The vector part of the neutral current coupling to $t\bar{t}$ contributes to toponium and open top production. Compared to lower energies this merely alters the normalization of the cross section.
- 2) Also the axial part of the neutral current coupling to $t\bar{t}$ contributes to the cross section. However, it is relevant only for open top production while axial vector resonance production /2/ is strongly suppressed. Though the axial charge is large, this channel is inhibited due to its v^3 dependence throughout the threshold region and becomes important only at about 10 GeV above $T\bar{T}$ threshold.
- 3) The narrow spacing (~ 100 MeV) between the high radial excitations together with the large energy spread of e^+e^- machines will lead to an apparent contribution to R that comes quite close to the

(QCD corrected) parton value already within the resonance region (nicely supporting the picture of local duality). Furthermore, the branching ratio for weak decays of a single quark is particularly large for high radial excitations. Since these events are practically indistinguishable from open top production, the apparent threshold will be lowered by about 500 MeV for a toponium mass around 80 GeV. This pattern complicates an accurate measurement of the t quark mass and precision tests of potential models.

We shall discuss these effects now in detail.

The production of 1^{--} resonances is characterized by their electronic widths which we have calculated for 1S to 12S for the Richardson potential /3/ with a top quark mass of 30, 40, 47 and 55 GeV. In Table 1 the values of $\Gamma_0 (1^{--} \rightarrow \gamma e^+e^-)$ are listed. This quantity denotes the toponium decay rate into e^+e^- through the photon channel without neutral current contributions, but includes first order perturbative QCD corrections,

$$\Gamma_0 (1^{--} \rightarrow \gamma e^+e^-) = \frac{4\alpha_t^2 e^2 |R_s(0)|^2}{m_V^2} \left[1 - \frac{16}{3} \frac{\alpha_s}{\pi} \right] . \quad (1)$$

For the high radial excitations ($n > 3$) level spacings and electronic widths are essentially the same for all potential models that describe $b\bar{b}$ and $c\bar{c}$ spectroscopy correctly. They depend only weakly on the quark mass value in the range relevant for our discussion. In the Richardson potential resonances with a binding energy $E_B \gtrsim 780$ MeV are lying above threshold /*1/. Their width into $T\bar{T}$ is expected to be comparable to the mass difference between subsequent levels or even larger. It is shown in Fig. 1a) that the contribution of resonances throughout the threshold region is locally dual to the cross section for quark production if the α_s correction to the latter is taken into

/*1/ The number of bound states below threshold can be derived in a semiclassical WKB approximation, see Ref. /4/ for details.

account. The correction factor to the parton result $3e_t^2$, first derived in QED by Schwinger /5/, is well approximated by

$$\kappa_V = \frac{v(3-v^2)}{2} \left\{ 1 + \frac{4}{3} \alpha_s \left[\frac{\pi}{2v} - \frac{(3+v)}{4} \left(\frac{\pi}{2} - \frac{3}{4\pi} \right) \right] \right\} \quad (2)$$

with

$$\alpha_s \approx 12\pi/25 \log \left[\frac{vm_t}{\sqrt{1-v^2}} / 200 \text{ MeV} \right]^2 .$$

To demonstrate how well local duality holds, we have smeared the resonances with Gaussian distributions whose dispersion of 60 MeV is chosen comparable to the level spacings. The contribution of the vector part of the neutral current (plus γ - z interference terms) increases these rates above and below threshold by a universal factor

$$\begin{aligned} \kappa_V^{\gamma+Z} = & \left| 1 + \frac{(1-4e_t \sin^2 \theta_w) (1-4\sin^2 \theta_w) G_F m_Z^2 m_V^2}{8\sqrt{2} \pi \alpha e_t |m_V^2 - m_Z^2 + im_Z \Gamma_Z|} \right|^2 \\ & + \left| \frac{(1-4e_t \sin^2 \theta_w) G_F m_Z^2 m_V^2}{8\sqrt{2} \pi \alpha e_t (m_V^2 - m_Z^2 + im_Z \Gamma_Z)} \right|^2 \end{aligned} \quad (3)$$

so that finally

$$R_V = 3e_t^2 \kappa_V^{\gamma+Z} \kappa_V . \quad (4)$$

The factor $\kappa_V^{\gamma+Z}$, however, does not change the characteristic pattern of the threshold behavior.

