CP-Non-Conservation and b-Quark Decays[†]

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Abstract

We show that in the left-right symmetric models of CP-violation with 3 generations, ϵ' vanishes in the limit of the Kobayashi-Maskawa angle $s_3 = 0$ and W_L - W_R mixing $\zeta = 0$. This model would appear to be better suited for the description of CP-violating phenomena, if the upper limits on the branching ratio for $b \rightarrow ue\bar{\nu}$ (which is proportional to s_3^2) as well as ϵ'/ϵ keeps getting smaller.

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Even though the phenomenon of CP-violation was discovered nearly twenty years ago, its origin remains a mystery. With the advent of gauge theories, it was realized that for two generations of quarks and leptons the V-A gauge interactions are always CP-conserving.^{1,2} In ref. 1 and 3 it was proposed that the CP-violation may have its origin in yet unobserved new physics such as possible right-handed weak currents. On the other hand, Kobayashi and Maskawa² proposed that the V-A gauge interactions can accomodate CP-non-conservation for three generations of quarks and leptons. The Kobayashi-Maskawa model, has become the focus of a great deal of theoretical activities, following the discovery of the b-quark and the τ -lepton, that indicated the existence of the third generation.

The KM model is described by the following Lagrangian:

$$\mathcal{L}_{WK} = \frac{ig}{\sqrt{2}} W_{\mu}^+ \bar{P}_L \gamma_{\mu} U_{KM} N_L + h.c. \quad (1)$$

where $P = (u, c, t, \dots)$ and $N = (d, s, b, \dots)$ and

$$U_{KM} = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix} \quad (2)$$

with $s_i = \sin \theta_i$ and $c_i = \cos \theta_i$. Of the three angles the first one, θ_1 is the Cabibbo angle and is determined to be .229. The recent results on the b-quark lifetime and decays⁴ have implied considerable restriction on the other angles θ_2 and θ_3 . More

quantitatively,⁵ one finds.

$$s_2^2 + s_3^2 + 2s_2 s_3 c_\delta \approx 4 \times 10^{-3} \left(\frac{10^{-12} \text{ sec.}}{\tau_b} \right) B(b \rightarrow c) \\ \approx 3 \times 10^{-3} (10^{-12} \text{ sec.} / \tau_b) \quad (3)$$

and

$$s_3^2 \approx 4 \times 10^{-2} (10^{-12} \text{ sec.} / \tau_b) \cdot B(b \rightarrow u) \\ \leq 10^{-3} (10^{-12} \text{ sec.} / \tau_b) . \quad (4)$$

Here we have used the experimental information that the ratio $r = B(b \rightarrow u)/B(b \rightarrow c)$ is less than 5%. The implications of eqn. (3) and (4) for CP-violation have been extensively studied in literature (for original references, see ref. 5). A consensus seems to be emerging that unless the $\Delta S = 2$ hadronic matrix element (the B-parameter) is large, the magnitude of the CP-violating parameter ϵ ($\epsilon \approx 2.3 \times 10^{-3}$), can not be understood for the s_3 parameter less than 1% (or $r < .01$). Thus, if r_{expt} gets smaller than 1% there must be new contributions to CP-violation in K^0 -decays.

A second prediction of the KM model is that the ratio ϵ'/ϵ is bigger than⁶ $(2-3) \times 10^{-3}$, although uncertainties exist here too. The most recent experimental search for ϵ'/ϵ has led to the result

$$\epsilon'/\epsilon = (-4.6 \pm 5.3 \pm 2.4) \times 10^{-3} . \quad (5)$$

It is thus clear that, if experimental value of ϵ'/ϵ goes below

2×10^{-3} or so the KM model will be ruled out and other sources for CP-violation must be looked for.

In this letter, we argue that the left-right symmetric model of CP-violation provides a natural framework for accomodating arbitrarily small ϵ'/ϵ as well as a small value of the parameter r while at the same time explaining the magnitude of the ϵ -parameter. The happens because ϵ' and ϵ receive dominant contributions from different sources: ϵ arises from the right-handed currents whereas ϵ' owes its origin to the s_3 -parameter of KM-model as well as the left-right mixing.^{3,7} The model requires an upper bound on the W_R -mass of about 65 Tev.

