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ON THE SU(3) AND PCAC EVALUATION OF THE K^0 - $\bar{K}^0$ MATRIX ELEMENT

AND $K \rightarrow 2\pi$ DECAYS

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A B S T R A C T

The only consistent procedure for using PCAC and SU(3) in the evaluation of B is the one which simultaneously explains $K \rightarrow 2\pi$ amplitudes. Such a procedure gives $B \approx 1$ in the most natural way.

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The main aim of this paper is to improve the understanding of the simultaneous evaluation of the $K^0-\bar{K}^0$ matrix element and $K \rightarrow 2\pi$ weak decay amplitudes.

First, we calculate $K \rightarrow 2\pi$ amplitudes by using a general decomposition of decay amplitudes to the leading order in meson momentum and by using the LSZ reduction of both pions^{1),2)}. Then we apply quark models to evaluate a typical $K-\pi$ matrix element³⁾. Second, we check the validity of the SU(3) relation between $K^0-\bar{K}^0$ matrix elements⁴⁾ and evaluate the parameter

$$B = \frac{\langle \bar{K}^0 | O_{LL} | K^0 \rangle}{\langle \bar{K}^0 | O_{LL} | K^0 \rangle_{vac. sat.}}$$

A simultaneous determination of the $K^0 \rightarrow \pi^+\pi^-$, $K^0 \rightarrow 2\pi^0$, $K^+ \rightarrow \pi^+\pi^0$ amplitudes and of parameter B is discussed.

A general decomposition of the decay amplitude to the leading order in the meson momenta is:

$$A(M(p) \rightarrow M_1(p_1) + M_2(p_2)) = A_0 + p_1(p_2 + p)A_1 + p_2(p_1 + p)A_2 + p(p_2 - p_1)A_3. \quad (1)$$

Momentum conservation gives, for the physical amplitude on the mass shell,

$$A(M \rightarrow M_1 + M_2)_{phys.} = A_0 + (A_1 - A_3)(m^2 - m_2^2) + (A_2 + A_3)(m_1^2 - m_2^2) \quad (2)$$

According to the well-known SU(3) theorem, the $K \rightarrow 2\pi$ amplitude vanishes at the SU(3) symmetric point, $A_0 = 0$. Combinations of the A_i constants are then fixed in the current algebra (CA) limit:

$$\lim_{p_1 \rightarrow 0} A = A_1^{CA} = (A_2 + A_3)(p p_2) \quad , \quad p = p_1 + p_2 \quad ,$$

$$\lim_{p_2 \rightarrow 0} A = A_2^{CA} = (A_1 - A_3)(p p_1). \quad (3)$$

Momentum conservation, which is the same in the CA limit and in the physical amplitude, gives, for pp_i :

$$p p_i = \frac{1}{2}(m^2 + m_i^2 - m_j^2); \quad i \neq j; \quad i, j = 1, 2. \quad (4)$$

For the $K \rightarrow 2\pi$, we have^{1),2)}:

$$A_{phys} \equiv A(K \rightarrow 2\pi) = 2 \left(1 - \frac{m_\pi^2}{m_K^2}\right) (A_1^{CA} + A_2^{CA}) = K (A_1^{CA} + A_2^{CA}) \quad (5)$$

where $K = 1.85$ is known as "the continuation to the physical amplitude" factor.

Let us first calculate the $K^+ \rightarrow \pi^+\pi^0$ amplitude:

$$\begin{aligned} A(K^+ \rightarrow \pi^+\pi^0) &= \langle \pi^+\pi^0 | H_W^{PV} | K^+ \rangle \sqrt{4E_\pi E_K} \Big|_{phys.} \\ &= \frac{iK}{\sqrt{2} f_\pi} \left(\langle \pi^+ | H_W^{PC} | K^+ \rangle + \sqrt{2} \langle \pi^0 | H_W^{PC} | K^0 \rangle \right) \sqrt{4E_\pi E_K} \end{aligned} \quad (6)$$

Application of any static quark model gives^{2),3)}:

$$A(K^+ \rightarrow \pi^+\pi^0) = K \frac{-i\tilde{G}_F}{4f_\pi} \left[24 c_4 (\alpha - 3b) \right] \sqrt{4E_\pi E_K}, \quad (7)$$

where $f_\pi = 0.13$ GeV, $c_4 = 0.4$, $E_K = m_K$, $E_\pi = \frac{1}{2}m_K$, $\tilde{G}_F = G_F \cos\theta_c \sin\theta_c$ and $(\alpha - 3b)$ is the integral over the quark-model wave functions²⁾ (MIT-bag, harmonic oscillator (HO), relativistically correlated harmonic oscillator (RCHO), quark models^{2),7),8)}).

