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MONTE CARLO RENORMALIZATION GROUP STUDIES OF
SU(3) LATTICE GAUGE THEORY*

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A B S T R A C T

Results are reported of Monte Carlo Renormalization Group studies of the approach to asymptotic scaling in SU(3) lattice gauge theory. By comparing measurements on 16^4 and 8^4 lattices, estimates are obtained for the shift $\Delta\beta$ in the fundamental plaquette coupling β , corresponding to a change of length scale by a factor of 2. The definition of block link variables contains a free parameter whose value can be optimized to minimize the transient flow to the renormalized trajectory. Our prediction $\Delta\beta(\beta=6.0)=0.35\pm 0.02$ is consistent with the results obtained previously with the improved ratio method which is discussed briefly also. In both methods simulation is performed with only the standard Wilson action. An important feature of the results is the appearance of a pronounced dip in $\Delta\beta$ which implies that in the presently accessible range of β the asymptotic value is approached from below, and its onset is delayed.

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1 INTRODUCTION

In the large cut-off limit of renormalizable theories it is possible to tune the cut-off and the coupling(s) together in such a way that the physical content of the theory remains unchanged. The functional relation between the coupling(s) and the cut-off is given by the β -function(s) of the theory. Perturbation theory suggests that in an $SU(N)$ gauge theory the leading corrections to this behavior are exponentially small in the inverse of the bare coupling constant g^2 . On the other hand the higher order corrections to the universal part of the β -function

$$\beta(g) = -b_0 g^3 - b_1 g^5 + \dots \quad (1)$$

are power like and are not necessarily small in the region where the cut-off dependent corrections are already negligible. There might be sizeable contributions to the β -function from non-perturbative phenomena also.

In the lattice formulation of Yang-Mills theories, where numerical studies are performed at moderate correlation lengths (i.e. at intermediate coupling constant values), the quantitative knowledge of the β -function is basically important for the correct interpretation of the results. One should confirm also that the β -function approaches the asymptotic form of Eq.(1) without passing through a phase transition, assuring a continuum limit with the expected properties of asymptotic freedom and confinement.

In this paper the first results obtained in an extended collaboration for a Monte Carlo renormalization group (MCRG) study of $SU(3)$ lattice gauge theory are reported. In this study block loop expectation values on 16^4 and 8^4 lattices are compared in order to determine the shift $\Delta\beta = \Delta\beta(\beta)$ in the fundamental plaquette coupling β ($=6/g^2$) of the standard Wilson action, corresponding to a change of scale by a factor of 2. The function $\Delta\beta(\beta)$ is directly related to the integral of the inverse of the β -function and contains the same information:

$$\int_{\beta-4\beta}^{\beta} \frac{dx}{x^{3/2} \beta_{\text{funct.}}(\sqrt{6/x})} = - \frac{2 \ln 2}{\sqrt{6}} \quad (2)$$

The definition of block link variables contains a free parameter whose value is optimized to minimize the transient flow to the renormalized trajectory [1]. This free parameter which is fixed non-perturbatively here helps to reduce and control the systematic errors in the matching predictions. Our main result is a single matching value at $\beta = 6.0$:

$$\Delta\beta(\beta=6.0) = 0.35 \pm 0.02 \quad (3)$$

where the error includes our estimate of both the statistical and systematic uncertainties. Results will be available soon at $\beta = 6.6$ and 7.2 also [2].

The result quoted above is consistent with the results obtained earlier by a different MCRG method, the improved ratio method (Fig. 1) [3]. The starting point of this latter procedure is the observation [4] that those ratios of Wilson loop expectation values from which the self-mass and corner contributions cancel satisfy the homogeneous renormalization group (RG) equation at least if the loops involved are large compared to the lattice spacing. Since the homogeneous RG equation is linear in these ratios, arbitrary linear combinations of the basic ratios satisfy it also. The method can be optimized by taking particular combinations of the basic ratios to cancel lattice artifacts order by order in perturbation theory. An advantage of this method is its simplicity. Any high statistics measurement of the potential or string tension can be extended easily to perform this analysis, and it is expected to work even at large correlation lengths. Its disadvantage is that it is difficult to see how to do the optimization non-perturbatively and the method requires very good statistics. The statistical error of the block loop matching result in Eq.(3) is about 4 times smaller than the corresponding error quoted in ref.[3].

2 THE BLOCK TRANSFORMATION

The block variable V_{AB} associated with the block link A-B in Fig. 2 is chosen with the probability

$$\text{prob.}(V_{AB}) \sim e^{\frac{P}{6} \text{Tr}(V_{AB}^{\dagger} X + h_c)} \quad (4)$$

where X is the sum of the matrix products along 7 different paths connecting the sites A and B : $U_5 U_8$, $U_1 U_2 U_3 U_4$, $U_7 U_8 U_9 U_{10}$ and the corresponding paths in the orthogonal planes [5]. The parameter P will be chosen in such a way as to force the renormalized trajectory to run close to the standard action at $\beta = 6.0$, where the calculation is performed [1].