To demonstrate our result, we start with the charged current electro-weak Lagrangian of the left-right symmetric model^{8,9} for three generations:

$$\begin{aligned} \mathcal{L}_{WK} = & \frac{ig_L}{2\sqrt{2}} W_{\mu,L}^+ \bar{P} \gamma_\mu (1 + \gamma_5) U_{KM}^N + \frac{ig_R}{2\sqrt{2}} W_{\mu,R}^+ \bar{P} \gamma_\mu (1 - \gamma_5) V^N \\ & + \frac{1}{2} m_{W_R}^2 \zeta e^{i\gamma} W_{\mu L}^+ W_{\mu R}^- + \text{h.c.} \end{aligned} \quad (6)$$

U_{KM} is defined in eqn. (2). In theories with pseudo-manifest left-right symmetry¹⁰ (i.e. with symmetric quark-mass matrices¹¹),

the mixing matrix V in the right-handed sector can be written as:

$$V = \begin{pmatrix} e^{i\alpha_1} & & \\ & e^{i\alpha_2} & \\ & & e^{-i(\alpha_1+\alpha_2)} \end{pmatrix} U_{KM}^* \begin{pmatrix} e^{i\beta_1} & & \\ & e^{i\beta_2} & \\ & & e^{-i(\beta_1+\beta_2)} \end{pmatrix} \quad (7)$$

The effective $\Delta S = 1$ four fermion weak Hamiltonian in this case receives contributions from W_L and W_R exchange graphs as well as the so-called Penguin graphs and can be written as a sum of three terms:¹²

$$H_{WK} = H_{WK}^{(0)} + H'_{WK} + H_{LR} \quad \dots \quad (8)$$

where

$$H_{WK}^{(0)} = \frac{4G_F}{\sqrt{2}} [O_L + \eta e^{i(\beta_1 - \beta_2)} O_R] \quad (9)$$

with

$$O_\sigma = \sum_{q=u,c,t} \lambda_q \bar{s}_\sigma \gamma_\mu q \bar{q}_\sigma \gamma_\mu s_\sigma + \frac{\alpha_s(\mu)}{24\pi} (\bar{u} \gamma_\mu \lambda^q_u + \bar{d} \gamma_\mu \lambda^q_d) \bar{s}_\sigma \gamma_\mu \lambda^a_d \sum_{q=u,c,t} \lambda_q \ln \frac{m_q^2}{\mu^2} \quad \sigma = L, R \quad (10)$$

$$\lambda_u = -c_1 s_1 c_3 ; \lambda_c = s_1 c_2 (c_1 c_2 c_3) ; \lambda_t = s_1 s_2 c_2 s_2 c_3 \quad (11)$$

$$\eta = (m_{W_L} / m_{W_R})^2$$

$$H'_{WK} = 4 \frac{G_F}{\sqrt{2}} c_2 s_1 s_2 s_3 \{ -e^{-i\delta} (\bar{s}_L \gamma_\mu c_L \bar{c}_L \gamma_\mu d_L - \bar{s}_L \gamma_\mu t_L \bar{t}_L \gamma_\mu d_L) - e^{-i\delta} \frac{\alpha_s(\mu)}{24\pi} (\bar{u} \gamma_\mu \lambda^a_u + \bar{d} \gamma_\mu \lambda^a_d) (\bar{s}_L \gamma_\mu \lambda^a_d) \ln \frac{m_c^2}{m_t^2} + \eta e^{i(\beta_1 - \beta_2)} (L \longleftrightarrow R, \delta \rightarrow -\delta) \} \quad (12)$$

H'_{LR} denotes the W_L - W_R mixing effect and is given by:

$$H'_{LR} = \frac{4G_F}{\sqrt{2}} \zeta e^{i\gamma} \sum_{q=u,c,t} \bar{s}_R \gamma_\mu q_R \bar{q}_L \gamma_\mu d_L U_{qd} U_{qs} e^{i\sigma_q} + (L \leftrightarrow R, q \leftrightarrow q') \quad (13)$$

where σ_q are simple linear combinations of α_i and β_i ; $\sigma_1 = \alpha_1 + \beta_2$; $\sigma_2 = \alpha_2 + \beta_2$; $\sigma_3 = \beta_2 - \alpha_1 - \alpha_2$ and $\sigma'_q = \sigma_q(\beta_2 \leftrightarrow \beta_1)$. Phenomenologically, it is known¹³ that $\zeta \lesssim 10^{-2} - 4 \times 10^{-3}$; its effect on CP-violation will be compared to the other terms below.