The same procedure gives the following results^{2),3)} for the $K^0 \rightarrow \pi^+\pi^-$, $2\pi^0$ amplitudes

$$\begin{aligned} A(K^0 \rightarrow \pi^+\pi^-) &= K \frac{-i\tilde{G}_F}{f_\pi} \left\{ \left[c_1 - 2(c_2 + c_3 + \frac{c_4}{-2c_4}) \right] (\alpha - 3b) \right. \\ &\quad \left. + \left(c_6 + \frac{16}{3}c_5 \right) \left[(\alpha + b) + (AC) \right] \right\} \sqrt{2} m_K. \end{aligned} \quad (8)$$

The values of the $(\alpha - 3b)$ and $(\alpha + b)$ integrals over the quark model wave functions for the MIT, HO and RCHO models are listed in Table 1.

The symbol (AC) in Eq. (8) denotes the so-called anomalous commutator term given by

$$(AC) = \frac{-f_\pi^2 m_\pi^2 m_K}{\sqrt{2} 6 (m_u + m_d)(m_d + m_s)} \quad (9)$$

From the recent analysis by Gavela et al.⁹⁾, Donoghue¹⁰⁾ and Bilic'-Guberina¹⁰⁾, we know that Eq. (9), for current quark masses, practically eliminates the penguin contribution to the $K^0 \rightarrow 2\pi$ amplitudes. We should demonstrate such properties by direct evaluation of the $K \rightarrow 2\pi$ amplitudes with and without penguin terms [without and with (AC) terms]. Note also that a very detailed analysis of the (AC) term in $K \rightarrow 2\pi$ amplitudes is not essential for this comment.

The amplitudes from Tables 2 and 3 are calculated for integrals from Table 1 and for the following QCD coefficients¹¹⁾:

$$c_1 = -2.74, \quad c_2 = c_3 = 0.08, \quad c_4 = 0.4, \quad c_5 = -0.13, \quad c_6 = -0.05.$$

Note that only absence of the (AC) term and HQ models lead to a reasonable good direction. With some additional quenches of c_4 ($\Delta I = 3/2$ part) and enhancement of $\Delta I = 1/2$ terms one can perfectly fit all $K \rightarrow 2\pi$ amplitudes. Unfortunately, c_4 is extremely stable under the variation of a typical QCD parameters. That is, the main reason why reasonable tuning of $\Delta I = 3/2$ (over $\Delta I = 1/2$) parts of the effective weak Hamiltonian¹¹⁾ does not exist. The CA (with quark models) analysis of the $K \rightarrow 2\pi$ amplitudes is relevant due to the close connection between the $K^0 - \bar{K}^0$ system and the $K \rightarrow 2\pi$ decays.

The second important part of our comment is the realization that the operator

$$O_{LL}^{|\Delta S|=2} = (\bar{d}_L^i \gamma_\mu s_L^i) (\bar{u}_L^j \gamma^\mu s_L^j)$$

belongs to the same SU(3) representations as the operator

$$O_4 = (\bar{d}_L^i \gamma_\mu s_L^i) (\bar{u}_L^j \gamma^\mu u_L^j) + (\bar{d}_L^i \gamma_\mu u_L^i) (\bar{u}_L^j \gamma^\mu s_L^j) - (\bar{d}_L^i \gamma_\mu s_L^i) (\bar{s}_L^j \gamma^\mu s_L^j)$$

responsible for the pure $\Delta I = 3/2$ transitions. Upon performing an SU(3) rotation on the $\langle \bar{K}^0 | O_{LL}^{\Delta S=2} | K^0 \rangle$ matrix element, followed by a V-spin rotation, we find that¹²⁾:

$$\langle \bar{K}^0 | O_{LL}^{|\Delta S|=2} | K^0 \rangle_{SU(3)} = \sqrt{2} \langle \pi^0 | O_4 | K^0 \rangle \quad (10)$$

in the SU(3) limit.

