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HAMILTONIAN TREATMENT OF SELF-DUAL ANTISYMMETRIC TENSORS

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A B S T R A C T

Self-dual antisymmetric tensors occur in certain $4n+2$ - dimensional supergravity models, in particular the one connected to type IIB superstrings. So far they have been quantized only using light front methods. Here we show that the existing covariant action for such fields leads to a consistent Hamiltonian system also for ordinary "timelike" dynamics, although rather complicated second class constraints are present.

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A vexing problem in superstring theories is the lack of a formalism which exhibits all their symmetries manifestly. This is not a problem of principle, but it leads to complications in practice. On the other hand it is important - also in principle - to be able to set initial data on spacelike surfaces, since the set of allowable initial data in the simple light front formulation¹⁾ is rather restricted. In the case of type IIB superstrings the field theory limit contains a self-dual antisymmetric tensor field which seems to be at the root of the problem of covariance²⁾. The problem is somewhat akin to the well known difficulties encountered in trying to exhibit all the supersymmetries manifestly in $N \geq 2$ supersymmetry³⁾. A manifestly covariant action for self-dual tensors was proposed in Ref. 4), where it was further shown that the light front gauge can be imposed. The purpose of this letter is to examine that action using canonical methods. We find that it leads to a consistent Hamiltonian system also in a "timelike" formulation, but the self-duality condition appears as complicated second class constraints, so that it is doubtful whether it can be used for covariant quantization. Similar statements can be made⁵⁾ about the covariant superstring action of Ref. 6).

To save some writing we consider the six-dimensional action⁴⁾ rather than the 10-dimensional one:

$$S = \int d^6x \left\{ -\frac{1}{12} F_{\mu\nu\sigma} F^{\mu\nu\sigma} - \frac{1}{2} G_{\mu\nu} \bar{G}^{\mu\nu} - \frac{1}{8} \epsilon^{\mu\nu\sigma\alpha\beta} A_{\mu\nu} \bar{G}_{\sigma\alpha} G_{\beta\gamma} + \frac{1}{4} \Lambda^{\mu\nu} F_{\mu\sigma\tau} F^{-\nu\sigma\tau} \right\} \quad (1)$$

where

$$F_{\mu\nu\sigma} = \frac{1}{2} \partial_{[\mu} A_{\nu\sigma]} + \frac{1}{2} \bar{B}_{\epsilon\mu} \overset{\leftrightarrow}{\partial}_\nu B_{\sigma]} \quad (2)$$

$$G_{\mu\nu} = \partial_{[\mu} B_{\nu]}$$

The self-duality condition (a possibility only when the dimension of space-time is twice odd) is

$$F_{\mu\nu\sigma}^- = \frac{1}{2} (F_{\mu\nu\sigma} - \frac{1}{6} \epsilon_{\mu\nu\sigma\alpha\beta\gamma} F^{\alpha\beta\gamma}) = 0 \quad (3)$$

The Lagrange multiplier $\Lambda_{\alpha\beta}$ is symmetric and traceless and interaction with a complex vector field has been included.

Using "timelike" dynamics, we find the following canonical momenta:

$$P_{\Lambda\alpha\beta} = P_{A\alpha 0} = P_{B\alpha 0} = P_{\bar{B}\alpha 0} = 0$$

$$P_{Aij} = -F_{ij0} + \frac{1}{2}\Lambda_{00}^{\mu} F_{\mu ij} - \frac{1}{2}\Lambda_{\alpha i}^{\mu} F_{j\mu} \gamma_0 + \frac{1}{4}\epsilon_{ijklm} \Lambda_{\mu k}^{\nu} F_{\nu l m} \quad (4)$$

$$P_{B_i} = \bar{G}_{i0} - \frac{1}{4}\epsilon_{iklmr} A_{kl} \bar{G}_{mr} - \frac{1}{2} \bar{B}_k P_{Aki}$$

$$P_{\bar{B}_i} = G_{i0} - \frac{1}{4}\epsilon_{iklmr} A_{kl} G_{mr} - \frac{1}{2} B_k P_{Aki}$$

We recognise 20+5+1+1 primary constraints. The canonical Hamiltonian becomes:

$$\begin{aligned} H_c = \int d^5x \left\{ \frac{1}{4} P_{ij} P_{Aij} + A_{0ij} P_{Aij} + \frac{1}{12} F_{ijk} F_{ijk} + P_{B_i} P_{\bar{B}_i} - B_{0,i} P_{B_i} - \bar{B}_{0,i} P_{\bar{B}_i} + \right. \\ \left. + \frac{1}{2} G_{ij} \bar{G}_{ij} + \frac{1}{4} B_{0i} \bar{G}_{ij} P_{Aij} + \frac{1}{4} \bar{B}_{0i} G_{ij} P_{Aij} - \frac{1}{2} B_j P_{B_i} P_{Aij} - \frac{1}{2} \bar{B}_j P_{\bar{B}_i} P_{Aij} - \right. \\ \left. - \frac{1}{8} P_{Aij} \epsilon_{iklmr} A_{kl} (G_{mr} \bar{B}_j + \bar{G}_{mr} B_j) - \frac{1}{4} P_{Aij} P_{Aki} (\bar{B}_j B_k + B_j \bar{B}_k) - \right. \\ \left. - \frac{1}{4} B_k P_{Aki} \bar{B}_s P_{Asi} + \frac{1}{16} \epsilon_{ijklm} \epsilon_{krstu} A_{ij} A_{rs} \bar{G}_{tu} G_{lm} + \right. \quad (5) \\ \left. + \frac{1}{4} \epsilon_{ijklm} [A_{ij} (G_{kl} P_{Bm} + \bar{G}_{kl} P_{\bar{B}m}) - A_{0i} \bar{G}_{jk} G_{lm}] - \right. \\ \left. - \frac{1}{2} \Lambda_{00} F_{0ij} \bar{F}_{0ij} - \frac{1}{4} \Lambda_{0k} \epsilon_{ijklm} F_{0ij} \bar{F}_{\alpha lm} + \Lambda_{ij} F_{0ik} \bar{F}_{0jk} - \right. \\ \left. - \frac{1}{16} (2\Lambda_{00} F_{0ij} + \Lambda_{0k} \epsilon_{ijklm} F_{0lm} + 2\Lambda_{k\alpha i} F_{j\mu} \gamma_0) \right\}^2 \end{aligned}$$