Of the three terms in eqn. (8), we note that $H'_{WK} \rightarrow 0$ as $s_3 \rightarrow 0$ and $H_{LR} \rightarrow 0$ as $\zeta \rightarrow 0$. For small s_3 and ζ we can approximate the effective $\Delta S = 1$ weak Hamiltonian only by the first term in eqn. (8). Our new point in this paper is that $H_{WK}^{(0)}$ satisfies the iso-conjugate relations even though the third generation is not totally decoupled from the first two, i.e. if we separate it into a P-conserving (S) and P-violating piece (P) and CP-conserving ($S^{(+)}, P^{(+)}$) and CP-violating pieces ($S^{(-)}$ and $P^{(-)}$) i.e.

$$H_{WK}^{(0)} = S^{(+)} + S^{(-)} + P^{(+)} + P^{(-)} \quad (14)$$

then, they satisfy the relations:³

$$[I_3, P^{(-)}] = i\eta \sin(\beta_1 - \beta_2) P^{(+)}$$

and

$$[I_3, S^{(-)}] = -i\eta \sin(\beta_1 - \beta_2) S^{(+)} \quad (15)$$

It was shown in ref. 3 that eqn. (15) implies that $\text{Im}A_2/\text{Re}A_2 = \text{Im}A_0/\text{Re}A_0$ where A_I denotes the $K^0 \rightarrow 2\pi$ decay amplitude in the channel with isospin, I . This in turn implies that $\epsilon' = 0$ and

$\epsilon \approx A \eta \sin(\beta_1 - \beta_2)$, where A is a dynamical factor. The entire CP-violation owes its origin to right-handed currents and vanishes as $m_{W_R} \rightarrow \infty$.

In ref. 3, it was observed that for two generations of quarks (i.e. $s_2 = s_3 = 0$) eqn. (15) holds and CP-violation arises entirely from right-handed currents. Here, we show that the same thing happens under more general conditions when $s_2 \neq 0$ but $s_3 = 0$. This is of phenomenological interest since s_3 is directly proportional to $B(b \rightarrow u)$; so if $B(b \rightarrow u)$ keeps getting smaller, CP-violation will arise dominantly from right-handed currents, and the iso-conjugate relations will keep getting better obeyed. On the other hand, $B(b \rightarrow c)$ is known to be nonzero which implies s_2 is nonzero. The scenario we envision is that if s_3 keeps getting smaller, ϵ will arise dominantly from V+A interactions and ϵ' from left-handed sector (and possibly from left-right mixing).

This ϵ -parameter can be determined in this model and receives dominant contribution from the left-right box graph¹⁴ and has been determined to be:¹²

$$\epsilon \approx \frac{m'}{\Delta m} \frac{e^{i\pi/4}}{\sqrt{2}} \quad (16)$$

where

$$m' \approx \frac{G_F^2}{12\pi^2} s_1^2 F_K^2 m_K^2 m_c^2 B [2s_2 s_3 s_\delta \{-\eta_1 + \eta_2 K \frac{m_t^2}{m_c^2} + \eta_3 \ln m_t^2/m_c^2\} - 60 I_{LR} \rho \cdot \eta \sin(\beta_1 - \beta_2)] \quad (17)$$

where I_{LR} is the QCD correction¹⁵ to the left-right box graph

estimated to be around 3; ρ is the ratio of the matrix elements $\langle K^0 | \bar{s}_R d_L \bar{s}_L d_R | \bar{K}^0 \rangle$ and $\langle K^0 | \bar{s}_L \gamma_\mu d_L \bar{s}_L \gamma_\mu d_L | \bar{K}^0 \rangle$, expected to be around 7 or 8 in vacuum saturation approximation; the factor $60 I_{LR} \rho \approx 1.4 \times 10^3$, providing an enhancement of the right-handed current contribution to ε -parameter. The Δm_K receives a similarly enhanced contribution from the L-R box graph leading to the bound¹⁴ of $m_{W_R} \geq 3$ Tev.

