CERN-TH.4113/85

COMPOSITE HIGGS FIELDS FROM A σ -MODEL FOR LEPTONS

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A B S T R A C T

We study a supersymmetric σ -model which contains the leptons of one family and an additional singlet as Goldstone superfields. Supersymmetry is softly broken through a scalar mass Δ . Quantum effects generate two composite chiral superfields with the quantum numbers of two Higgs doublets. Their vacuum expectation values break the local $SU(2)_W \times U(1)_Y$ symmetry to $U(1)_{EM}$. The mass spectrum of the theory is calculated as a function of Δ and the σ -model scale f .

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February 1985

The standard model of electroweak interactions¹⁾ provokes the question of the origin of families and fermion masses. One approach which addresses this question assumes that quarks and leptons have a further substructure. If quarks and leptons are composite, their size must be very small compared to their Compton wavelength. The only known bound states of this kind are the pions, and it is therefore suggestive to suppose that quarks and leptons belong to the "Goldstone sector" of the underlying preon theory.

It is indeed possible to construct supersymmetric models where quarks and leptons are identified as quasi-Goldstone fermions, i.e., as fermionic partners of Goldstone bosons²⁾. At energies below the substructure scale the interactions of the Goldstone superfields are described by supersymmetric σ -models which can be constructed once the pattern of spontaneous symmetry breaking has been specified. Non-renormalizable effective Lagrangians for composite particles have often been used to obtain further bound states of the underlying theory: pions and ρ -mesons have been obtained from nucleons³⁾; nucleons^{4),5)} and ρ -mesons⁶⁾ have been constructed from pions.

In this paper we consider a gauged supersymmetric σ -model whose Goldstone superfields contain the leptons of one family and an additional gauge singlet. Supersymmetry is explicitly softly broken through a scalar mass Δ . In spite of the non-renormalizability of the interaction we assume that it makes sense to compute quantum corrections using a cut-off of the order of the scale of the σ -model. These quantum effects generate as bound states two Higgs superfields, H_1 and H_2 , and the local $SU(2)_W \times U(1)_Y$ symmetry is spontaneously broken to $U(1)_{EM}$. The mass spectrum is determined in terms of the σ -model scale f and the supersymmetry breaking scale Δ . Our treatment of bound states is closely related to previous work⁷⁾ on the supersymmetric Nambu-Jona Lasinio (SNJL) model⁸⁾. In this letter we concentrate on the main features of our model. A more detailed discussion will be presented elsewhere⁹⁾.

Our starting point is the coset space $U(3) \times U(2) / U(2) \times U(1)^2$. The Grassmann manifolds $U(3) / SU(2) \times U(1)_a \times U(1)_b$ and $U(2) / U(1)_c \times U(1)_d$ yield the Goldstone chiral superfields L^i ($i=1,2$) and E_c , which correspond to a left-handed doublet and the charge conjugate of a right-handed singlet of one family of leptons. A connection between the left- and right-handed sectors is obtained by breaking $U(1)_b \times U(1)_d$ to the diagonal subgroup. This gives rise to a superfield S which contains one Goldstone and one quasi-Goldstone boson. The unbroken subgroup can be identified with the electroweak $SU(2)_W \times U(1)_Y$ symmetry and the global $U(1)_L$ of lepton number.

The generators of hypercharge (Q_Y) and lepton number (Q_L) read:

$$\begin{aligned} Q_Y &= -\frac{1}{2} Q_a + Q_c, \\ Q_L &= Q_a - Q_c. \end{aligned} \quad (1)$$

$(Q_b - Q_d)$ is the generator of the unbroken diagonal $U(1)$ subgroup and $(Q_a - Q_c + Q_b - Q_d)$ acts trivial on all fields. Our toy model for leptons described above is similar to technicolour-type preon models based on $U(6) \times U(6) / U(4) \times U(4) \times SU(2)_D^{10), 11)}$, where the breaking of the two $U(6)$ factors to $U(4) \times SU(2)$ generates quarks and leptons, whereas the breaking of $SU(2) \times SU(2)$ to the diagonal subgroup, which connects the left- and right-handed sectors, yields the technipions.

