MULTIPLICITIES OF CHARGED HADRONS IN 280 GeV/cMUON-PROTON SCATTERING

The European Muon Collaboration

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Abstract

Properties of the hadron multiplicity distributions in 280 GeV/c μ^+p interactions have been investigated. The c.m. energy dependence in the range from 4 to 20 GeV of the total charged multiplicities are presented. No variation faster than logarithmic is seen in the energy range of this experiment. Comparison with νp and $\bar{\nu} p$ data at lower energy has been made and shows good agreement between μ^+p and $\bar{\nu} p$ total charged multiplicities. It has been found that the average forward multiplicity (charged hadrons with $x_F > 0$) exceeds the average backward multiplicity (charged hadrons with $x_F < 0$) in the whole energy range and presents a different energy variation. The average forward multiplicity has been compared to e^+e^- data and shows a similar dependence on energy. Little correlation was observed between the forward and backward multiplicities indicating that the current and target regions fragment almost independently.

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INTRODUCTION

The study of the multiplicity distributions of hadrons produced in high energy collision processes provides a test of the models of hadronization. In the framework of the Quark Parton Model (neglecting the sea quarks) the particles produced in μ^+p deep inelastic scattering originate from the fragmentation of the struck quark and from the remaining diquark. In addition to the Quark Parton Model predictions one may test QCD ideas such as the emission of hard gluons which would change the energy dependence of the mean multiplicities. In particular, the number of forward particles ($x_F > 0$) is a measure of the multiplicity per fragmenting quark and the extra multiplicity due to gluon bremsstrahlung. Similarly, the number of backward particles ($x_F < 0$) is a measure of the multiplicity of the fragments of the remaining diquark.

In this paper we report measurements of the charged hadron multiplicities in μ^+p interactions at hadronic center of mass (c.m.) energies between 4 and 20 GeV and compare with the results of other experiments. The measured multiplicities available so far in lepton nucleon scattering were obtained in ν and $\bar{\nu}$ experiments at c.m. energies lower than 10 GeV [1,2,3] and are directly comparable to μ^+p data. The e^+e^- annihilation experiments also provide a large amount of information on charged hadron multiplicities [4,5] over a wide energy range but only quark fragmentation can be studied with these data which can thus be directly compared with the forward μ^+p results.

1. Experimental Procedure.

a) The General Selection Criteria

The data presented here were taken by the EMC (NA9) experiment, in the M2 muon beam at the SPS at CERN. This experiment consisted of a vertex magnet containing a streamer chamber and a 1 meter long liquid hydrogen target, large angle track chambers and a forward spectrometer which was used to detect and measure the scattered muon and fast charged hadrons. The complete system detected tracks from energies of 280 GeV down to less than 200 MeV, and gave almost 4π coverage in the hadronic c.m. system. In addition, time-of-flight hodoscopes plus four threshold Cerenkov counters gave good particle identification over a wide range of momentum. More details of the experimental set-up and analysis procedures can be found elsewhere [6,7].

The streamer chamber pictures were measured for all events where the reconstructed scattered μ satisfied the trigger conditions, and these measurements were then passed through the geometrical reconstruction programmes. The streamer chamber tracks (SC) covered the low momentum or large emission angle part of the particle spectrum which represented on average 70% of all tracks. The high momentum, small angle tracks were generally not measurable in the streamer chamber as they were obscured by extra beam tracks occurring within the live time of the chamber. These hadron tracks were detected in the forward spectrometer (FS) and reconstructed starting from the most downstream drift chambers placed after the FS-magnet. The large angle part of the spectrometer fulfilled two functions: firstly to improve the parameters of the SC-tracks and FS-tracks and secondly to reconstruct those tracks which were not measured in either the streamer chamber or the forward spectrometer. The primary vertex position was used as an extra constraint in the reconstruction of these large angle spectrometer tracks (called vertex-system (VS) tracks), which contributed mainly to the intermediate region of the particle momentum spectrum.

As the primary vertex was not visible in the streamer chamber, the main vertex position was obtained by fitting the incident and scattered muons and the reconstructed SC and FS tracks to a common interaction point. A search for V^0 decays and γ conversions as well as for secondary interaction vertices was also made amongst the charged hadron tracks detected in the FS and SC. Only such tracks assigned to the primary vertex were used for the analysis presented here.

