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HOW ACCURATELY CAN THE DEUTERON RADIUS BE DETERMINED?

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ABSTRACT

The uncertainty in the correction term due to nucleon finite size is found to be of the order of 1-2% (i.e., $\pm 0.1\%$ in r_d), which is still negligible at the present experimental precision. Most of the contribution is of long range.

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The deuteron charge radius $r_e = \langle r^2 \rangle^{\frac{1}{2}} \approx 2.095$ fm has been determined to very high precision ($\pm 0.2\%$) from the extrapolation of the form factor in electron scattering at low energy¹⁾. From this value, one can deduce the "intrinsic" deuteron radius $r_d \approx 1.963$ fm, assuming that the deuteron consists of nucleons only and that their charge radii are unaffected by the interaction²⁾:

$$r_d^2 = r_e^2 - \langle r_p^2 \rangle - \langle r_n^2 \rangle. \quad (1)$$

Here $\langle r_p^2 \rangle^{\frac{1}{2}} \approx 0.80$ fm is the proton r.m.s radius, and $\langle r_n^2 \rangle = 6[dG_n(q^2)/dq^2]_{q^2=0}$ is the neutron squared r.m.s radius (which is negative). It is given mainly by the Foldy term from the magnetic moment. The third term on the right-hand side of Eq. (1) is the Darwin-Foldy term due to the Zitterbewegung of the proton. The corrections for nucleon structure represent about 6% of the charge radius. In order to match the experimental accuracy, it is therefore necessary to understand these terms to a precision of about 3%, and to control their contributions on this level of precision. The three terms contribute as follows: proton form factor - 83%, neutron form factor - 14%, Darwin-Foldy term - 3%.

In view of the smallness of the last term, we will not discuss any correction to it. Within the framework of standard meson-exchange theory, these will only be very small additional corrections to the "intrinsic" radius ($\approx 0.1\%$)³⁾, which can safely be neglected (the "meson-exchange current" corrections).

The main concern at the very high level of accuracy required even in the correction term is that the nucleon contributions may be modified at short range, and in particular that the nucleon form factors may differ from the free ones during close encounters with another nucleon. A specific situation of this type would occur in the case of interpenetrating quark bags⁴⁾⁻⁷⁾. As a consequence, we will evaluate here the model dependence of the "intrinsic" deuteron radius as deduced from Eq. (1).

A first estimate can be made as follows. Owing to the short-range interactions, there is a certain probability ($< 100\%$) that the nucleons contribute with their free form factors. The nucleons may instead occur as Δ -isobars^{8),9)} for example, or even be dissolved into a 6-quark bag⁷⁾. As an extreme situation, we can assume that the new component in the wave function does not contribute at all to the correction term. From the experimental precision, we can therefore

immediately conclude that an anomalous component must have a probability $P_{an} \gtrsim 3\%$, in order to concern us here. Since both the fact that the magnetic moment is accurately the sum of the nucleon values to 2%, and that explicit experiments searching for additional components give lower limits to an anomalous component^{3),10)}, we can safely conclude that such a modification of the nucleons will not contribute appreciably to r_d .

As a next step, we investigate in a more model-dependent way which regions in r -space give the important contributions to the intrinsic deuteron radius r_d and to the nucleon size correction. Figure 1 gives the characteristic shape of the S-wave contribution $r^2 u^2$ and the D-wave one $r^2 w^2$ to the square of the intrinsic deuteron radius $r_d^2 \equiv \langle r^2 \rangle$ for a typical modern deuteron wave function¹¹⁾. As is well known, this distribution extends over large distances with only small contributions inside of 1 fm. The D-state contribution to r_d is only 1.2%. Its model dependence is insignificant on the scale of the experimental uncertainty, $\pm 0.2\%$ in r_d , as is obvious from Fig. 1. Consequently, this term will be omitted in the following discussion.