The most general Kähler potential⁹⁾ for the Goldstone superfields L^i , E_c , and S is an arbitrary function $F(K_0)$, with

$$K_0 = f^2 + f(S + \bar{S}) + \bar{L}_i L^i + \bar{E}_c E_c + \bar{S} S + \frac{1}{f^2} \bar{L}_i L^i \bar{E}_c E_c \left[\left(1 + \frac{1}{f} S \right) \left(1 + \frac{1}{f} \bar{S} \right) \right]^{-1}. \quad (2)$$

F is only constrained by the requirement to yield correctly-normalized kinetic terms for the Goldstone superfields. The appearance of an arbitrary function in the Kähler potential is due to the breaking of the $U(1)$ factor and the related presence of one quasi-Goldstone boson. Different examples^{12), 13)} of Kähler manifolds of this type have previously been discussed in the literature. If the $U(1)$ subgroup were unbroken the Kähler potential for L^i and E_c would be uniquely determined¹⁴⁾:

$$K_0' = f^2 \ln \left(1 + \frac{1}{f^2} \bar{L}_i L^i \right) + f^2 \ln \left(1 + \frac{1}{f^2} \bar{E}_c E_c \right). \quad (3)$$

We choose, for simplicity, the function F to be the identity. Gauging the unbroken $SU(2)_W \times U(1)_Y$ subgroup yields the action^{*})

$$\begin{aligned} I_{\sigma} &= \int d^4x d^4\theta \left\{ \left[\bar{L}_i (e^{2gW - g'B})^i_j L^j + \bar{E}_c e^{2g'B} E_c \right] (1 - \delta) + \bar{S} S \right. \\ &\quad \left. + \frac{1}{f^2} \bar{L}_i \bar{E}_c (e^{2gW + g'B})^i_j L^j E_c \left[\left(1 + \frac{1}{f} S \right) \left(1 + \frac{1}{f} \bar{S} \right) \right]^{-1} \right\}, \\ \delta &= \Delta^2 \theta \theta \bar{\theta} \bar{\theta} \quad ; \end{aligned} \quad (4)$$

$W = \frac{1}{2} \tau^{IJ} W^{IJ}$, B , g and g' are the electroweak vector superfields and coupling

^{*}) We use the conventions of Wess and Bagger¹⁵⁾.

constants^{*}), Δ is a softly supersymmetry breaking scalar mass for fields with non-vanishing $SU(2)_W \times U(1)_Y$ quantum numbers. The action (4) is similar to the SNJL model and one therefore expects the occurrence of bound states, which are most conveniently described by introducing auxiliary superfields⁷⁾. The action (4) is equivalent to

$$\begin{aligned} I_4 = \int d^4x \{ & \int d^4\theta ([\bar{L}_i (e^{2gW - g'B})^i_j L^j + \bar{E}_c e^{2g'B} E_c] (1-\delta) + \bar{S} S \\ & + \bar{H}_{1i} (e^{2gW + g'B})^i_j H_1^j) \\ & + \int d^2\theta [f H_1^i H_{2i} + H_1^i H_{2i} S + L^i H_{2i} E_c] + c.c. \} , \end{aligned} \quad (5)$$

where the doublet H_2 acts as a Lagrange multiplier field whose elimination turns (5) into (4). The interaction term in (4) corresponds to the kinetic term of H_1 in (5). The equations of motion for H_2 and H_1 yield

$$H_1^i = -\frac{1}{f} (1 + \frac{1}{f} S)^{-1} L^i E_c , \quad (6a)$$

$$H_{2i} = \frac{1}{f^2} (1 + \frac{1}{f} S)^{-1} (-\frac{1}{4}) \bar{D}^2 [(1 + \frac{1}{f} \bar{S})^{-1} \bar{L}_j \bar{E}_c (e^{2gW + g'B})^j_i] . \quad (6b)$$