The kinematic cuts defined in terms of the deep inelastic scattering variables were as follows:

$$\begin{aligned} Q^2 &> 4 \text{ GeV}^2 \\ 4 &< W < 20 \text{ GeV} \\ 20 &< \nu < 260 \text{ GeV} \\ E_{\mu}^{\prime} &> 20 \text{ GeV} \\ y &< 0.9 \\ \theta_{\mu} &> 0.75^\circ \end{aligned}$$

where $-Q^2$ is the square of the 4-momentum transferred between the incident and scattered leptons, ν is the corresponding energy transferred in the laboratory

system, W is the total c.m. energy of the final state hadrons, E_μ' is the energy of the scattered muon in the laboratory system, $y = v/E_\mu$ with E_μ the energy of the incident muon and θ_μ the angle of the scattered muon. These cuts were applied in order to avoid regions where corrections for the acceptance, radiative effects or smearing are large. The number of events remaining after these cuts was 11,700.

b) Acceptances and Track Selection

The distributions presented below were corrected for acceptance losses, detector inefficiencies, incorrect vertex assignments and radiative effects. A detailed Monte Carlo simulation of the experiment was performed in which events were generated using the Lund string model [8], imposing the kinematic cuts given above. A simulation of the raw data was made including simulation of the film measurements as well as the effects of chamber and hodoscope inefficiencies. The tracks produced by γ conversions, K_S^0 , Λ and $\bar{\Lambda}$ decays and secondary interactions of hadrons in the target were also added to the simulated events. The mean values of the acceptance which accounted for losses of tracks were typically 0.90 and incorrect vertex assignments gave a contribution of $\sim 20\%$ extra tracks. Thus the overall apparent acceptance was somewhat greater than 1.0 and dependent on the kinematic range.

The reliability of the Monte Carlo simulation was checked in two ways: firstly, the distributions of all kinematic quantities such as Q^2 , W^2 , momentum and angular distributions of hadrons in the laboratory were compared to those measured. Secondly, comparisons were made between the predicted and observed distributions of hits in the reference planes of the detectors. The data were reasonably well simulated by the Monte Carlo.

These studies led to the following selection criteria for the tracks:

- all streamer chamber (SC) and forward spectrometer (FS) tracks attached to the main vertex were accepted;
- the large angle spectrometer tracks (VS) measured in at least two detectors were accepted.

The uncorrected, measured average multiplicity was 6.93 ± 0.05 charged particles per event, compared to 6.82 ± 0.05 found in the Monte Carlo simulation.

c) Correction of the Multiplicity Distributions

The corrected multiplicity distributions were determined by unfolding the measured multiplicity distributions using a method based on the Monte Carlo simulation. The probability $P(m)$ to observe a given multiplicity m is related to the true multiplicity spectrum, $P(n)^{\text{true}}$, by:

$$P(m) = \sum_n C(n,m) P(n)^{\text{true}}$$

where $C(n,m)$ is the frequency factor (probability) of measuring the total charged multiplicity n as m .

These frequency factors, which accounted for the acceptance and radiative effects, were calculated from the Monte Carlo simulation. The true probabilities $P(n)^{\text{true}}$ were fitted (since charge conservation restricts n to odd values) by solving the matrix equation derived from the minimum χ^2 condition. The calculated values of χ^2 per degree of freedom allowed a test of the reliability of the method. Systematic errors were studied using different track selection criteria and treating the high multiplicity tails in different ways as their frequency factors were poorly determined from the Monte Carlo. The estimated systematic error (which includes the uncertainties in the correction for K_s^0 , Λ , $\bar{\Lambda}$ decays) on the multiplicity is ~ 0.4 .

2. Results

a) Multiplicities of Charged Hadrons

The multiplicity distribution was studied as a function of the c.m. energy W where the true probabilities $P(n)^{\text{true}}$ were determined by the method described above. Table 1 gives the values of $P(n)^{\text{true}}$ for different W ranges where the errors were determined from the diagonal terms of the covariance matrix.

