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B_d^0 TRANSITIONS IN $N = 1$ SUPERGRAVITY

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ABSTRACT

We study experimental tests for current $N = 1$ supergravity models in the $B^0 - \bar{B}^0$ system, stemming from flavour-changing couplings between gluinos, quarks and squarks. For light gluinos the $B_d^0 - \bar{B}_d^0$ mixing is large, contrary to the $SU(3)_c \times SU(2) \times U(1)$ prediction. Present and future colliders could observe a rate for $\mu^+\mu^+/\mu^+\mu^-$ as high as 25% for certain mass ranges where $m_{\tilde{b},\tilde{s}} \lesssim 85$ GeV and $m_{\tilde{g}} < 25$ GeV, and smaller but observable effects when relaxing these bounds. Besides, for very light $m_{\tilde{g}}$ and $m_{\tilde{b},\tilde{s}} \lesssim 85$ GeV, the super-penguin contribution enhances channels such as $B_d^0 \rightarrow \Phi K_s$, reaching a rate $R \equiv \text{Br}(B_d^0 \rightarrow \Phi K_s)/\text{Br}(B_d^0 \rightarrow \Psi K_s) > 1$, in opposition to the $SU(3)_c \times SU(2) \times U(1)$ value $10^{-2} < R < 10^{-1}$ due to the ordinary penguin mechanism. The bound on $m_{\tilde{q}}^4/m_{\tilde{g}}$ recently claimed by Goldberg is discussed.

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The coupling $\tilde{g}$ - q - $\tilde{q}$ (gluino-quark-squark) presents a non-diagonal component in current $N = 1$ supergravity (Sugra) models. As a consequence, $\tilde{g}$ exchange plays a competitive or even dominant role in certain traditional weak interaction processes. Such couplings are directly proportional to the square of quark masses times Kobayashi-Maskawa angles, which means that when the top quark mass is involved, the situation becomes particularly interesting. The implications for the ε parameter in the K^0 - $\bar{K}^0$ system have been pointed out by several authors^{1),2)}. Similarly, Gérard et al.³⁾ have explored the enhanced mixing of the B^0 - $\bar{B}^0$ system through this mechanism, both in the real and imaginary parts of the matrix elements, but restricted to gluino masses of ~ 40 GeV. These nice mixing effects are reconsidered here for a large sector of masses. The reason is that while some interpretations of recent CERN events⁴⁾ favour $m_{\tilde{g}} > 40$ GeV or $m_{\tilde{g}} < 3-10$ GeV, others⁵⁾ favour light gluinos which may even be in between these bounds⁶⁾. Our study allows a comparison of the short-distance box diagram effects with recent studies by Goldberg⁷⁾ on the contribution of intermediate $\tilde{g}\tilde{g}$ bound states; this discussion will be developed at the end of the paper. Finally, we point out that the effect of such couplings is also neat in some decay channels such as $B \rightarrow \Phi K_S$: amusingly enough, these channels happen to be also ideal for studying effects related to the $\Delta I = \frac{1}{2}$ rule in kaon decays⁸⁾, as they can only occur through the penguin mechanism in the standard model. We try to present the results in the form of rates of quantities independent of the values of $\sin\delta$, τ_b , etc., as long as possible.

Following the notation of Ref. 2) as well as their choice of Sugra parameters, the mixing between left-handed squarks of different flavours i, j and charge $-1/3$ is given by $(K^+(M_U^2/M_W^2)K)a$, where K is the Kobayashi-Maskawa matrix and M_U corresponds to m_u, m_c, m_t . This translates into a left-handed $q_L^i - \tilde{q}_L^j - \tilde{g}$ coupling of the form $i g_s \sqrt{2} (K^+ M_U^2 K / M_W^2)_{ij} ((\tilde{m}_i^2 + \tilde{m}_j^2) / 2(\tilde{m}_i^2 - \tilde{m}_j^2))$. It has been assumed here that the parameter c which often signals the left-handed squark mixing in the mass matrix is $c = (m_{3/2}/m_W)^2 a$, where a is independent of the gravitino mass $m_{3/2}$. In terms of the standard A of minimal $N = 1$ Sugra, $a = (3 + |A|^2)/7 = 1$ for $|A| = 2$. Smaller values of c , recently proposed, will be discussed later.

