CERN-TH.4119/85
LAPP-TH-128

PATTERNS OF SYMMETRY RESTORATION IN GAUGE-SCALAR THEORIES:
A MONTE CARLO SIMULATION OF THE SO(3) GEORGI-GLASHOW MODEL

G. Schierholz

CERN - Geneva

and

Deutsches Elektronen-Synchrotron (DESY)
Notkestrasse 85, D-2000 HAMBURG 52, Fed. Rep. Germany

J. Seixas^{*)}

LAPP, Annecy-le-Vieux

and

M. Teper

CERN - Geneva

ABSTRACT

We enumerate several ways in which the symmetry of the SO(3) (lattice-regularized) Georgi-Glashow model can be restored. We focus on the symmetry restoration in the radial Higgs degree of freedom, which occurs for small but positive values of the ϕ^4 coupling. We try to elucidate the underlying dynamics using some novel techniques that probe the vacuum (anti-) monopole condensate.

^{*)} On leave of absence from CFMC/INIC, Lisbon, Portugal.

The symmetry breaking that scalar fields can induce by the Higgs mechanism, forms a crucial ingredient in attempts to understand physics at the scale of the weak interactions and beyond. Although there is surely much to be learned at the tree- and one-loop level, the phase transitions associated with the breaking/restoration of a local gauge symmetry are intrinsically non-perturbative phenomena which await non-perturbative methods for their better understanding. In obtaining such a better understanding of the dynamics one might hope to discipline the more naive scenarios and to discover interesting new possibilities.

In this paper we report on a study of the dynamics of such symmetry breaking in the context of the $SO(3)$ Georgi-Glashow model¹⁾. The non-perturbative methods we use centre on the monte carlo simulation of the Euclidean theory regularised on a space-time lattice. The monte carlo technique has been extensively used to extract the non-perturbative physics of QCD²⁾; it has also been used to investigate some limited aspects of the phase diagrams of theories in which one couples scalars to non-abelian gauge fields (as herein)³⁻⁶⁾.

The Georgi-Glashow model couples an adjoint Higgs multiplet to a set of $SU(2)$ non-abelian gauge fields. The Euclidean action has the familiar form

$$S = \int d^4x \left\{ \frac{1}{4g^2} F_{\mu\nu}^a F_{\mu\nu}^a + |D_\mu \phi|^2 + \lambda(\phi^2 - v^2)^2 \right\} \quad (1)$$

To make sense of the theory we regularise it by replacing continuous Euclidean space-time by a four-dimensional lattice of points. The action now becomes

$$\begin{aligned} S = & -\frac{\beta_G}{2} \sum_{\substack{n \\ \mu \neq \nu}} \text{Tr} \{ U_\mu(n) U_\nu(n+\mu) U_\mu^\dagger(n+\nu) U_\nu^\dagger(n) \} \\ & + \beta_H \sum_{n,\mu} \left[\text{Tr} \{ \phi(n) U_\mu(n) \phi^\dagger(n+\mu) U_\mu^\dagger(n) \} - \phi(n) \cdot \phi^\dagger(n) \right] \\ & + \beta_R \sum_n \left[\phi(n) \cdot \phi^\dagger(n) - 1 \right]^2 \end{aligned} \quad (2)$$

where the three terms of eq.(2) correspond to the three terms in eq.(1). We remark that much of the interesting physics in the present work will arise because we retain the radial Higgs degree of freedom (which has been "frozen" out in almost all previous work). The vacuum expectation value, v , has been absorbed into the field ϕ for convenience, and we are using the 2×2 unitary matrix representation for the scalars. The elementary $SU(2)$ gauge connections, $U_\mu(n)$, are represented by $SU(2)$ matrices. This way of writing the couplings has been chosen to conform to standard lattice usage²⁻⁶⁾; so (with a the lattice spacing)

$$\beta_G = \frac{4}{g^2} ; \quad \beta_H = (av)^2 ; \quad \beta_R = \lambda(av)^4 \quad (3)$$

For a given set of values of these bare couplings we can use the monte carlo technique to generate the typical field configurations that dominate the path integral: from these field configurations we can extract the physics that interests us, just as in QCD. However we would like to emphasise an important difference of philosophy with respect to similar QCD studies. In the latter case the interesting physics presumably lies solely in the neighbourhood of the asymptotically free continuum limit, $g^2 \rightarrow 0$. In contrast scalar fields are most likely composite, not elementary, with a compositeness scale that may even be comparable to the symmetry breaking scale. So, a priori, every point in the coupling-constant space of the lattice-regularised theory is of interest, and (treated with care) can produce an insight into the dynamics. Of course to obtain such insights one must supplement the monte carlo with numerical techniques that focus on the relevant vacuum fluctuations. Part of the purpose of this paper is to introduce methods, based on our studies of magnetic monopoles⁶⁾, which directly probe the (anti-) monopole condensate of the vacuum.