where

$$F_{0ij} = -\frac{1}{2} (\phi_{ij} + \Lambda_{00} F_{0ij} + \frac{1}{2} \Lambda_{0k} \epsilon_{ijklm} F_{0lm} + \Lambda_{k\alpha i} F_{j\mu} \gamma_0)$$

$$\phi_{ij} = P_{Aij} + \frac{1}{4} \epsilon_{ijklm} (2A_{klm} + \bar{B}_k G_{lm} + B_k \bar{G}_{lm}) \quad (6)$$

We have not been able to solve explicitly for F_{oij}^- , but fortunately we do not have to. Consistency requires the following secondary constraints:

$$\{P_\Lambda, H\} = 0 \Rightarrow \phi_{ij} = 0 \Rightarrow F_{oij}^- = 0$$

$$\{P_{A\alpha}, H\} = 0 \Rightarrow \partial_i \phi_{ij} = 0$$

$$\{P_{B\alpha}, H\} = 0 \Rightarrow \partial_i P_{Bj} + \frac{1}{4} \bar{G}_{ij} P_{Aij} = 0 \quad (7)$$

$$\{P_{\bar{B}\alpha}, H\} = 0 \Rightarrow \partial_i P_{\bar{B}j} + \frac{1}{4} G_{ij} P_{Aij} = 0$$

Using the naive Poisson brackets, we find that

$$\{\phi_{ij}, \phi_{kl}\} = 2 \epsilon_{ijklm} \partial_m \delta^{(5)}(x-y) \quad (8)$$

Hence the constraints do not obey a closed algebra, as would be the case if only first class constraints were present (a first class constraint generates gauge transformations in the Hamiltonian formalism). In fact, six of the ϕ_{ij} 's are second class constraints. Such constraints can be thought of as first class constraints with canonical gauge fixing conditions already "built in". Their presence leads to an additional factor

$$\pi_k \delta(\phi_k) \sqrt{\det |\{\phi_k, \phi_l\}|} \quad (9)$$

in the path integral measure⁷⁾. Second class constraints occur in a number of field theories⁷⁾, but usually the factor (9) is either an innocuous scalar, as in massive Yang-Mills or the Wess-Zumino model with auxiliary fields, or else they

occur when manifest covariance is not an issue, as in light front field theory. Here the situation is less trivial, but the fact that the constraint matrix (8) is field-independent may mean that they do less harm than one might think.

In order to isolate the second class constraints we give up manifest rotation invariance. If we let the index $a, b, \dots$ range from 2 to 5, we find that

$$\{\phi_{ab}, \phi_{cd}\} = 2\epsilon_{abcd} \partial_i \delta^{(s)}(x-y) \quad (10)$$

Assuming suitable boundary conditions, this matrix can be inverted and the corresponding Dirac brackets compatible with the constraints can be defined:

$$\{f, g\}^* = \{f, g\} - \{f, \phi_{ab}\} \{\phi_{ab}, \phi_{cd}\}^{-1} \{\phi_{cd}, g\} \quad (11)$$

It is easily checked that the Dirac brackets among the remaining constraints vanish so that they are all first class. To conclude that we have a consistent Hamiltonian system we must also check that the Hamiltonian "commutes" with the constraints; this is somewhat more awkward and was checked in the free case only.

Note that the self-dual tensor contributes six degrees of freedom to phase space, twice the number in a light front treatment, as it should. The Lagrange multiplier $\Lambda_{\mu\nu}$ is a pure gauge field.

In conclusion, we have found that the action of Ref. 4) indeed leads to a consistent Hamiltonian picture of self-dual antisymmetric tensors, so that a theory containing such fields does admit spacelike initial data. The drawback is that the self-duality condition appears as rather complicated second class constraints.

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REFERENCES

- 1) J.H. Schwarz, Phys. Reports 89 (1982) 223;
M.B. Green, J.H. Schwarz and L. Brink, Nucl. Phys. B218 (1983) 43;
M.B. Green, Caltech preprint CALT-68-1219 (November 1984).
- 2) N. Marcus and J.H. Schwarz, Phys. Lett. 115B (1982) 111.
- 3) W. Siegel and M. Roček, Phys. Lett. 105B (1981) 275;
B.E.W. Nilsson, Göteborg preprint 81-6 (February 1981);
V.O. Rivelles and J.G. Taylor, Phys. Lett. 121B (1983) 37.
- 4) W. Siegel, Nucl. Phys. B238 (1984) 307.
- 5) I. Bengtsson and M. Cederwall, Göteborg preprint 84-21 (June 1984);
T. Hori and K. Kamimura, Univ. Tokyo preprint INS-Rep. 504 (August 1984).
- 6) M.B. Green and J.H. Schwarz, Phys. Lett. 136B (1984) 367.
- 7) H. Yabuki, Kyoto preprint RIMS-183 (May 1975);
P. Senjanovic, Ann. of Phys. 100 (1976) 227.