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CONTINUOUS SPIN IN THREE DIMENSIONS

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ABSTRACT

Field theories for continuous spin in two + one dimensions are studied, using light front methods. In the interacting case, the Poincaré algebra requires the 'spin one' and 'spin two' models to be identical with topologically massive Yang-Mills and gravity respectively, while the 'spin three' model turns out to be massless.

1. INTRODUCTION

It is not known whether it is possible to define interacting field theories for massless particles with spin greater than 2, but there are some positive indications. The actions for free fields of arbitrary spin and zero mass in four dimensions have been constructed [1], as well as the three-point couplings for arbitrary spin [2]. The latter construction did not employ tensor fields; instead, it was constructed directly in terms of the physical degrees of freedom, using light front methods. A covariant three-point coupling for spin 3 was found recently [3], at the level of the equation of motion.

In this paper we note that in three dimensions, a pure 3-index symmetric tensor field for 'spin 3' (which is used when $D=4$) is ruled out, and we investigate whether there is any obstacle to the light front approach as well. The suspicion, not quite confirmed as we will see, was that there might be a connection between the possibility of introducing interaction in the light front form and the existence of a manifestly covariant form of the free theory.

2. THE SITUATION IN THREE DIMENSIONS

In three dimensions massless particles do not carry discrete spin, but continuous spin representations exist in both the massive and the massless case [4], and it has been pointed out that these are interestingly analogous to massless particles in four dimensions [5,6], since they are described by gauge theories. Thus, the covariant action for three dimensional Yang-Mills augmented with a 'topological' mass term [7] is

$$S_1 = \int d^3x \left\{ -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \frac{\mu}{4} \epsilon^{\mu\nu\rho} \left(F_{\mu\nu}^a A_\rho^a + \frac{1}{3} g f^{abc} A_\mu^a A_\nu^b A_\rho^c \right) \right\} \quad (1)$$

If we impose the light front gauge $A^+ = 0$ and solve for the auxiliary field through the appropriate constraint equation [we use the front form of dynamics, $A^\pm \equiv (1/\sqrt{2}) (A^0 \pm A^1)$, $A \equiv A^1$, and x^+ is the 'time'] the action becomes

$$S_1 = \int d^3x \left\{ \frac{1}{2} A(\Box - m^2) A + g f^{abc} \partial_- A^a A^b \frac{\partial}{\partial_-} A^c + \text{quartic coupling} \right\} \quad (2)$$

The Lorentz transformations are those of a continuous spin representation, i.e.

$$\begin{aligned}\delta_{J^+} A &= (x \partial^+ - x^+ \partial) A \\ \delta_{J^-} A &= (x^- \partial^+ - x^+ \partial^-) A \\ \delta_{J^3} A &= (x \partial^- - x^- \partial - \lambda \frac{\mu}{\partial_-} + \delta_S) A\end{aligned}\quad (3)$$

In the massive case, $\mu = m$ and the parameter λ must be integer if we demand the wave function to be single valued. For topologically massive Yang-Mills, $\lambda = 1$ (massless Yang-Mills describes a spinless excitation). The non-linear spin transformation $\delta_S A$ is zero in the free case.

A topological mass term for gravity was given by Deser et al. [5]. After suitable gauge fixing of the free theory, one finds that the dynamical Lorentz transformation of the single physical degree of freedom becomes

$$\delta_{J^3} h = (x \partial^- - x^- \partial - 2 \frac{m}{\partial_-}) h \quad (4)$$

Thus, this is a 'spin 2' theory (in the massless case, Einstein's equations in three dimensions are -- almost -- trivial). In the interacting case, the light front action for gravity becomes

$$\begin{aligned}S_2 = \int d^3x \{ & \frac{1}{2} h (\Box - m^2) h + \kappa \partial_-^2 h \left[\frac{\partial_-^2}{2} h h - \frac{\partial_-}{2} h \frac{\partial_-}{2} h + \right. \\ & \left. + 4\mu \left(\frac{\partial_-^2}{2} h h - \frac{\partial_-}{2} h \frac{1}{2} h \right) + 3\mu^2 \left(\frac{1}{2} h h - \frac{1}{2} h \frac{1}{2} h \right) \right] + \\ & \left. + \text{higher order couplings} \right\}.\end{aligned}\quad (5)$$

The form of this light front action was not determined by gauge fixing -- this would be rather awkward -- but by the requirements that the Poincaré algebra should close and reduce to (4) in the free case, by the dimension of the available parameters, and by demanding that the action be nonpolynomial. These requirements fix Eq. (5) uniquely; Poincaré invariance demands $\mu^2 = m^2$. Incidentally, the probable renormalizability of the model [5] is not manifest in this formulation.