Similar to charm and bottom, top events are also expected to look markedly different above and below threshold as long as $m_t \lesssim 25 \text{ GeV}$. Hard leptons and multijet topologies would then be characteristic for events above threshold, two and three-jet topologies for those below threshold. This distinction is no longer valid for

heavier toponia. Radial excitations will have particularly large branching ratios for weak single quark decays (SQD), and their signature can practically not be distinguished from $T\bar{T}$ decays. Neglecting transitions within the toponium system (they barely affect the subsequent results) the branching ratios for all resonances are given by

$$\text{Br}(\text{SQD}) = \frac{\Gamma_{\text{SQD}}}{\Gamma_{\text{SQD}} + \Gamma_0 \sum_i r_i} \quad (5a)$$

where /6/

$$\Gamma_{\text{SQD}} = 18 \frac{G_F^2 m_t^5}{192 \pi^3} f([m_t/m_w]^2, [m_b/m_t]^2) \quad (5b)$$

$$f(\rho, \mu) = 2 \int_0^{(1-\sqrt{\mu})^2} \frac{du}{(1-u\rho)^2} \cdot \frac{[(1-\mu)^2 + u(1+\mu) - 2u^2]}{[1+\mu^2 + u^2 - 2(\mu u + \mu + u)]^{1/2}} \quad (5c)$$

The ratios r_i ($i = ggg, \gamma g g, f\bar{f}$) characterize the relative strength of the various annihilation decays /6,7/. They are independent of the wave function. The kinematical factor f is of order 1 for $m_t = 30$ to 60 GeV. Γ_0 decreases for increasing radial quantum number whereas Γ_{SQD} is the same for all states. The branching ratio for SQD therefore increases dramatically for high radial excitations as shown in Table 1.

The axial vector contribution is suppressed by a factor v^3 , but sufficiently far above threshold (s large enough) it dominates over the vector contribution due to its large neutral current coupling $[(g_A/g_V)^2 \approx 5.6]$. The order of magnitude of J^{++} resonance production can be estimated through axial vector $t\bar{t}$ production in the parton model plus first-order QCD correction /8/. This cross-section is well approximated by /*2/

/*2/ Since α_s is small, $R \sim v^2$ only very close to threshold.

$$\mathcal{K}_A = v^3 \left\{ 1 + \frac{4}{3} \alpha_s \left[\frac{\pi}{2v} - \left(\frac{19}{10} - \frac{22}{5} v + \frac{7}{2} v^2 \right) \left(\frac{\pi}{2} - \frac{3}{4\pi} \right) \right] \right\} . \quad (6)$$

They are shown in Fig. 2 together with the corresponding vector current quantities. α_s has been evaluated for a top mass of 40 GeV. The axial contribution is evidently small throughout the threshold region. Even if the larger axial-vector t coupling is taken into account,

$$\mathcal{K}_A^Z = \left\{ 1 + (1 - 4 \sin^2 \theta_w)^2 \right\} \left| \frac{G_F m_Z^2 m_V^2}{8\sqrt{2} \pi \alpha e_t (m_V^2 - m_Z^2 + i m_Z \Gamma_Z)} \right|^2 \quad (7)$$

so that

$$R_A = 3 \mathcal{K}_A^Z \mathcal{K}_A , \quad (8)$$

the axial-vector contribution is always smaller than the vector contribution within the first 10 GeV above threshold and quite negligible below.