Let us now study the one-loop corrections to (5) which involve the chiral superfields L^i and E_c . Loops with H_1 and S are not taken into account because in the action (5) H_1 does not correspond to an independent degree of freedom [cf. (6a)]. The graphs of Fig. 1 yield a negative mass squared for H_2 ,

$$V_{1q} = -2Z \int d^4\theta \bar{H}_2^i H_{2i} \delta , \quad (7)$$

with

$$Z = \frac{1}{(6\pi)^2} \left(\ln \frac{f^2}{m^2} + O(1) \right) , \quad (8)$$

where the logarithmically divergent integral for Z has been cut off at the σ -model scale f . The mass m is equal to the vacuum expectation value of H_2 which has to be determined from a minimization of the complete scalar potential. Graphs of the type shown in Fig. 2 yield a gauge invariant kinetic term for H_2 ⁹⁾,

^{*}) Without quarks the $SU(2)_W \times U(1)_Y$ gauge interactions are anomalous. These anomalies are, however, irrelevant for the dynamical mechanism which is the subject of this paper.

$$\mathcal{L}_{1l} = \int d^4\theta \bar{H}_i (e^{-2gW - g'B})^i_j H_{lj} \quad , \quad (9)$$

where terms of relative order $1/\ln(f^2/m^2)$ have been neglected.

From Eqs. (5), (7) and (9) one obtains for the complete action including the leading one-loop corrections:

$$\begin{aligned} I = \int d^4x \{ & \int d^4\theta [\bar{L}_i (e^{2gW - g'B})^i_j L^j + \bar{E}_c e^{2g'B} E_c] (1-\delta) + \bar{S} S \\ & + \bar{H}_{li} (e^{2gW + g'B})^i_j H_l^j + \int d^4\theta \bar{H}_i (e^{-2gW - g'B})^i_j H_{lj} (1+2\delta) \\ & + \int d^4\theta [f H_i^j H_{lj} + H_i^j H_{lj} S + L^i H_{li} E_c] + \text{c.c.} \} \quad . \end{aligned} \quad (10)$$

The quantum effects have generated a kinetic term for H_2 . As a consequence, H_1 and H_2 correspond to physical degrees of freedom; they are bound states whose "wave functions" are given by Eqs. (6). H_1 and H_2 have the quantum numbers of Higgs fields and interact with the leptons through a superpotential familiar from Fayet's model¹⁶⁾ for $SU(2)_W \times U(1)_Y$ breaking. The appearance of a superpotential and composite chiral superfields is a non-trivial consequence of the structure of the Kähler potential (2).

In order to obtain the action (10) we had to anticipate the existence of a non-zero v.e.v. for H_2 which rendered the fermionic one-loop integrals of Figs. 1 and 2 infra-red finite. All vacuum expectation values can be computed from (10) in terms of the two parameters f and Δ . The scalar potential for the fields S , H_1 and H_2 reads^{*)}

$$\begin{aligned} V = & \frac{1}{8} g^2 (\tilde{H}_1^\dagger \tilde{H}_1 - \tilde{Z} \tilde{H}_2^\dagger \tilde{H}_2^*)^2 + \frac{1}{8} g^2 (\tilde{H}_1^\dagger \tilde{H}_1 - \tilde{Z} \tilde{H}_2^\dagger \tilde{H}_2^*)^2 \\ & + \frac{1}{2} (\tilde{H}_1^\dagger \tilde{H}_1 + \tilde{Z} \tilde{H}_2^\dagger \tilde{H}_2^*) (f + \tilde{S}) (f + \tilde{S}^*) + (\tilde{H}_1^\dagger \tilde{H}_2^*) (\tilde{H}_2^\dagger \tilde{H}_1) \\ & - 2\Delta^2 \tilde{Z} \tilde{H}_2^\dagger \tilde{H}_2^* \quad . \end{aligned} \quad (11)$$

*) $\tilde{A}$, a and $\hat{A}$ denote scalar, fermionic and auxiliary components of the chiral superfield A .