We now consider the corresponding distribution for the size correction $\delta \langle r^2 \rangle = r_e^2 - r_d^2$, which is obtained by folding the proton and neutron charge density distributions with the deuteron matter density distribution $\rho_d(r)$. The result is seen in Fig. 2 for the same wave function as in Fig. 1, assuming that the nucleon form factors are short-range, so that

$$F_{p,n}(q^2) \approx 1 - \frac{q^2}{6} \langle r_{p,n}^2 \rangle + \dots \quad (2)$$

In this limit, the change in the charge density $\rho_c(r)$ has the convenient form

$$\delta \rho(\vec{r}) \equiv \rho_c(\vec{r}) - \rho_d(\vec{r}) \approx \frac{\langle r_p^2 \rangle + \langle r_n^2 \rangle}{6} \nabla^2 \rho_d(\vec{r}). \quad (3)$$

The overall change in $\langle r^2 \rangle$ retains the correct value for inert nucleons:

$$\delta \langle r^2 \rangle = \langle r_p^2 \rangle + \langle r_n^2 \rangle. \quad (4)$$

The distribution $\delta J(r)$ of the correction term averaged over angles has the form

$$\delta J(r) \approx \frac{4\pi}{6} \left(\langle r_p^2 \rangle + \langle r_n^2 \rangle \right) r^3 \frac{d^2}{dr^2} (r \rho_d(r)). \quad (5)$$

In Fig. 2, we show the density distribution for the nucleon size correction $\langle r^2 \rangle$ using the Reid soft core wave function¹¹⁾ renormalized to correct asymptotic behaviour for comparison with the PARIS wave function¹⁰⁾. The integrated deviations are (-3%) in the positive region and (+2%) in the negative region. We have checked, using analytically soluble expressions for $\rho_d(r)$, closely similar in shape to the ones from modern deuteron distributions^{11),12)}, that nucleon form factors of monopole and dipole shape are well-approximated by the simple approach (3) inside of present experimental precision.

From Fig. 2 we note that the nucleon size correction is very long-range, with 20% contributions from the region outside 10 fm. The characteristic oscillation near 1 fm occurs because the actual deuteron S-wave function vanishes rapidly in the inner region with an inflexion point, which produces the sign change. Examining the region inside 1.7 fm, we note that two-thirds of the (-12%) negative contribution comes from outside 1 fm. Even if all of this (-12%) correction were totally false and should be neglected, it would only change r_d by 0.7%. (The small positive contribution at short range is even smaller and irrelevant to the present discussion.) For comparison purposes, we give also the result for the Reid soft core wave function¹²⁾ in Fig. 2. It is commonly used but of insufficient accuracy for the present discussion: its effective range is over 2% (i.e., eight standard deviations) smaller than the present experimental value. The integrated deviations are (-3%) in the positive density region and (+2%) in the negative one, so that they are responsible for -0.18% and +0.12% contributions to r_d respectively. We conclude that the $\delta\langle r^2 \rangle$ -density is stable with modern wave function, and that uncertainties of this origin are negligible.

In the perspective that the nucleonic structure may change inside ≈ 1.5 fm, the contributions from this region will be partially modified. It is difficult, however, to see how such corrections could change the area of the negative term by more than 15-20% by any reasonable modifications of the nucleon form factor. We conclude therefore that the overall uncertainty in the deuteron deduced intrinsic radius is only 0.1%, i.e., within the present experimental uncertainties of 0.2%. The nucleon size correction is at present still model-independent, but it will no longer be so if experiments are improved with a factor 2-3 better precision.

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FIGURE CAPTIONS

- Fig. 1 : Contributions to the $\langle r^2 \rangle$ for the deuteron for the S-state density $[r^2 u^2(r)]$ and the D-state density $[r^2 w^2(r)]$ for a realistic deuteron wave function¹¹⁾.
- Fig. 2 : Density distribution for the nucleon size correction $\delta\langle r^2 \rangle$ to the square of the charge radius according to Eq. (5) for a modern deuteron wave function¹¹⁾. The corresponding distribution for the Reid soft core wave function¹²⁾ renormalized to the correct asymptotic behaviour is given for comparison.

Fig. 1

Fig. 2