Depending on the precise mass value of the top quark we anticipate a large variety of possible threshold pattern which all can be derived from the expressions given above. To illustrate the different types, we present in Fig. 3 the contribution to R from all narrow resonances below threshold. Above threshold resonance production is then joined smoothly to the continuum contribution defined in eqs. (2), (3), (6), (7). Local duality together with the beam spread and the natural line width for resonances /9/ above threshold insure that the precise location of the junction is not crucial. We have smeared the cross section by a Gaussian energy spread with dispersion δW . The contribution of a resonance to R is thus given by

$$\Delta R(\text{RES}) = \frac{9\pi}{2\alpha^2} \Gamma(V \rightarrow e^+ e^-) \frac{e^{-Z^2/2}}{\sqrt{2\pi} \delta W} \quad (9)$$

where $Z = (E_{\text{CM}} - m_V)/\delta W$. The shaded area denotes the fraction of events with single top quark weak decays. Four typical regions are shown.

- a) $M(t\bar{t}) \approx 60 \text{ GeV}$: The production rate is barely influenced by neutral current effects. Single quark decays dominate the highest resonance decays and diffuse the threshold region. We have chosen $\delta W = 40 \text{ MeV}$ in this region.
- b) $M(t\bar{t}) \approx 80 \text{ GeV}$: The vector part of the neutral current increases the resonance and threshold cross section roughly by a factor 1.3. The contribution of the axial part remains negligible throughout the threshold region. Despite its larger coupling it equals the vector part only above 90 GeV.
- c) $(t\bar{t})$ nearly degenerate with Z : If the threshold region happens to coincide with the Z^0 peak within $\pm 2 \text{ GeV}$, decays through Z^0 will dominate even for the highest radial excitations below threshold, resulting primarily in 2-jet hadronic final states. The threshold would be quite distinct in this case. Note that, open top hadronic decays would produce high sphericity multijet final states in the continuum. For these dominant decay channels the interference between the continuum and the toponium state is important. It can turn the resonance peak even into a dip. If the resonance is very close to Z , the formula for the enhancement is drastically changed as far as Z^0 mediated decays are concerned [11/,

$$\Delta R(\text{RES}) = \frac{9\pi}{2\alpha^2} \Gamma(V \rightarrow e^+ e^-) \frac{1}{\sqrt{2\pi} \delta W} \left[\frac{1-\gamma^2}{1+\gamma^2} e^{-Z^2/2} + \frac{2\gamma}{1+\gamma^2} f(Z) \right] \quad (9')$$

$$f(Z) = \frac{2}{\sqrt{\pi}} e^{-Z^2/2} \int_0^{Z/\sqrt{2}} dy e^{y^2}$$

$$\gamma = \Gamma_Z m_Z / (m_V^2 - m_Z^2)$$

[For direct channels like the single quark decay eq. (8) remains valid.] Below threshold large fluctuations in the total cross section above and below the continuum level would be observed.

d) $M(t\bar{t}) \approx 110$ GeV: Single quark decays dominate all toponium decays.

Production above and below threshold are practically indistinguishable.

With SQD's being dominant resonances below and above top threshold cannot be distinguished any longer through differences in final state topologies. Is there still a way to establish experimentally the presence of a threshold? One possibility would be to look for forward-backward asymmetries in the angular distribution of leptons from semileptonic decays. For leptons originating from top quarks within toponium, a sizeable asymmetry can be expected /10/. In forming top mesons (T or T^*) this polarization is partly lost and, as a consequence, the asymmetry is presumably different above and below threshold.

The cross sections given above can also be used to calculate the decay rate of Z^0 into top mesons. Normalized to the muon decay rate,

$$\frac{\Gamma(Z \rightarrow t\bar{t})}{\Gamma(Z \rightarrow \mu^+\mu^-)} = \frac{3 \{ (1 - \frac{8}{3} \sin^2 \Theta_w)^2 \mathcal{M}_V + \mathcal{M}_A \}}{(1 - 4 \sin^2 \Theta_w)^2 + 1} \quad (10)$$

The ratio is shown in Fig. 4 for m_t between 23 and 45 GeV. Note that for a measurement of the top mass through such a ratio it is crucial to take the radiative QCD corrections into account /5,8,12/.