The first term within the bracket in eqn. (17) will become smaller if s_3 gets smaller and may eventually become too small to explain the magnitude of ε . In that case, the right-handed current contribution provide bulk of the contribution to ε as mentioned earlier. This implies an upper bound on the value of m_{W_R} of about 65 Tev. For lower values of m_{W_R} , we must have $\beta_1 - \beta_2 \ll \pi/2$.

The ε'/ε in this model owes its origin to two sources: (i) $s_3 \neq 0$ and (ii) $\zeta \neq 0$. Which of these plays the important role can be determined from independent experiments: case (i) by studying $b \rightarrow ue\bar{\nu}$ decay and case (ii) by measuring the electric dipole moment of the neutron.^{7,16} An accurate prediction of ε'/ε is plagued by uncertainties in hadronic matrix elements. In any case, we can write down $I_m A_I / R_e A_I$ in terms of matrix elements of $\Delta S = 1$ operators as follows:

Denoting $P \equiv \frac{1}{2} (O_L - O_R)$, $O^5 \equiv s_1 \frac{\alpha_s(\mu)}{24\pi} (\bar{u} \gamma_\mu \lambda^a u + \bar{d} \gamma_\mu \lambda^a d) \times \bar{s} \gamma_\mu \lambda^a \gamma_5 d$ and $O_{LR} \equiv -i \sum_{q=u,c,t} \sin(\gamma + \sigma_q) \lambda_q (\bar{s} \gamma_\mu q \bar{q} \gamma_\mu \gamma_5 d - \bar{s} \gamma_\mu \gamma_5 q \bar{q} \gamma_\mu d)$. Neglecting terms of order $\zeta \cdot s_1 s_2 s_3$ compared to terms of order ζs_1 and $\zeta s_1 s_2^2$, we find that, for $\beta_1 - \beta_2 \ll 1$

$$H'_{LR} = \frac{4G_F}{\sqrt{2}} \zeta O_{LR} \quad (18)$$

It is interesting that the left-right mixing contribution to the parity violating weak Hamiltonian is purely CP-violating in the limit of $s_3 \rightarrow 0$.¹⁷ We, then, find (denoting by a subscript I, the isospin projection of an operator)

$$\frac{\text{Im}A_0}{\text{Re}A_0} = \eta \sin(\beta_1 - \beta_2) + s_2 s_3 s_\delta \ln \left(\frac{m_c^2}{m_t^2} \right) \frac{\langle O^5 \rangle}{\langle P_{1/2} \rangle} + \zeta \frac{\langle O_{LR,1/2} \rangle}{\langle P_{1/2} \rangle} \quad (19)$$

and

$$\frac{\text{Im}A_2}{\text{Re}A_2} = \eta \sin(\beta_1 - \beta_2) + \zeta \frac{\langle O_{LR,3/2} \rangle}{\langle P_{3/2} \rangle} \quad (20)$$

This implies that

$$\epsilon' \approx \frac{\omega}{\sqrt{2}} \left(s_2 s_3 s_\delta \frac{\alpha_s(\mu)}{24\pi} \ln \frac{m_c^2}{m_t^2} \times - \zeta (y_{1/2} - y_{3/2}) \right) \quad (21)$$

where $\times = \langle O^5 \rangle / \langle P_{1/2} \rangle$; $y_I = \langle \text{Re}O_{LR,I} \rangle / \langle P_{1/2} \rangle$. The first term is identifiable as the contribution of the KM model whereas the second term is the left-right contribution. Existence of the unknown phase $(\gamma + \sigma_q)$ prevents us from giving an estimate of the L-R term. However, since $H'_{LR}{}^{P.V.}$ is purely CP-violating, dominant contribution to the $\Delta I = 3/2$ matrix element comes from the W_L -exchange graphs and thus is a product of left-left currents. On the other hand, $O_{LR,3/2}$ involves left-right operators, whose matrix elements are enhanced (at least in the vacuum saturation approximation¹⁸). As a result, we expect¹⁹ $y_{3/2} \approx 10 \sin(\gamma + \sigma_a)$ where $y_{1/2} \approx \sin(\gamma + \sigma_a)$ i.e. $y_{3/2} \gg y_{1/2}$. Nevertheless, a small ϵ'/ϵ can be easily accommodated by adjusting $\gamma + \sigma_q$.