In Fig. 1 we display the (positive) coupling constant space of the theory. (Here we ignore that there may be interesting physics for some couplings negative). The top face corresponds to $\beta_R = \infty$, i.e. $|\phi|$ frozen to unit length, and the phases of the theory have, in this case, been previously investigated³⁾. As shown, they are two at zero physical temperature: a symmetry broken Higgs phase and, at sufficiently small β_G or β_H , a confining phase in which the symmetry is restored. Previously⁶⁾ we showed that in the symmetry restored phase a condensate of (anti-) monopoles forms in the vacuum. In both phases the length of ϕ oscillates around a value close to unity if β_R is large⁶⁾. In the present paper we focus on a different symmetry restoring phase transition, one that occurs as β_R is decreased at fixed β_G and β_H (the presence of such a transition was first noted in ref.6)) and which is characterised by a shift in the distribution of the length of ϕ ; from fluctuating around a non-zero value it shifts to fluctuating around zero. In Fig. 1 we show a cross-section of the phase diagram at fixed large β_H : the heavy line represents the first-order transition in β_R just discussed; the light lines represent (presumably) second-order transitions. The three phases are labelled by some of their main characteristics. We now describe how we obtain all this, and attempt to elucidate the underlying dynamics.

To begin with we start with some typical couplings in the Higgs phase, for example $\beta_G = 2.0$, $\beta_H = \beta_R = 10.0$. We now equilibrate (with respect to the action in eq.(2)) a 4^4 lattice at these couplings and then decrease β_R in small steps. At each value of β_R we first equilibrate (typically 30 sweeps) and then we measure (typically the same number of sweeps). In particular we measure the length of the Higgs field at each point, fold in the path integral integration measure $|\phi|^2 d|\phi|$, and plot the resulting number density $N(|\phi|)$. The histogram peaks at a value ≈ 1

for large β_R , and this peak moves slowly to smaller values as we decrease β_R . At a certain value of β_R , however, the peak suddenly jumps from an appreciable non-zero value to zero: as shown in Fig. 2. (Folding the measure is crucial to see this clearly). When the scalar field ceases to oscillate about a non-zero value, we expect that we lose the Higgs phase. What is the order of this phase transition? At the same value of β_G and β_H begin with a small β_R , so that $N(|\phi|)$ has a distribution as in Fig. 2a, and gradually increase β_R . Again at some point there is a sudden change and the peak of $N(|\phi|)$ moves to non-zero values. However this occurs for a very different value of β_R . In Fig. 3a we plot $\langle\phi^2\rangle$ as a function of β_R ; as the peak moves from zero to non-zero, $\langle\phi^2\rangle$ jumps from small to large values (and vice-versa). We see revealed a very clean hysteresis effect that is typical of first order transitions. (We have performed some runs on 6^4 and 8^4 lattices to confirm this hysteresis and to rule out appreciable finite volume effects). We simultaneously measure expectation values of thermal loops (products of links joined through the periodic boundaries), $\langle L \rangle$: when $\langle L \rangle$ oscillates about zero the vacuum is confining, when it acquires a non-zero value the vacuum is non-confining²⁾ (as in the Higgs phase). In Fig. 3b we see that the β_R transition shown in Fig. 2 and Fig. 3a is associated with the loss/onset of confinement: and hence with the onset/loss of the Higgs phase. (This can also be seen using the inverse monopole lifetime as an order parameter).

Similar calculations at various β values give us the picture in Fig. 1 : we have a first-order transition line that separates, at larger β_G , the Higgs phase from a symmetry restored phase characterised by the scalar field fluctuating around zero, and at smaller β_G two differing symmetric phases. What is the dynamics driving this pattern of phase transitions? Since we are considering a phase-transition that (in part) separates the Higgs phase from a symmetry restored phase, and since the cleanest gauge-invariant way to characterize the Higgs phase is by the fact that it supports isolated 't Hooft-Polyakov magnetic monopoles, it will be helpful to introduce a way of measuring the presence of a monopole⁶⁾. The simplest way to measure the presence of a monopole within a closed surface, is by measuring the total magnetic flux out of the surface: in the classical continuum case

$$\text{flux monopole} = \oint_S \mathbf{B} \cdot d\mathbf{S} = \oint_S \frac{1}{2} \text{tr} (\phi \cdot \mathbf{F}_{\mu\nu}) \cdot d\mathbf{S} = 4\pi \quad (4)$$