To summarize. The apparent threshold behavior expected for top production is quite different from the pattern in the charmonium and bottonium system. This is due to the neutral current contribution to the production amplitude and due to the weak decays of a quark inside toponium, independent of the second quark. These effects, distinctly different for toponium masses below, above or close to the Z mass, must be carefully taken account of if the t quark mass is extracted from the toponium spectrum and if QCD inspired potential models, such as the Richardson potential, are experimentally scrutinized.

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Table 1. Binding energies of toponium nS , for $1S$ up to threshold, decay rates into e^+e^- through a photon and branching ratios for single quark decays.

top quark mass: $2m_t = 60 \text{ GeV}$				$2m_t = 80 \text{ GeV}$			
nS	E_B/MeV	Γ_0/keV	$\text{Br}(SQD)$	nS	E_B/MeV	Γ_0/keV	$\text{Br}(SQD)$
1	-1259	6.71	0.10	1	-1485	7.34	0.27
2	- 444	1.91	0.30	2	- 572	1.97	0.58
3	- 109	1.09	0.43	3	- 218	1.08	0.72
4	109	0.80	0.52	4	4	0.77	0.78
5	278	0.65	0.57	5	172	0.61	0.82
6	420	0.56	0.61	6	310	0.52	0.84
7	545	0.51	0.64	7	431	0.46	0.85
8	659	0.46	0.66	8	539	0.42	0.86
9	764	0.43	0.68	9	638	0.39	0.87
				10	731	0.37	0.88

$2m_t = 94 \text{ GeV}$				$2m_t = 110 \text{ GeV}$			
nS	E_B/MeV	Γ_0/keV	$\text{Br}(SQD)$	ns	E_B/MeV	Γ_0/keV	$\text{Br}(SQD)$
1	-1625	7.75	0.016	1	-1755	8.21	0.36
2	- 648	2.02	0.028	2	- 728	2.08	0.72
3	- 281	1.08	0.043	3	- 344	1.09	0.83
4	- 55	0.75	0.058	4	- 114	0.74	0.88
5	113	0.59	0.073	5	55	0.58	0.91
6	250	0.50	0.087	6	192	0.49	0.92
7	368	0.44	0.10	7	309	0.43	0.93
8	474	0.40	0.12	8	412	0.38	0.94
9	571	0.37	0.13	9	507	0.36	0.94
10	660	0.35	0.14	10	596	0.35	0.94
				11	686	0.37	0.94

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FIGURE CAPTIONS

- Fig. 1 Production cross section for 1^{--} resonances up to 10S (Richardson potential) in the threshold region for $m_t = 40$ GeV without neutral current effects, smeared with a Gaussian distribution of width $\delta W = 60$ MeV. $3e_t^2 4_V$ is shown for comparison.
- Fig. 2 Threshold factors for heavy quark production through vector and axial-vector current.
- Fig. 3 Threshold behavior of toponium and open top production, including neutral current effects. The dashed area indicates the fraction of SQD events. The contribution from axial-vector current to the continuum is indicated by the dotted area.
 (a) $2m_t = 60$ GeV, $\delta W = 40$ MeV; (b) $2m_t = 80$ GeV, $\delta W = 32$ MeV;
 (c) $2m_t = 94$ GeV, $\delta W = 48$ MeV [the solid line denotes the change of the fermion-antifermion cross section alone - normalized to $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$]; (d) $2m_t = 110$ GeV, $\delta W = 80$ MeV.
- Fig. 4 Decay rate of Z^0 into top quarks, normalized to $\Gamma(Z \rightarrow \mu^+\mu^-)$. The dashed line indicates the rate without QCD corrections.

- Figure 1 -

- Figure 2 -

- Figure 3a -

- Figure 3b -

- Figure 3c -

- Figure 3d -

- Figure 4 -