In conclusion, we note that if the ratio $B(b \rightarrow u)/\bar{B}(b \rightarrow c)$ decay becomes smaller 1% an appealing framework for the description of CP-violation is provided by left-right symmetric model.

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Footnotes and References

1. R.N. Mohapatra, Phys. Rev. D6, 2023 (1972).
2. M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).
3. R.N. Mohapatra and J.C. Pati, Phys. Rev. D11, 566 (1975).
4. N.S. Lockyer et. al., Phys. Rev. Lett. 51, 1316 (1983); E. Fernandez et. al., Phys. Rev. Lett. 51, 1022 (1983).
5. See excellent recent reviews by: L.L. Chau, Phys. Rep. 95C, 1 (1983); A.J. Buras, Max Planck Preprint (1984); B. Holstein, Univ. of Mass. Preprint (1984); L. Wolfenstein, CERN Preprint (1984).
6. F. Gilman and M. Wise, Phys. Lett. 83B, 83 (1979); Phys. Rev. D20, 2392 (1979); B. Guberina and R.D. Peccei, Nucl. Phys. B163, 289 (1980); F. Gilman and J. Hagelin, Phys. Lett. 126B, 111 (1983).
7. G. Beall and A. Soni, Phys. Rev. Lett. 47, 552 (1981).
8. J.C. Pati and A. Salam, Phys. Rev. D10, 275 (1974); R.N. Mohapatra and J.C. Pati, Phys. Rev. D11, 566, 2558 (1975); G. Senjanović and R.N. Mohapatra, Phys. Rev. D12, 1502 (1975).
9. For a recent review see, R.N. Mohapatra, Lectures at the NATO Advanced Summer Institute on Particle Physics, München (1983).
10. R.N. Mohapatra, F.E. Paige and D.P. Sidhu, Phys. Rev. D17, 2642 (1978).
11. For the minimal Higgs model, left-right symmetry implies two kinds of restrictions on the quark mass matrices: (i) M_q

$= M_q^T$; (ii) $M_q = M_q^+$. In case (ii), $U_{KM} = V$ and this has been called manifest left-right symmetry in the literature: M.A.B. Bég, R. Budny, R.N. Mohapatra and A. Sirlin, Phys. Rev. Lett. 38, 1252 (1977).

12. We emphasize that our model is more general than the specific 3-generation extensions of the model of ref. 3 already existing in the literature: G. Branco, J.M. Frere, and J.M. Gerard, Nucl. Phys. B221, 317 (1983); D. Chang, Nucl. Phys. B214, 435 (1983).
13. M.A.B. Bég et. al. ref. 11; I.I. Bigi and J.M. Frere, Phys. Lett. 110B, 255 (1982); J. Donoghue and B. Holstein, Phys. Lett. 113B, 383 (1982).
14. G. Beall, M. Bender and A. Soni, Phys. Rev. Lett. 48, 848 (1982).
15. I.I. Bigi and J.M. Frere, Univ. of Michigan Preprint (1983).
16. G. Ecker, W. Grimus and H. Neufeld, Nucl Phys. B (1983).
17. The parity conserving part of this Hamiltonian is therefore CP-conserving; therefore $K_S \rightarrow 3\pi$ decay received no CP-violating contribution from left-right mixing. This means that for $s_3 \rightarrow 0$, $K_S \rightarrow 3\pi$ decay satisfies the relation $\eta_{+-0} = \eta_{000}$ and $\eta_{+-} - \eta_{+-0} \approx \epsilon'$.
18. M. Shifman, A. Vainshtein and V. Zakharov, Nucl. Phys. B120, 315 (1977).
19. D. Chang, ref. 12.