A special role is played by the singlet S whose v.e.v. is given by

$$\langle \tilde{S} \rangle = -f \quad , \quad (12)$$

provided $\langle (\tilde{H}_1^\dagger \tilde{H}_1 + Z \tilde{H}_2^\dagger \tilde{H}_2) \rangle \neq 0$. S acts as a "sliding singlet"^{17),18)} whose v.e.v. cancels the large tree-level mass f for H_1 and H_2 . The presence of this "sliding singlet" is crucial in order to obtain a non-zero v.e.v. for H_2 . Without S the positive mass squared would always dominate the radiatively induced negative mass squared, unless one were to replace the cut-off f in Eq. (8) by $\Lambda \gg f$ [cf. Ref. 7)], such that

$$\frac{1}{8\pi^2} \Delta^2 \ln \frac{\Lambda^2}{\Delta^2} > f^2 \quad . \quad (13)$$

It appears difficult, however, to justify such a large cut-off in a σ -model which is valid only at distances larger than $1/f$. We note that a v.e.v. $\langle \tilde{S} \rangle = -f$ leads to singularities in Eqs. (2), (4) and (6). This can be avoided by introducing a small scalar mass for $\tilde{S}$ in (4), which will be discussed in more detail in Ref. 9).

The v.e.v.'s of H_1 and H_2 are easily determined from the scalar potential (11) and can be written as

$$\langle \tilde{H}_1 \rangle = \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \quad , \quad \sqrt{Z} \langle \tilde{H}_2 \rangle = \begin{pmatrix} 0 \\ v_2 \end{pmatrix} \quad , \quad (14)$$

i.e., they break $SU(2)_W \times U(1)_Y$ to $U(1)_{EM}$ as desired. The resulting values for v_1 and v_2 depend on Z as follows. For $1/4(g^2 + g'^2)Z > 1$ we find

$$\begin{aligned} v_1^2 &= \frac{2(\frac{1}{4}(g^2 + g'^2)Z - 1)Z\Delta^2}{(\frac{1}{2}(g^2 + g'^2)Z - 1)} \quad , \\ v_2^2 &= \frac{(g^2 + g'^2)Z^2\Delta^2}{2(\frac{1}{2}(g^2 + g'^2)Z - 1)} \quad , \end{aligned} \quad (15a)$$

whereas for $1/4(g^2 + g'^2)Z < 1$ we get

$$v_1^2 = 0 \quad , \quad v_2^2 = \frac{8\Delta^2}{g^2 + g'^2} \quad . \quad (15b)$$

In the second case the particle spectrum contains two massless charged fermions (one wino, one higgsino); hence we will present the spectrum for the more interesting first case.

The mass spectrum of scalars, fermions and vector bosons can be obtained in the standard manner from the action (10). With

$$\begin{aligned} \cos \theta_w &= \frac{g}{\sqrt{g^2 + g'^2}}, \quad \cos \varphi = \frac{V_2}{\sqrt{V_1^2 + V_2^2}}, \quad L = \begin{pmatrix} N \\ E \end{pmatrix}, \\ Z &= \cos \theta_w W^3 - \sin \theta_w B, \quad h = \cos \varphi h_1^0 + \sin \varphi \sqrt{2} h_2^0, \\ h^1 &= -\sin \varphi h_1^0 + \cos \varphi \sqrt{2} h_2^0, \end{aligned} \quad (16)$$

one obtains the result:

$$M_W^2 = \frac{1}{2} g^2 (V_1^2 + V_2^2), \quad M_Z^2 = \frac{1}{2} (g^2 + g'^2) (V_1^2 + V_2^2), \quad (17a)$$

$$m_{W^-, h_1^+} = \sqrt{2} \sin \varphi M_W, \quad m_{W^+, h_1^-} = \sqrt{2} \cos \varphi M_W, \quad (17b)$$

$$m_{h, S} = \sqrt{2} \Delta, \quad m_{h^1, \tilde{Z}} = M_Z, \quad (17c)$$

$$m_{e, e_c} = m = \sqrt{2} \Delta \cos \varphi, \quad m_\nu = 0, \quad (17d)$$

$$M_{\tilde{N}}^2 = \Delta^2 (1 + 2 \cos^2 \varphi), \quad M_{\tilde{E}}^2 = \Delta^2 (1 + 4 \cos^2 \varphi \sin^2 \theta_w), \quad (17e)$$

$$M_{\tilde{E}_c}^2 = \Delta^2 (1 + 2 \cos^2 \varphi (1 - 2 \sin^2 \theta_w)), \quad M_{\tilde{S}}^2 = 2 \Delta^2 \cos^2 \varphi, \quad (17f)$$

$$M_{\tilde{H}^+}^2 = M_W^2 - 2 \Delta^2, \quad M_{\tilde{H}_0}^2 = 0, \quad (17g)$$

$$M_{\tilde{H}_{1,2}}^2 = \Delta^2 \left[X \pm \left(\frac{X(X^2 - 5X + 8)}{X - 1} \right)^{\frac{1}{2}} \right], \quad X = \frac{1}{2} (g^2 + g'^2) Z = \frac{M_Z^2}{2 \Delta^2} \geq 2. \quad (17h)$$