It is clear that while the contribution of the SUSY box diagram (Fig. 1) does not dominate the standard $SU(2) \times U(1)_{em}$ result for the real part of the K^0 - $\bar{K}^0$ mass matrix, it becomes competitive when considering its imaginary part. Thus, ε_m could in principle be saturated by supersymmetry (SUSY). Nevertheless, ε_m is proportional to $\sin\delta$ and the actual value of δ is unknown, so that there is

not necessarily a problem with respect to the experimental value of ϵ . Further attempts to set bounds on this SUSY contribution from other inputs, such as τ_B , seem to us at least hazardous, given the uncertainties involved, and will not be considered here.

We find for the ratio of the imaginary part of the $\tilde{g}$ box (Fig. 1) to the imaginary part of the standard one, in the short-distance approximation and neglecting crossed diagrams which are suppressed by a factor m_g^2/m_q^2 :

$$\frac{\text{Im} \langle k^0 | H_{\text{eff}} | \bar{k}^0 \rangle_{\text{SUSY-}\tilde{g} \text{ BOX}}}{\text{Im} \langle k^0 | H_{\text{eff}} | \bar{k}^0 \rangle_{\text{STANDARD}}} =$$

$$= \frac{22}{27} \frac{\alpha_s^2(m_{\tilde{g}}^2)}{\alpha_s^2} \frac{m_c^2}{m_s^2} f'\left(\frac{m_{\tilde{g}}^2}{m_s^2}\right) \frac{[(m_t^2 - m_c^2) + (s_2^2 + s_2 s_3 c_3) m_t^4/m_c^2]}{[m_c^2 (\ln \frac{m_t^2}{m_c^2} - 1) + (s_2^2 + s_2 s_3 c_3) m_t^2]} \quad (1)$$

where we set $c_1 = c_2 = c_3 = 1$, $m_u = 0$; $f'(z)$ is the function:

$$f'(z) = 2 \int_0^1 dx (x^3(1-x)/(x+z(1-x))^3)$$

which takes the value 1 when $m_{\tilde{g}} \rightarrow 0$, and 1/10 when $m_{\tilde{g}} \approx m_q$ [this function is three times larger than the function f of Ref. 2)]. Remark that the ratio of the angle factors is of the order of 1.

In the $B^0 - \bar{B}^0$ system, both the real and imaginary SUSY matrix elements are important because they are approximately proportional to m_t^2 :

$$\frac{(m_{B_{dL}} - m_{B_{dS}})_{\text{SUSY-}\tilde{g} \text{ BOX}}}{(m_{B_{dL}} - m_{B_{dS}})_{\text{STANDARD}}} \approx \frac{22}{27} \frac{\alpha_s^2(m_{\tilde{g}}^2)}{\alpha_s^2} f'\left(\frac{m_{\tilde{g}}^2}{m_s^2}\right) \cdot \frac{m_t^2}{m_s^2} \quad (2)$$

and

$$\frac{\text{Im} \langle B_d^0 | H_{\text{eff}} | \bar{B}_d^0 \rangle_{\text{SUSY-}\tilde{g} \text{ BOX}}}{\text{Im} \langle B_d^0 | H_{\text{eff}} | \bar{B}_d^0 \rangle_{\text{STANDARD}}} \simeq \frac{22}{27} \frac{\alpha_s^2(m_{\tilde{g}}^2)}{\alpha_z^2} f' \left(\frac{m_{\tilde{g}}^2}{m_S^2} \right) \frac{m_t^2}{m_S^2} \quad (3)$$

The number of same-sign dileptons from $B^0 - \bar{B}^0$ production is given through the ratio $(m_{B_L} - m_{B_S})/\Gamma_B$ which measures the importance of oscillations.