One can transcribe this into our lattice variables by

$$\text{flux} = \sum_{\text{box}} \frac{1}{2} \text{tr} (\hat{\phi}^+ U_{p1}) = 4\pi \quad (5)$$

where $\phi = |\phi| \cdot \hat{\phi}$ and

$$U_{p1} = U_{\mu}(n) U_{\nu}(n+\mu) U_{\mu}^+(n+\nu) U_{\nu}^+(n) \quad (6)$$

(using the notation of eq.(2), and with $|\phi|^2 = \phi^a \phi_a$) and where the sum is over the plaquettes that form the surface enclosing the monopole. When one has quantum fluctuations,

$$D \cdot \phi \neq 0$$

and

$$\text{tr}|\hat{\phi}^+ \cdot U_{p1}| \neq \text{tr}|\hat{\phi} \cdot F_{\mu\nu}| \quad (7)$$

so that the numerical value of the flux measured by eq.(5) is not 4π but typically about 3π in practice⁶⁾. This stands sufficiently far above the random vacuum background "noise" as to provide an unambiguous measure of the presence or absence of a magnetic monopole⁶⁾.

Using the measure in eq.(5) we obtain the net flux B out of each elementary spatial cube on the lattice, and the corresponding number density $N(B)$ (symmetrised in B). If we compare this distribution at typical couplings in the Higgs phase, say $\beta_{G,H,R} = (2.4, 15.0, 25.0)$, with the corresponding distribution in the symmetric large β_R phase (region II in Fig. 1), say $\beta_{G,H,R} = (0.5, 10.0, 10.0)$, then we find curves (i) and (ii) respectively in Fig. 4a, and we see that in the latter phase $N(B)$ possesses a tail of cubes that possess large fluxes (the large B cut-off is due to a bound on the cube with our compact variables). This tail can be naturally interpreted⁶⁾ as being due to virtual (anti-) monopoles, and so we understand the symmetry restoration obtained by decreasing β_G at large β_H and β_R as being due to the condensation into the vacuum of monopoles and anti-monopoles: this, of course, goes well together with the confining character of this phase.

We now begin with a coupling in this symmetric phase, say $\beta_{G,H,R} = (0.5, 10.0, 10.0)$, and gradually decrease β_R until we pass through the transition in the length of the Higgs field. If we now measure $N(B)$ - say at $\beta_{G,H,R} = (0.5, 10.0, 5.0)$ - we obtain a distribution, curve (iii) in Fig. 4a, which, although it extends to quite large B , shows no secondary tail of cubes with large flux; the overall distribution is roughly a single Gaussian from smallest to largest B , in contrast to curve (ii). One understands this as follows. The scalar field maps out the structure in the gauge field because of the coupling $\beta_H |D\phi|^2$. For the flux out of a cube, as in eq.(5), the relevant piece of $D\phi$ is the piece $D\hat{\phi}$ which relates to the coupling of the gauge and scalar directions in the internal symmetry space. This has an effective form:

$$\beta_H |D\phi|^2 \rightarrow \beta_H \langle \phi^2 \rangle |D\hat{\phi}|^2 \quad (8)$$

In the type of phase transition we are considering here, $\langle \phi^2 \rangle$ becomes small, so that, effectively, the scalar and gauge directions in group space decouple: curve (iii) in Fig. 4a represents the resulting random "noise". To further confirm this interpretation observe that a cube embedded in a monopole configuration should be relatively stable over several monte carlo sweeps. On the other hand if the flux is simply random noise there should be no special stability. So we calculate at each value of β_R the number of cubes which have a flux that is greater than 1.40 (from curve (ii) in Fig. 4a this value appears to separate "monopoles" from background noise) when averaged over all 30 sweeps; $N(\bar{B} > 1.40; \beta_R)$. This quantity is shown in Fig. 4b - the solid points - and as we expected, in the low β_R phase there are no stable large-flux cubes.