Figure 2

3 CONFIGURATIONS, STATISTICS

The 16^4 , $SU(3)$, $\beta = 6.0$ configurations were created on a DAP at Edinburgh. After every 112 pseudo heat-bath [6] sweeps the configuration was stored for later blocking and other measurements [2]. The first 1500 sweeps were discarded. The limited memory of the DAP's configuration at Edinburgh forced us to store the link variables as 16 bit integers with a scale factor $N = 32000$, while the matrix multiplications in the updating were done in $real*3$. The corresponding rounding errors introduce additional randomness into the system resulting in a slight systematic error: the configuration looks somewhat "hotter" than $\beta = 6.0$ would require. However this effect is very small. Theoretical considerations and test runs (with artificially decreased scale factor N) suggest that the corresponding error in the plaquette expectation value is $O(10^{-8})$ - well below our statistical accuracy.

The 8^4 configurations at $\beta = 5.4$, 5.6 and 5.7 were created at CERN and DESY starting from the last, well-equilibrated configurations of earlier studies. These configurations were separated by 10 pseudo heat-bath sweeps.

The block transformations were performed at the CERN IBM machines at four different values of the free parameter P : 20, 30, 35 and 40. These values were picked after a few preliminary runs.

The result quoted in Eq.(3) is based on $O(50)$ 16^4 configurations, while 64 , 96 and $32 \cdot 8^4$ configurations were blocked at $\beta = 5.7$, 5.6 and 5.4 respectively. Linear interpolation was used to get intermediate β values.

The statistical errors were estimated by measuring time correlations and also by the usual "bunching".

4 THE OPTIMAL VALUE OF THE PARAMETER P

If for some block transformation the renormalized trajectory runs along the line of the standard Wilson action in the multi-parameter coupling constant space [7], then after the first blocking step the effective action is a standard action again at some coupling $\beta' = \beta - \Delta\beta$. In this case the block loop expectation values are equal to the corresponding Wilson loop expectation values of the standard action at coupling β' on a lattice of half the size. The parameter P is fixed by requiring that one gets as close as possible to this situation. To say it in another way: p^{optimal} is obtained by requiring the best possible consistent matching for many observables after the first blocking step.

As is illustrated in Fig. 3 the matching values of 12 different loops (1×1 , 1×2 , ..., 4×4 , ,) after the 1st blocking step suggest:

$$p^{\text{opt}} = 35 \pm \frac{10}{5} \quad . \quad (5)$$

It should be emphasized: in principle any value of P is appropriate. The different block transformations should give the same final prediction for $\Delta\beta$. As we shall see the P dependence of the matching predictions becomes weaker and weaker as the number of blocking steps is increased. However at $P \sim p^{\text{optimal}}$ the coefficient of the subleading eigenoperator is small resulting in a better control of the systematic errors.

5 RESULTS

The predictions for $\Delta\beta(\beta=6.0)$ obtained from matching four different block loops (plaquette, $6_1(=\square)$, $6_2(=\triangle)$ and $6_3(=\nabla)$) are given for $P = 30, 35$ and 40 at subsequent blocking levels in Table 1. As the statistical errors of the block loops 6_1 and 6_3 after the 3rd blocking step are very large no matching value is quoted there. The results obtained from the 1×1 block loop are shown in Fig. 4, where the tendencies can clearly be seen. The P dependence weakens as the number of blocking steps is increased. The P dependence is linear in $1/P$ - as suggested by perturbation theory - for $P \geq 30$, which makes it possible to follow the predictions for large P even without actually measuring them. For the 1×1 block loop matchings $p^{\text{optimal}} \sim 50$ - this is the value where the predicted $\Delta\beta$ is the same after the first and second blocking step: $\Delta\beta = 0.35 \pm 0.01$. This value is consistent with the extrapolated 3rd blocking result. A simple averaging of the 3rd blocking step predictions at $P = 30, 35$ and 40 gives $\Delta\beta = 0.359 \pm 0.013$.

The following simple consideration helps to gain confidence that the systematic errors are really under control. It is expected that in the continuum limit the subleading eigenoperator is of dimension 6 with an eigenvalue $1/4$ (up to negligible logarithmic corrections). This implies the behavior:

$$\Delta\beta^{(n)} = \Delta\beta^{(n=\infty)} + \alpha(P) \left(\frac{1}{4}\right)^n \quad (6)$$

where n is the number of block steps, $\Delta\beta^{(n=\infty)}$ is the P independent result after $n=\infty$ blocking steps^{*)}. Eq (6) gives

$$\Delta\beta^{(n=\infty)} = \frac{4\Delta\beta^{(n=2)} - \Delta\beta^{(n=1)}}{3} \quad (7)$$

If this procedure is consistent the predicted $\Delta\beta^{(n=\infty)}$ should be independent of P and the block loop considered. The numbers are summarized in Table 2, where the errors quoted are statistical. On the basis of this table and of the previous considerations we feel that the error estimate in Eq.(3) is rather conservative.