The topological mass terms are in fact the Chern-Simons secondary characteristic classes. For dimensional reasons, this device does not work for tensor fields with more than two indices; e.g. the Chern-Simons term for the field $\varphi_{\mu\nu\sigma}$ contains five derivatives [8], yet the action must

contain the dimensioned parameter raised to one, possibly two, but not exclusively higher powers in order for it to yield the free theory correctly. Gauge-invariant additions can nevertheless be made to the equation of motion for such a field, namely

$$\square \varphi^{\mu\nu\sigma} - \sum \partial^\mu \partial_\alpha \varphi^{\alpha\nu\sigma} + \sum \partial^\mu \partial^\nu \varphi_\alpha^{\mu\sigma} + \frac{1}{m} \sum \epsilon^{\sigma\rho\alpha} \partial_\rho (\square \varphi_\alpha^\mu - \partial^\mu \partial_\rho \varphi^\rho_\alpha - \partial^\nu \partial_\rho \varphi^\rho_\alpha + \partial^\mu \partial^\nu \varphi^\sigma_{\sigma\alpha}) = 0, \quad (6)$$

but it can be shown that this does not lead to sensible propagation. Without the second term, Eq. (6) is of course empty in three dimensions. Therefore, a pure $\varphi_{\mu\nu\sigma}$ field cannot be used to describe a 'spin 3' theory in three dimensions. We have not investigated whether one can add auxiliary fields to Eq. (6), say, and obtain a sensible equation of motion in that way, as is the case for the 'spin 5/2' field [6].

We now ask whether the construction of a light front action for interacting 'spin 3' is impossible as well. Motivated by the form of the spin 3 action in four dimensions [2], we start from the action

$$S_3^0 = \int d^3x \left\{ \frac{1}{2} \varphi^a \square \varphi^a + 2\alpha f^{abc} \partial_-^3 \varphi^a \left(\frac{\partial_-^3}{\partial_-^2} \varphi^b \varphi^c - 3 \frac{\partial_-^2}{\partial_-} \varphi^b \frac{\partial_-}{\partial_-} \varphi^c \right) \right\}, \quad (7)$$

which is invariant, as far as cubic terms are concerned, under

$$\delta_{\eta} \varphi^a = (\eta \partial^- - \eta^- \partial + \delta_S^0) \varphi^a, \quad (8)$$

where $\delta_S^0 \varphi^a$ is generated by the dynamical spin generator

$$S^0 = 6 \int d^3x f^{abc} \partial_-^3 \varphi^a \left[\frac{\partial_-^3}{\partial_-^2} \varphi^b \varphi^c - 2 \frac{\partial_-^2}{\partial_-} \varphi^b \frac{\partial_-}{\partial_-} \varphi^c + \frac{1}{2} \varphi^b \frac{\partial_-^2}{\partial_-} \varphi^c \right] \quad (9)$$

through the canonical Poisson bracket. The question is now whether Eqs. (7) and (9) can be augmented with additional terms to yield a continuous spin representation of the Poincaré group.

This question has a unique answer. The answer is that the action

$$S_3 = S_3^0 + 2\mu \int d^3x f^{abc} \partial_-^3 \varphi^a \left(3 \frac{\partial_-^2}{\partial_-^2} \varphi^b \varphi^c - 6 \frac{\partial_-^2}{\partial_-^2} \varphi^b \frac{\partial_-}{\partial_-} \varphi^c + 3 \frac{\partial_-^2}{\partial_-^2} \varphi^b \frac{1}{\partial_-} \varphi^c \right) \quad (10)$$

is invariant, up to higher order couplings, under the Lorentz transformation

$$\delta_J \varphi^a = (x \partial^- - x^- \partial - 3 \frac{J}{\partial_-} + \delta_S) \varphi^a \quad (11)$$

provided that the dynamical spin generator is chosen to be

$$S = S^0 + \int d^3x \, f^{abc} \partial_-^3 \varphi^a \left\{ 12\mu \left(\frac{\partial_-^3}{\partial_-^3} \varphi^b \varphi^c - \frac{1}{\partial_-^2} \varphi^b \frac{\partial_-}{\partial_-} \varphi^c + \right. \right. \\ \left. \left. + 3 \frac{1}{\partial_-} \varphi^b \frac{\partial_-}{\partial_-^2} \varphi^c \right) + 36\mu^2 \varphi^b \frac{1}{\partial_-^3} \varphi^c \right\} \quad (12)$$

Note that Poincaré invariance demands $m^2 = 0$ in this case. Thus, it is indeed impossible to construct a massive 'spin 3' continuous spin field theory in three dimensions, but, surprisingly, a massless one seems to exist instead, unless something goes wrong in higher orders of the interaction. It was argued in Ref. [4] that such a theory cannot be given a manifestly covariant formulation in terms of finite dimensional tensors.

3. CONCLUSIONS

We have investigated whether the continuous spin representations in three dimensions can be used to produce interacting theories which are analogous to massless particles with spin in four dimensions. In the massive case, the answer is yes for spins 1 and 2, but no for spin 3. A manifestly covariant formulation of the tree theory has been given only in the first two cases. In the massless case, where no covariant formulation exists, the answer is no for spins 1 and 2, but seems to be yes for spin 3.

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