What is the driving dynamics of this transition? A clue can be found in eq.(8): reducing $\langle \phi^2 \rangle$ allows larger values of $|D\hat{\phi}|^2$ at a given β_H . The cost is $O(\beta_R)$ per site and comes from the last term in the action of eq.(2): the gain is in entropy, in the increased number of configurations that can contribute if the restriction on the size of $|D\hat{\phi}|^2$ is relaxed. The phase-transition occurs as the gain in entropy wins over the $O(\beta_R)$ loss in the action. To see this more explicitly we plot, in Fig. 4c, at $\beta_{G,H} = (0.5, 10.0)$, both $\langle \square \rangle \equiv \frac{1}{2} \text{tr} U_\mu(n) U_\nu(n+\mu) U_\mu^\dagger(n+\nu) U_\nu^\dagger(n)$ and $|D\phi|^2 / \langle \phi^2 \rangle$ ($\approx |D\hat{\phi}|^2$). At the transition point, as we decrease β_R , $\langle \square \rangle$ suddenly changes in the direction of greater disorder (note that $\langle \square \rangle = 1$ when the gauge links are completely frozen), and $|D\phi|^2 / \langle \phi^2 \rangle$ jumps to a much larger value, showing the same effect. This same pattern of behaviour is repeated at all points along this phase-transition line.

We have identified the low β_R vacuum as being qualitatively more disordered than the large β_R vacuum. Is it completely disordered or does the vacuum still retain a remnant of the (anti-) monopole condensate? Since $|D\hat{\phi}|^2$ is now large, the dynamical scalar field no longer maps out any structure in the internal symmetry space of the gauge vacuum. To address this question we introduce a supplementary scalar field which we shall use to provide a "contour map" of the topological structure in the pure gauge vacuum. That is to say, at a given β_R below the phase transition we generate say 30 sequential field configurations. We now take each gauge field configuration and apply the following procedure: couple an auxiliary adjoint Higgs field, $\tilde{\phi}$, to the gauge fields; bring $\tilde{\phi}$ into equilibrium with the fixed gauge field, with the corresponding $\tilde{\beta}_H$ and $\tilde{\beta}_R$ large enough that this Higgs field is in its large $\tilde{\beta}_R$ phase; now perform an adequate number of sweeps on $\tilde{\phi}$, with the Metropolis algorithm (trivially) modified so as to drive the system to the minimum of the action, i.e. close to

$$|D\tilde{\phi}| = 0 \quad ; \quad \tilde{\phi}^2 = 1 \tag{9}$$

The resulting $\tilde{\phi}$ maps out any monopole-like structure in the gauge vacuum. (The choice of scalar field was made with this in mind. Other topological structures will require other, suitably tailored, choices of auxiliary fields). Calculate the flux out of each cube on the lattice, for the given fixed gauge field configuration and the final $\tilde{\phi}$ configuration. Repeat this "gauge staining" procedure for the gauge part of each of the 30 configurations we have previously sequentially generated. Construct now the corresponding distribution $N(\bar{B} > 1.40; \beta_R)$ i.e. the number of cubes at that β_R with an average flux (over all 30 sweeps) greater than 1.40. This distribution is plotted - open circles - in Fig. 4b. We find that the low β_R vacuum contains a weaker, but significant, (anti-) monopole condensate.

We can summarise what we have learned as follows.

- A) For large β_R the last term in the action, eq.(2) forces $|\phi| \approx 1$. If β_H is large one is forced to $|D\phi| \approx 0$ and the natural phase is a Higgs phase.
- B) If we go to small β_G , however, the gauge fields favour disorder. A more disordered field configuration than the Higgs one, but still having $|D\phi| \approx 0$ over most of space, is obtained by having a "gas" of (anti-) monopoles in the vacuum: at small enough β_G such a condensate forms, disorders the vacuum, thus restoring the symmetry and confinement. (Recall that in continuous semi-classical Higgs models, the monopole mass is $0(1/g^2) = 0(\beta_G)$ and so becomes light at small β_G). At this point we still have $|\phi| \approx 1$.
- C) If we decrease β_R , at some point the gain in entropy generated by freeing the $|D\hat{\phi}| \approx 0$ constraint between the scalar and gauge fields, offsets the $0(\beta_R)$ cost in action when $|\phi| \approx 1$ shifts to $|\phi| \approx 0$: (in this shift the coefficient of $|D\hat{\phi}|^2$ changes effectively from β_H to $\beta_H|\phi|^2 \approx 0$). The vacuum becomes more disordered, as characterised by a sudden increase in the pure gauge action. However, our "staining" of the gauge fields with auxiliary scalar fields reveals, surprisingly, that the gauge part of this disordered vacuum still retains a (weaker) (anti-) monopole condensate.