We will not consider in detail $B_s^0 - \bar{B}_s^0$, as the mixing is already very important in the standard model [even $\gtrsim 25\%$ ⁹⁾], and we could not get a distinctive SUSY signal out of it. This is not the case for the B^0 system. Let us consider the parameter

$$x_{\text{SUSY-}\tilde{g} \text{ BOX}}(B_d) \equiv \frac{(\Delta M)_{B_d \text{ SUSY-}\tilde{g} \text{ BOX}}}{(\Delta M)_{B_d \text{ STANDARD}}} \cdot \frac{(\Delta M)_{B_d \text{ STANDARD}}}{\Gamma_B}$$

being conservative we will take for $x_{\text{STANDARD}} \equiv ((\Delta M)_{B_d \text{ STANDARD}}/\Gamma_{B \text{ exp}})$ the values proposed by Buras et al¹⁰⁾:

$$\frac{x_{\text{STANDARD}}}{B} = \begin{cases} \leq 0.2 \left(\frac{45}{35} \right)^{1.2} \frac{22}{27} f' \left(\frac{m_{\tilde{g}}^2}{m_S^2} \right) \frac{\alpha_s^2(m_{\tilde{g}}^2)}{\alpha_z^2} \cdot \frac{m_t^2}{m_S^2} & \text{for } \delta \text{ IN } 2^{\text{ND}} \text{ QUADRANT} \\ \geq 0.02 \left(\frac{45}{35} \right)^{1.2} \frac{22}{27} f' \left(\frac{m_{\tilde{g}}^2}{m_S^2} \right) \frac{\alpha_s^2(m_{\tilde{g}}^2)}{\alpha_z^2} \cdot \frac{m_t^2}{m_S^2} & \text{for } \delta \text{ IN } 1^{\text{ST}} \text{ QUADRANT} \end{cases} \quad (4)$$

B is the parameter with value 1 when the vacuum insertion approximation is used in the estimate of the $K^0 - \bar{K}^0$ matrix elements. In consequence, observable oscillations appear for $m_{\tilde{g}}/m_{\tilde{q}} < 1$ as the function $f'(m_{\tilde{g}}^2/m_{\tilde{q}}^2) \rightarrow 1$ when $m_{\tilde{g}}^2 \ll m_{\tilde{q}}^2$. The results are presented in Fig. 2 for $B = 1$ and $m_t = 45$ GeV. The "pregnancy" curves (thus named for evident reasons) indicate the region where the SUSY gluino

box dominates the standard one as well as the amount of same-sign dileptons to be expected. For δ in the second quadrant, 25% of the total number of dileptonic events should have equal sign for a region bounded by $m_{\tilde{q}} < 85$ GeV, $m_{\tilde{g}} < 25$ GeV (although in the extreme case $B = 1/3$, the 25% is reduced to 5%); the prediction comes down to a still observable 10% for $m_{\tilde{q}} < 120$ GeV, $m_{\tilde{g}} < 40$ GeV. For δ in the first quadrant, 25% of the total dilepton events should have equal sign for a region bounded by $m_{\tilde{q}} < 30$ GeV, $m_{\tilde{g}} < 10$ GeV; the prediction comes down to 10% for $m_{\tilde{q}} < 40$ GeV and $m_{\tilde{g}} < 13$ GeV.

Let us now consider B decays. Recently, we have pointed out⁸⁾ that unlike the case of kaon decays, there are exclusive modes in $\Delta S = 1$ B decays (such as $B_d^0 \rightarrow \Phi K_s$, $K^- \pi^+$), not suppressed by Kobayashi-Maskawa mixing angles, whereas in the standard $SU(3)_c \times SU(2) \times U(1)_{em}$ model only penguin diagrams contribute. We had found that in the standard model $Br(B_d \rightarrow \Phi K_s^0)/Br(B_d \rightarrow \Psi K_s) \approx 3 \times 10^{-2}$. The class of Sugra models considered here provides new contributions to these modes which necessarily involve loops of heavy particles. They are depicted in Fig. 3. Gluino superpenguins have been previously considered for the kaon system, but they are negligible in front of the standard penguin¹¹⁾. For B decays, the dependence on m_t^2 is once more determinant. Diagram 3a gives an order of magnitude estimate

$$\frac{\langle B_d^0 | H | \Psi K_s \rangle_{\text{SUSY-}\tilde{g} \text{ ONE LOOP}}}{\langle B_d^0 | H | \Psi K_s \rangle_{\text{STANDARD}}} \approx \frac{3}{2} \frac{\alpha_s(m_B^2)}{\alpha_2} \frac{m_t^2}{m_{\tilde{b}}^2} \frac{\ln(m_B^2/m_{\tilde{g}}^2)}{\ln(m_t^2/m_c^2)} \cdot P\left(\frac{m_{\tilde{g}}^2}{m_{\tilde{b}}^2}\right) \approx$$