Let us check the validity of relation (10) by direct a calculation. Using first the vacuum-saturation method, one finds for Eq. (10),

$$(-ip_\mu^K)(ip_\mu^K) f_K^2 \frac{1}{2E_K} = (-ip_\mu^\pi)(ip_\mu^\pi) f_\pi f_K \frac{1}{\sqrt{4E_\pi E_K}} \quad (11)$$

For the momentum transfer at the $E_\pi = \frac{1}{2}m_K$ point and with the natural assumption that (in the transition π -K matrix element) $p_\mu^\pi = p_\mu^K$, we have for (10) [from (11)]:

$$f_K = \sqrt{2} f_\pi, \quad (\text{experiment: } f_K = 1.25 f_\pi). \quad (12)$$

So relation (10) is satisfied within 25%.

The next proof for the validity of the SU(3) relation (10) is the calculation in the quark model. For any quark model (with $E_\pi = \frac{1}{2}m_K$), we have

$$(a_s - b_s - c_s)_{SU(3)} = (a - 3b), \quad (13)$$

where the integrals a_s, b_s, c_s are known from Refs. 7) and 13). Notice that the SU(2) V-spin rotation [in the SU(3) limit] in the sense of Eq. (10) means:

$$(f_K, m_s) \longrightarrow (f_\pi, m_u) \quad (14)$$

and

$$a_s \xrightarrow{(s,d) \rightarrow (u,u)} a \quad (15a)$$

$$(b_s + c_s) \xrightarrow{(s,d) \rightarrow (u,u)} (3b) \quad (15b)$$

The values of the integrals a, b, a_s, b_s and c_s , the ratio $(a_s - b_s - c_s)/(a - 3b)$ and the parameter B are listed in Table 4 for various quark models. It is clear, from Table 4, that the MIT bag model hardly breaks Eq. (10), while HO quark models (HO and RCHO) satisfy (10) within 25%!

The figure shows nicely the stability of the integrals $(a-3b)_{\text{RCHO}}$ and $(a_S-b_S-c_S)_{\text{RCHO}}$ within the variation of the HO-strength parameter κ , and also hardly confirms Eqs. (10) and (13).

Some additional comments are in order.

- (i) It is important to note that the dependence of the $|\Delta S| = 2$ typical integrals $[(a-3b) \text{ and } (a_S-b_S-c_S)]$ on the typical quark-model parameters (radius for the MIT-bag, harmonic oscillator strength for the HO's) are completely equivalent. This is demonstrated in Table 4 and more dramatically in the Figure.

- (ii) Equation (9) of Ref. 13)

$$\text{sign } M_i^{\text{QM}}(s(r \rightarrow 0)) = \text{sign } M_i^{\text{VS}}, \quad \forall \text{QM}$$

also holds for the $|\Delta S| = 1$, π -K four-quark operator $[(\bar{d}u\bar{u}s)_{\text{VV+AA}}]$ matrix element.

Let us now calculate the matrix element $K^0 \rightarrow \bar{K}^0$ by using Eqs. (7), (10) and (13) and determine the parameter B:

$$\begin{aligned} \langle \bar{K}^0 | O_{\text{LL}}^{\Delta S=2} | K^0 \rangle_{\text{SU}(3)} &= \sqrt{2} \langle \pi^0 | O_4 | K^0 \rangle = 4(a-3b) = \\ &= \frac{f_\pi A \exp(\kappa^+ \rightarrow \pi^+ \pi^0)}{3\sqrt{2} \left(1 - \frac{m_K^2}{m_\pi^2}\right) m_K c_4 \tilde{G}_F} = \frac{0.13 \times 1.83 \times 10^{-8} (\text{GeV}^2)}{3\sqrt{2} \times 0.12 \times 10^{-5} c_4 (\text{GeV}^{-1})} = \\ &= \frac{1}{c_4} 0.468 \times 10^{-3} \text{ GeV}^3 = 1.17 \times 10^{-3} \text{ GeV}^3, \end{aligned} \quad (16)$$

$$B_{\text{SU}(3)}^{\text{PCAC}} = \frac{\langle \bar{K}^0 | O_{\text{LL}}^2 | K^0 \rangle_{\text{SU}(3)}^{\text{PCAC}}}{\langle \bar{K}^0 | O_{\text{LL}}^2 | K^0 \rangle_{\text{v.s.}}} = \frac{0.00117}{0.00428} = 0.273. \quad (17)$$