*) In this context the optimization consists in minimizing $\alpha(P)$.

There is one trivial type of systematic error which we did not check however: the error coming from the linear extrapolation between $\beta = 5.6$ and 5.7 on the 8^4 lattice. This error is very easy to avoid almost completely (by blocking 8^4 configurations at $\beta' = \beta - 0.35 = 5.65$) and we intend to do so in the future. The effect is expected to be small.

6 DISCUSSION

Fig. 1 shows a pronounced dip in $\Delta\beta(\beta)$ around $\beta \sim 6.0$ which implies that in the accessible range of β the asymptotic value is approached from below and its onset is delayed. For $\beta = 6.6$ this deviation seems to be rather small, which is supported by recent MCRG measurements by Gupta and Patel [8] using a special " $\sqrt{3}$ " block transformation [9] at $\beta = 6.5$ and 7.0 . Recent precise string tension [10] and critical temperature [11] measurements show the same qualitative behavior for $\Delta\beta(\beta)$. A similar structure seems to be emerging in $SU(2)$ according to string tension [12] and preliminary MCRG results [13]. It is an interesting theoretical problem to understand the origin of this "unnatural" behavior, which is unlike the one found in the $d=2$ standard non-linear σ model, where the asymptotic value is approached smoothly from above [1,14]. It is even more important to determine $\beta_{\min}$ above which the different methods give quantitatively the same $\Delta\beta(\beta)$. For $\beta \geq \beta_{\min} - \Delta\beta(\beta_{\min})$ a unique β -function can be defined and the theory reflects continuum properties.

Concerning the first question it is a natural assumption that the dip in $\Delta\beta$ is related to the critical point at the end of the first order transition line in the fundamental-adjoint coupling constant plane. The flow from the spurious critical point is expected to slow down the flow from $\beta_f = \infty$ until the flow has passed the neighbourhood of the spurious critical point when the two flows reinforce and speed up the flow towards the fixed point at $\beta_f = \beta_a = 0$. The slowing down implies that $\Delta\beta_f$ approaches its asymptotic value from below. A related explanation was suggested by Makeenko and Polikarpov recently [15]. According to these ideas smoother behavior and earlier onset of asymptotic scaling is expected along the lines $\beta_a/\beta_f = -c$ ($c>0$). It is known that the peak of the specific heat is strongly reduced in this region [16].

The explanation of the dip in $\Delta\beta$ in terms of higher order perturbative terms of the β -function is very improbable [17].

The answer to the second question requires precise data. It is exciting and reassuring that the kind of precision quoted here and in related works might pin down $\beta_{\min}$ and predict the β -function with a reasonable precision.

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TABLE CAPTIONS

- Table 1: The matching predictions $\Delta\beta(\beta=6.0)$ are summarized for 4 different block loops at different blocking levels and different values of P .
- Table 2: $\Delta\beta(n=\infty)$ is given as obtained by assuming that the subleading eigenvalue of the linearized RG transformation is $1/4$.

FIGURE CAPTIONS

- Figure 1: The shift $\Delta\beta$ as a function of β obtained from 1-loop improved ratios (squares) [3] and from optimized blocking (dot) in this work. In the case of the ratio results the thin error bars refer to the statistical error while the thick error bars refer to the average fluctuation of the large number of different ratios included in the analysis. The dashed line is the asymptotic prediction.
- Figure 2: Construction of block link (see text).
- Figure 3: The matching predictions obtained from 12 different block loops after the first blocking step. P is the free parameter in the block transformation, while $\beta' = \beta - \Delta\beta$. For $P \geq 30$ the predictions obtained from a given loop are linear in $1/P$. The mean deviation of the matching predictions has a broad minimum in the region $P = 35 \pm \frac{10}{5}$.
- Figure 4: The matching predictions obtained for the 1x1 block loop at subsequent blocking levels as a function of $1/P$.

P	blocking step
30	1	0.461(3)	0.471(3)	0.414(3)	0.392(3)
	2	0.382(6)	0.375(7)	0.372(5)	0.368(5)
	3	0.359(13)	-	0.383(25)	-
35	1	0.420(2)	0.444(2)	0.386(3)	0.370(3)
	2	0.371(6)	0.369(5)	0.365(5)	0.362(5)
	3	0.368(13)	-	0.348(21)	-
40	1	0.388(2)	0.424(2)	0.363(3)	0.352(3)
	2	0.361(5)	0.360(6)	0.367(5)	0.355(5)
	3	0.351(13)	-	0.342	-

TABLE I

P
30	0.358(8)	0.346(9)	0.358(7)	0.360(7)
35	0.356(8)	0.346(7)	0.358(7)	0.359(7)
40	0.353(7)	0.340(8)	0.355(7)	0.356(7)

TABLE II

Fig. 1

Fig. 3

Fig. 4