$$\approx 6 \frac{m_t^2}{m_{\tilde{b}}^2} \cdot \frac{\ln(m_B^2/m_{\tilde{g}}^2)}{\ln(m_t^2/m_c^2)} \cdot P\left(\frac{m_{\tilde{g}}^2}{m_{\tilde{b}}^2}\right) \quad (5)$$

for the left-handed contribution. $P(z)$ is a function

$$P(z) = \frac{6}{\ln(1/z)} \int_0^1 dx \int_0^x dy \frac{y(x-y)(1-x+y/2)}{[y+z(1-y)]^2}$$

which takes the value 1 when $m_{\tilde{g}} \rightarrow 0$ and $1/[10 \ln(1/z)]$ when $m_{\tilde{g}} \approx m_{\tilde{b}}$.

In consequence, for $m_{\tilde{b}} < 80$ GeV and light gluinos (< 10 GeV), the supersymmetric penguin dominates the standard one, and this decay channel is reinforced. For instance, for $m_{\tilde{b}} = m_t$ and light gluinos ($m_{\tilde{g}} < 3$ GeV),

$$BR(B_d^0 \rightarrow \psi K_S)_{\text{SUSY}} \gtrsim 10 BR(B_d^0 \rightarrow \psi K_S)_{\text{STANDARD}}$$

and may even reach

$$\left\{ BR(B_d^0 \rightarrow \Psi K_S) / BR(B_d^0 \rightarrow \Psi K_S) \right\}_{\text{SUSY}} \geq 1$$

in opposition to the standard $SU(3)_c \times SU(2) \times U(1)$ prediction⁸⁾. Diagrams of the type 3c are of the order

$$\frac{| \langle B_d^0 | H | \psi K_S \rangle |_{\text{SUSY-}\tilde{g} \text{ BOX}}}{| \langle B_d^0 | H | \psi K_S \rangle |_{\text{STANDARD}}} \approx \frac{11}{16} \cdot \frac{\alpha_s^2(m_{\tilde{g}}^2)}{\alpha_2 \alpha_3(\mu^2)} \cdot \frac{m_t^2}{m_{\tilde{b}}^2} \cdot \frac{C(\frac{m_{\tilde{g}}^2}{m_{\tilde{b}}^2})}{\ln(m_t^2/\mu^2)} \cdot \left\{ \frac{2}{1 + \frac{\langle B_d^0 | (\bar{s} \gamma_\mu \frac{\lambda^c}{2} L b)(\bar{s} \gamma_\mu \frac{\lambda^c}{2} R s) | \psi K_S \rangle}{\langle B_d^0 | (\bar{s} \gamma_\mu \frac{\lambda^c}{2} L b)(\bar{s} \gamma_\mu \frac{\lambda^c}{2} L s) | \psi K_S \rangle}} \right\} \quad (6)$$

where $\mu \approx m_c$ ⁸⁾, and $C(Z) \equiv 2 \int_0^1 dx (x^2(1-x)/[x(1-z)+z]^2)$ with value 1 for $m_{\tilde{g}} \rightarrow 0$, and 1/6 for $m_{\tilde{g}} = m_{\tilde{q}}$.

Diagrams of the type 3c contribute less than those of 3b and may be neglected. More detailed calculations of these and other decay channels relevant as possible experimental tests for $N = 1$ supergravity models are under way¹²⁾.

A renormalization group improved estimate¹³⁾ of a flavour-changing gluino coupling leads to a smaller value of c (a factor of the order of 2 for light gluinos; notice nevertheless that some freedom is left in the choice of the parameter A). The masses in Fig. 2 scale as c . The dotted curve in Fig. 4 is independent of c , since it compares two SUSY contributions.