Optimistically using the fact that total uncertainties (uncertainties in the application of CA and PCAC⁴⁾, the continuation procedure^{1),2)} and the possibility that one part of the $K^0 \rightarrow 2\pi^0$ amplitudes comes from $K^0 \rightarrow \eta\pi^0$ followed by an $\eta \rightarrow \pi^0$ transition¹⁴⁾) in the derivation of (17) are within 17%, we find

$$B_{SU(3)}^{PCAC} = 0.33 , \quad (18)$$

and recover the standard result⁴⁾.

If we are now going back to Eq. (7) and Tables 2 and 3, we see that the RCHO model provides a consistent, reasonable description of all $K \rightarrow 2\pi$ amplitudes, and that, unfortunately, in this case, the amplitude $A(K^+ \rightarrow \pi^+ \pi^0)$ is 6.4×10^{-8} GeV, which is about three times too large. However, if we return (and repeat) the procedure of the evaluation of the $A(K^+ \rightarrow \pi^+ \pi^0)$ amplitude and make, according to the recent upper bound¹⁵⁾ on the matrix element $|\langle \pi | O_4 | K \rangle| < 1.1 \times 10^{-3}$ GeV⁴, by about a factor of three, quench of the $\langle c_4 O_4 \rangle$ matrix element, we roughly obtain

$$A(K^+ \rightarrow \pi^+ \pi^0) \simeq 2 \times 10^{-8} \text{ GeV} , \quad (19)$$

and, consequently,

$$B_{SU(3)}^{PCAC} \simeq 1 . \quad (20)$$

Taking into account that the SU(3) relation (10) is reasonably valid under 25%, and that (CA) and (PCAC) are valid under 10%, we may conclude that:

$$B_{SU(3)}^{PCAC} = 1 \pm 0.35 . \quad (21)$$

This result is strongly supported by a recent analysis by Cea et al.¹⁶⁾.

From the above facts, it is quite clear that the only consistent procedure of using CA, PCAC and SU(3) in the evaluation of B is the one which simultaneously explains $A(K \rightarrow 2\pi)$ amplitudes. Such a procedure gives $B \simeq 1$ in the most natural way.

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Table 1 - Numerical values for quark-model integrals^(a)

Quark model integral [10 ⁻³ GeV ³]	MIT bag model ^(b)			HO quark models ^(c)	
	A	B	C	HO	RCHO
(a-3b)	-1.19	-0.22	-0.09	2.24	1.03
(a+b)	7.52	1.91	4.27	2.24	1.95

- (a) Notice that the negative of the (a-3b)^{A,B,C}_{MIT} integral is a consequence of the dominance of the small components of the confined-quark wave function. Namely,

$$a = \int_{\text{Bag}} d^3r (\mu_u^3 \mu_s + \nu_u^3 \nu_s)$$

contains a pure large component (and a pure small component which we can neglect), while

$$b = \int_{\text{Bag}} d^3r (\mu_u^2 \nu_u \nu_s + \nu_u^2 \mu_u \mu_s)$$

contains mixed large and small components. Such a situation is not present in the case of HO quark models.

- (b) It is possible to find a set of parameters such that the combination (a-3b) vanishes. For example, see the paper by Donoghue et al., Ref. 5). The sets A, B and C of the MIT model parameters⁶⁾ are the same as those used in Ref. 7).
- (c) Here we have used parameters which describe the static properties of hadrons reasonably well:

$$(g_A/g_V)^{pn}, \mu(p), \langle r^2 \rangle_p / \langle r^2 \rangle_\pi, f_\pi, \text{ etc.}$$

Precisely, we have used the harmonic oscillator shape parameter α_ρ to be 0.291 GeV³.

Table 2 - $K \rightarrow 2\pi$ amplitudes calculated for various quark models without penguins.