The symmetry restoration obtained by decreasing β_R would appear, superficially, to be in some sense the reverse of the well-known Coleman-Weinberg mechanism: quantum corrections restore the symmetry despite a negative (bare) mass-squared term, which naively should drive the theory into a Higgs phase. However the reader is reminded that our parameters are the bare couplings of the cut-off theory. In any case what happens as $\beta_H \rightarrow 0$ (and $\beta_R = \lambda\beta_H^2 \rightarrow 0$) is of special interest in this context. Is there a first (as here) or second order transition? Does symmetry breaking depend on the value of the ϕ^4 coupling? Is there a continuum limit? We remark only that as one decreases β_H the transition is much harder to see, so we delay its detailed exploration to a subsequent publication.

It is worth re-emphasising that much of the interesting physics of gauge-scalar theories is lost if one drops the radial Higgs degree of freedom⁸⁾ (as has been done in most previous lattice studies). In a longer paper⁹⁾ we shall go in more detail into the phase-structure of the Georgi-Glashow model: in particular into the symmetry restoration that is obtained at high physical temperatures. We believe that it is necessary to explore these various patterns of symmetry breaking/restoration, and to understand the dynamical mechanisms that drive them, if the Higgs mechanism is to realise its full potential in unified model building. At the same time we expect that what one learns about possible gauge vacuum structures will shed light upon the important phenomenon of dynamical symmetry breaking without scalars. At present we are applying the methods of this paper to two familiar, but still mysterious, examples of this phenomenon: confinement and chiral symmetry breaking in QCD¹⁰⁾. The extent of our success therein should clarify the ultimate usefulness of these ideas.

ACKNOWLEDGEMENTS

One of us (M.T.) thanks LAPP for its hospitality during the course of much of this work.

The numerical simulations were performed on the VAX 11/780 at LAPP ; we are grateful for the generous computing support.

REFERENCES

- 1) H. Georgi, S.L. Glashow, Phys. Rev. Lett. 32, 438 (1974).
- 2) For recent reviews see:
G. Schierholz, Lectures at Scottish Universities Summer School;
M. Teper, Talk given at the 1984 Aspen Lattice/Glueball Meeting, preprint LAPP-TH-122.
- 3) For some studies of SU(2) gauge fields coupled to adjoint scalars (with the length of the scalar field frozen to unity) see:
C.B. Lang, C. Rebbi, M. Virasoro, Phys. Lett. 104B, 294 (1981);
R. Brower, D. Kessler, T. Schalk, H. Levine, M. Nauenberg, Phys. Rev. D25, 3319 (1982);
F. Karsch, E. Seiler, I. Stamatescu, preprint CERN.TH.3612.
- 4) For some studies with fundamental scalars see ref. 3) and:
H. Kuhnelt, C.B. Lang, G. Vones, Nucl. Phys. B230, 31 (1984);
I. Montvay, Phys. Lett. 150B, 441 (1985).
- 5) A study of SU(3) with adjoint scalars:
K. Olynyk, J. Shigemitsu, preprint OSU DOE/ER/01545-353.
- 6) G. Schierholz, J. Seixas, M. Teper, Phys. Lett. 151B, 69 (1985).
- 7) S. Coleman, E. Weinberg, Phys. Rev. D7, 1888 (1973).
- 8) A similar β_R phase transition occurs in the case of SU(2) gauge fields coupled to fundamental scalars.
- 9) In preparation.
- 10) I. Barbour, I. Fox, M. Laursen, G. Schierholz, J. Seixas, M. Teper, in preparation.

FIGURE CAPTIONS

- Fig. 1 The 3-dimensional phase diagram of the zero-temperature Georgi-Glashow model: shown are the phase transition lines on the surfaces $\beta_R = \infty$ and $\beta_H = 10$. The three phases are labelled by some of their characteristics.
- Fig. 2 The distribution of the length of the Higgs field (a) just below, (b) just above, the (heavy) phase transition line in Fig. 1.
- Fig. 3 Hysteresis plots of (a) $\langle \phi^2 \rangle$ and (b) the thermal loop average $|\langle L \rangle|$, as functions of β_R .
- Fig. 4 (a) The number of spatial cubes versus the magnetic flux out of the cube for typical couplings in the three phases in Fig. 1 : curve (i) from phase I, curve (ii) from phase II, curve (iii) from phase III.
- (b) The number of spatial cubes, versus β_R , which have a large magnetic flux (> 1.4) when averaged over 30 sequential sweeps :
 (●) with respect to the dynamical scalar fields ;
 (o) with respect to auxiliary scalars introduced to map the topological structure of the gauge vacuum.
- (c) Decreasing β_R suddenly disorders (o) the gauge fields and (●) the relative orientations of gauge and scalar fields.

Fig. 1

Fig. 2

Fig. 3a

Fig. 3b

Fig. 4a

Fig. 4b

Fig. 4c