$\tilde{g}\tilde{g}$ bound states contribution

Goldberg⁷⁾ has claimed that in the minimal $N = 1$ supergravity models, virtual gluino-gluino 0^{-+} bound states make potentially very large contributions to the $K^0 - \bar{K}^0$ mass matrix. He claims that for $m_{3/2} < 110$ GeV and $m_{\tilde{g}} > 40$ GeV, they dominate in magnitude those of the supersymmetric two-gluino box diagram. We

have plotted both results in Fig. 4. Goldberg considers his result safe for $m_{\tilde{g}} < 5 m_{\tilde{q}}$. This seems to us at best optimistic, and we are thoroughly studying this point¹²⁾. Should he be right, supersymmetry would dominate under the dotted and dashed curves of Fig. 4. In such a region, SUSY would saturate ϵ_m and there would be $B_d^0 - \bar{B}_d^0$ oscillations. In the subregion where the bound-state effects would dominate, the sign of Δm_d and the sign of $\sin\delta$ would be opposite to the standard expectations, but there is no a priori experimental constraint on such a sign. Only very close to the dashed line and for $m_{\tilde{g}} > 60$ GeV would the cancellation between the standard box and the Goldberg mechanism prevent the fitting of ϵ_{exp} . Anyhow, the relation between the SUSY $\tilde{g}$ box estimates and those with $\tilde{g}\tilde{g}$ bound states raises questions connected with duality [presently under study¹²⁾]. We do believe that the local box approximation is reasonable as long as the masses involved are large compared to external momenta: $m_{\tilde{q}} \gg 5$ GeV and $m_{\tilde{g}} \gg 5$ GeV. In the small $m_{\tilde{g}}$ region caution is advisable, and long-distance effects à la Goldberg should be looked at in detail.

Our conclusion is that such an easy experimental probe of SUSY models as looking for same-sign dileptons above $B_d^0, \bar{B}_d^0$ thresholds in e^+e^- collisions is most promising. Their eventual detection simultaneously with the measurement of an enhanced $B_d^0 \rightarrow \Phi K_s$ decay rate could be a test for $N = 1$ supergravity models with not-too-heavy gluinos. Conversely, their absence would set limits on the combined values of $m_{\tilde{q}}$ and $m_{\tilde{g}}$ which do not depend heavily upon subtle parameter fittings.

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Note Added:

In any case, the pseudoscalar contribution to $i \int d^4x e^{ikx} \langle 0 | T^* (J_\mu(x) J_\nu(0)) | 0 \rangle$ in Goldberg's notation⁷⁾ should give a contribution proportional to $k_\mu k_\nu$ [see, for example, formulae (4.1) and (4.5) in Ref. 14)] and not to $g_{\mu\nu}$, as stated in Ref. 7). It should vanish when $k_\mu \rightarrow 0$ ¹²⁾. This lowers dramatically the contribution of the intermediate $\tilde{g}\tilde{g}$ bound states, since only the axial ones give a non-zero result and these, according to Goldberg himself, are negligible. Thus, bound state effects are smaller than the box diagram contribution, when external momenta are neglected.

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FIGURE CAPTIONS

- Fig. 1 : Left-handed box diagram contributing to $K^0-\bar{K}^0$ ($B_d^0-\bar{B}_d^0$) transition, due to flavour mixing $\tilde{g}$ couplings.
- Fig. 2 : Pregnancy curves.
a) $B_d^0-\bar{B}_d^0$ analysis - the full curve separates two areas: inside the curve the SUSY gluino box dominates the standard box; the reverse is true outside. The area between the two dashed curves corresponds to 10% of the total number of dileptons being of the same sign for all possible values of $\sin\delta$. The area between the dotted curves corresponds analogously to 25% of same-sign dileptons. $B = 1$ is assumed here, except for the plain curve.
b) $B_d^0 \rightarrow \Phi K_s$ analysis - the full (dashed) curve corresponds to the gluino superpenguin (Fig. 3a) being equal to 1(10) times the standard penguin.
- Fig. 3 : Diagrams contributing to $B_d^0 \rightarrow \Phi K_s$ decay through flavour non-diagonal gluino exchange.
- Fig. 4 : $\text{Im}\langle K^0 | H | \bar{K}^0 \rangle$ analysis - inside the full curve the SUSY $\tilde{g}$ box dominates the standard box; the reverse is true outside. Below the dashed curve, Goldberg's estimate dominates the standard box; the reverse is true above. The Goldberg estimate dominates the SUSY gluino box to the right of the dotted curve.

Fig. 1

Fig. 2a

Fig. 2b

Fig. 3

Fig. 4