The mean values of the charged particle multiplicities and the values of χ^2 per degree of freedom for each W range are presented in Table 2. The values of this estimator show that the Monte Carlo simulation of the multiplicities does not fully reproduce the data. However, the acceptance corrections are in general small (less than 10%) so that it is still justified to use the Monte Carlo simulation to correct the data.

Fig. 1 shows some examples of the fitted distributions for different W regions. Two Poisson curves are superimposed. The dashed line corresponds to a Poisson distribution of the total number of charged particles:

$$P(n) = 2 \exp(-\langle n \rangle) \frac{\langle n \rangle^n}{n!}$$

while the solid line is obtained from charge conservation under the assumption that the number of negative (positive) particles follows a Poisson law

$$P(n) = \exp\left(-\frac{\langle n \rangle - 1}{2}\right) \frac{\left(\frac{\langle n \rangle - 1}{2}\right)^{\frac{n-1}{2}}}{\left(\frac{n-1}{2}\right)!}$$

where $\langle n \rangle$ is the average charged multiplicity and n an odd integer.

The data are rather well described by the simple Poisson distribution which does not account for charge conservation. A similar behaviour was observed in e^+e^- interactions at comparable energies [4].

In Fig. 2 the average charged multiplicity is shown as a function of W^2 . A linear function of $\ln W^2$ was fitted to the data. As the values of χ^2/NDF (Table 2) are somewhat larger than 1.0, the errors were increased by multiplying by factors equal to $(\chi^2/\text{NDF})^{\frac{1}{2}}$ for this and subsequent fits. These factors varied from 1 to 2 depending on W. A fit was also made including a $\ln^2 W^2$ dependent term. The fit showed that the coefficient for this term was consistent with zero within the error. Thus an extra term proportional to $\ln^2 W^2$ is not required to describe the average charged multiplicity as a function of energy within the energy range of this experiment. The results of the fit, together with ν , $\bar{\nu}$ and e^+e^- results are presented in Table 3. The systematic error on the slope parameter has been evaluated to be ± 0.12 . A comparison with ν , $\bar{\nu}$ data [1,2] shows good agreement between the μ^+p multiplicities and the $\bar{\nu}p$ results

The comparison with e^+e^- [4,5,10], pp and $p\bar{p}$ [14,15] data is shown in Fig. 3. The e^+e^- average charged multiplicity shows a different energy dependence from the μ^+p data which may well be related to the contribution of diquark fragmentation (see below). A difference in the energy dependence is also seen when comparing these data with pp, $p\bar{p}$ results. Note, however, that the results from the e^+e^- , pp and $p\bar{p}$ experiments are not corrected for K_s^0 , Λ , $\bar{\Lambda}$ decays. There is a significant difference in the number of K_s^0 , Λ and $\bar{\Lambda}$ particles produced in e^+e^- and μ^+p interactions. For example the mean number of K_s^0 in e^+e^- annihilations at $W = 14$ GeV is 0.58 ± 0.5 [4] compared to a value of 0.19 ± 0.02 in μ^+p at a similar energy [7].

The dispersion D , defined as $D = (\langle n^2 \rangle - \langle n \rangle^2)^{\frac{1}{2}}$, and the ratio $\langle n \rangle / D$ for each distribution are presented in Table 4 and Figs. 4 and 5. Data from e^+e^- are also shown in Figs. 4 and from e^+e^- , pp and $p\bar{p}$ in Fig. 5 for comparison. The contributions from K_s^0 , Λ and $\bar{\Lambda}$ have been subtracted from the e^+e^- data [10].

A fit of the following form was made to the variation with W of the dispersion observed in this experiment,

$$D = C_1 + C_2 \ln W^2.$$

This fit gave $C_1 = 0.89 \pm 0.53$, $C_2 = 0.30 \pm 0.12$ with $\chi^2/NDF = 0.2$. The ratio $\langle n \rangle / D$ is almost independent of W , consistent with the results from e^+e^- , pp and $p\bar{p}$ experiments. The QCD parton model [13] predicts an asymptotically constant ratio, $\langle n \rangle / D$, independent of W .