$m_{a,b}$ denotes a Dirac mass for the Weyl fermions a and b ; $\tilde{H}^+$, $\tilde{H}_0$ and $\tilde{H}_{1,2}^0$ are the

charged and neutral scalars of the Higgs doublets H_1 and H_2 which are not absorbed by the gauge bosons.

In Eqs. (17) we have expressed the mass spectrum in terms of Δ and the v.e.v.'s v_1 and v_2 . The σ -model scale f enters only through the wave function normalization constant Z [cf. (8), (15a)]:

$$\ln \frac{f^2}{\Delta^2} = 8\pi^2 \frac{v_1^2 + v_2^2}{\Delta^2} = \frac{2\sqrt{2}\pi^2}{\Delta^2 G_F}, \quad (18)$$

where we have neglected terms of relative order $1/\ln(f^2/\Delta^2)$. From Eq. (18), it is clear that the SUSY breaking scale Δ has to be of the order of the Fermi scale $G_F^{-1/2}$, whereas the σ -model scale f is only restricted to be larger than Δ , which is required already by the self-consistency of our approach. The inequality $1/4(g^2 + g'^2)Z > 1$, i.e., $\Delta < \frac{1}{2}M_Z$, which we have used in order to obtain the v.e.v.'s (15a), leads, together with Eq. (8), to unrealistically large values for f . In a realistic model, however, we expect a modification of Eq. (8) because more leptons and quarks will contribute to the wave function renormalization of H_2 . From (17d), we deduce that the fermion mass m cannot be identified with any of the known lepton masses. It should rather be interpreted as the fermion which, in an extended model including quarks, couples most strongly to the composite Higgs fields.

Given the composite Higgs fields, the mechanism for $SU(2)_W \times U(1)_Y$ symmetry breaking discussed in this paper is similar to the one proposed by Ibáñez and Ross¹⁸⁾, where the large Yukawa coupling of the t -quark plays a crucial role. We emphasize, however, that in our model the superpotential is dynamically generated and that its parameters are calculable in terms of the σ -model scale f and the SUSY breaking scale Δ . This implies in particular that the fermion masses are also calculable!

An unconventional feature of our model is that the Higgs doublets are bound states of leptons and that the $SU(2)_W \times U(1)_Y$ symmetry is therefore broken through lepton condensates. If we consider the σ -model not as a fundamental Lagrangian but only as an effective Lagrangian for the interaction of composite leptons at distances larger than $1/f$, the lepton condensates correspond to higher dimensional preon condensates. Such multi-preon condensates, and in particular their possible relevance for fermion mass generation, are currently also being studied by Mizrachi and Peccei¹⁹⁾. Similar ideas have been pursued by Harari and Seiberg²⁰⁾ and Napoly²¹⁾.

We have presented a supersymmetric σ -model for leptons in which soft supersymmetry breaking induces the formation of composite Higgs fields whose vacuum expectation values break the chiral $SU(2)_W \times U(1)_Y$ symmetry. The entire mass spectrum, including fermion masses, can be calculated in terms of the σ -model scale f and the SUSY breaking scale Δ . The applicability of this mechanism to a realistic theory including quarks as well as families remains to be investigated.

ACKNOWLEDGEMENTS

We would like to thank D. Amati, A. Masiero, R.D. Peccei and G. Veneziano for helpful discussions.

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FIGURE CAPTIONS

- Fig. 1 Supergraphs yielding the negative mass squared for H_2 . The cross denotes the insertion of $\delta = \Delta^2 \theta \theta \bar{\theta} \bar{\theta}$.
- Fig. 2 Supergraphs contributing to the gauge invariant kinetic term for H_2 .

- Figure 1 -

- Figure 2 -