$A(K \rightarrow 2\pi)$ [10^{-8} GeV]	MIT bag model			HO quark models		Exp.
	A	B	C	HO	RCHO	
$K^+ \rightarrow \pi^+\pi^0$	- 7.41	-1.37	-0.56	13.95	6.41	1.83
$K^0 \rightarrow \pi^+\pi^-$	-17.0	-3.14	-0.71	31.98	14.56	27.96
$K^0 \rightarrow \pi^0\pi^0$	- 6.43	-1.19	-0.27	12.10	5.50	25.36

Table 3 - $K \rightarrow 2\pi$ amplitudes calculated for various quark models with penguins included. The presence of the (AC) term^(a) in $K^0 \rightarrow 2\pi$ amplitudes is indicated^(b).

$A(K \rightarrow 2\pi)$ [10^{-8} GeV]		MIT bag model ^(c)			HO quark models		Exp.
		A	B	C	HO	RCHO	
$K^+ \rightarrow \pi^+\pi^0$		- 7.41	- 1.37	-0.31	13.95	6.41	1.83
Without (AC)	$K^0 \rightarrow \pi^+\pi^-$	3.68	2.14	11.00	38.14	20.07	27.96
	$K^0 \rightarrow \pi^0\pi^0$	14.25	4.09	11.44	18.26	10.92	25.36
With (AC)	$K^0 \rightarrow \pi^+\pi^-$	-16.92	-18.49	-10.16	17.47	-0.57	27.96
	$K^0 \rightarrow \pi^0\pi^0$	- 6.38	-16.54	-9.37	-2.38	-9.70	25.36

(a) Here we have used $m_u+m_d = 15$ MeV, $m_s+m_d = 165$ MeV.

(b) Note here that the presence of the (AC) term (9) in amplitudes (8) together with very low masses of light current quarks, totally destroy our present picture and understanding of $K \rightarrow 2\pi$ decays. This kind of behaviour may be an indication that [in the evaluation of (8) and (9)] one can use not the pure current quark masses, but some kind of "effective and/or constituent" quark masses. That was noted in Ref. 5) five years ago!

(c) Inspection of the results for the MIT bag sets A, B, C shows a large degree of instability. Also, the set B with (AC) gives, up to the over-all factor ~ -1.5 , in good agreement with experiments. This particular set is also unacceptable for the explanations of $K \rightarrow 2\pi$ decays because it destroys the present picture of non-leptonic hyperon and Ω^- decays³⁾.

Table 4 - Mesonic $|\Delta S| = 1$ and $|\Delta S| = 2$ typical integrals for various quark models and the relevant parameter B.

Quark model integrals [10^{-3} GeV^3]		MIT bag model			HO quark models	
		A	B	C	HO	RCHO
$ \Delta S = 1$	a	5.34	1.38	3.18	2.24	1.72
	b	2.18	0.53	1.09	0	0.23
$ \Delta S = 2$	a_S	5.71	1.52	3.41	2.31	1.93
	b_S	2.07	0.50	1.02	0	-0.92
	c_S	4.09	0.97	2.02	0	1.57
$\frac{(a_S - b_S - c_S)}{(a - 3b)}$		0.38	-0.27	-6.76	1.03	1.25
B		-0.42	0.35	0.05	2.16	1.20

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FIGURE CAPTION

Fig. 1 Values of typical $|\Delta S| = 1$ and $|\Delta S| = 2$ integrals are plotted as a function of harmonic oscillator strength κ [see Refs. 2) and 7)]. Solid lines correspond to the $|\Delta S| = 1$ integral

$$I_{\pi^0 \kappa^0}^{QM} = (\alpha - 3b)_{\pi^0 \kappa^0}^{QM}$$

while the dashed lines describe the $|\Delta S| = 2$ integral

$$I_{\kappa^0 \bar{\kappa}^0}^{QM} = (\alpha_s - b_s - c_s)_{\kappa^0 \bar{\kappa}^0}^{QM}$$

where QM; = HO and RCHO quark models. The terms a , b , a_s , b_s and c_s are defined in previous papers^{1)-3), 7)}. The vertical lines (1), (2) and (3) correspond to $\alpha_\rho = 0.25$, 0.291 and 0.32 GeV, respectively. Here we used $m_u = 0.33$ GeV. The value $\alpha_\rho = 0.32$ GeV corresponds to the experimental fit value $\kappa = 0.0106$ GeV³ and was used in the weak non-leptonic interactions¹⁾⁻³⁾. The value $\alpha_\rho = 0.291$ GeV was used in the calculations of Tables 1-4.