In order to test KNO scaling [11], Fig. 6 shows $P(n)\langle n \rangle$ versus $n/\langle n \rangle$ for different W regions. KNO scaling holds to within 20% and the scaled distributions are close to those observed in e^+e^- annihilation [5] and $p\bar{p}$ interactions [15]. The KNO scale breaking recently observed in the UA5 experiment [9] is not seen at the energies of this experiment. The solid curve in Fig. 6 shows a fit to $p\bar{p}$ data [15] and the other curves are KNO functions as predicted by two different models. In the geometrical model of reference [16] (dashed curve) the total multiplicity distribution is a weighted sum of the multiplicity distributions at each impact parameter. In lepton-nucleon reactions this impact parameter is defined as the transverse distance between the struck parton and the centre of mass

of the target remnant. The dashed-dotted curve was originally proposed by Levy [17]. More recently Hayot and Sterman showed [12] that this calculation represents a simplified form of a KNO function, following QCD arguments. In this model a small, Poisson distributed number of clusters is produced. The masses of these clusters grow with energy while the average multiplicity of the clusters stays approximately constant, each cluster decaying into a sharply peaked number of hadrons. Both these models give a good representation of the measured KNO distributions.

b) The Forward and Backward Multiplicities

The charged multiplicities have been studied separately in the forward and backward hemispheres. In the framework of the Quark Parton Model the forward multiplicity is related to the particles coming from the fragmentation of the scattered quark and the backward multiplicity is related to the target remnant system. It is therefore of interest to compare these two multiplicities.

The forward jet was defined as all particles with $x_F > 0$ in the γ^*p c.m. system and the backward jet as those with $x_F < 0$. Each particle was assigned the π mass but the correction for wrong mass assignments has been included in the Monte Carlo. The study of identified particles [7] has shown that these effects were estimated correctly by the Monte Carlo simulation. Radiative effects which change the c.m. energy for some events were taken into account automatically in the Monte Carlo simulation and mostly affected the highest W regions.

For each W range the average forward and backward multiplicities were determined by applying a correction factor to each measured multiplicity separately. This factor was computed from the ratio of the generated Monte Carlo distribution to that obtained after experimental simulation. The total average charged multiplicity was then used as a constraint on the separate forward and backward multiplicities. The values obtained for the forward and backward average multiplicities as a function of W^2 are presented in Table 5 and in Fig. 7.

The average forward multiplicity (fragmenting quark jet) exceeds the average backward multiplicity (diquark jet) in the whole range of W and shows a stronger increase with energy. At the energies available in this experiment this cannot be explained simply by kinematic limitations from the strong production of baryons in

the backward direction. The solid lines in Fig. 7 are fits of the form $\langle n_{F,B} \rangle = a + b \ln W^2$ which describe the data well. The results of the fits are given in Table 6.

In the framework of the Quark Parton Model the virtual photon couples approximately eight times more often to a u quark than to a d quark. Neglecting the contribution of the nucleon sea the remaining system is mostly a (ud) diquark which fragments into particles going backwards in the hadronic c.m. system. Therefore the average forward μ^+p multiplicity is compared in Fig. 8 with that obtained in νp scattering and the backward μ^+p multiplicity with that obtained in $\bar{\nu}p$. For both multiplicities the μ^+p data are in reasonable agreement with the lower energy νp , $\bar{\nu}p$ data [1,2].

In Fig. 9 and Table 5 the dispersions of the forward and backward multiplicities are given as a function of $\langle n_{F,B} \rangle$. The multiplicity distributions resulting from diquark fragmentation seem to be wider than those for a quark jet. This fact is in agreement with other data presented recently by Wroblewski [18]. The dashed line shown in Fig. 9 was obtained from fits to the νp , $\bar{\nu}p$ and e^+e^- results [18] and gives a good representation of the forward data presented here.

Fig. 10 shows the average forward multiplicities multiplied by 2 and the e^+e^- data [4,5,10] from which the contribution of K_s^0 , Λ , $\bar{\Lambda}$ has been subtracted. Good agreement between the two sets of data is obtained. The combined data indicates that the mean multiplicity rises faster than $\ln W^2$. This fact can be explained by QCD effects such as gluon emission. It should be noted that in the energy range of this experiment a linear function in $\ln W^2$ fits the data adequately and there is no need for an additional $\ln^2 W^2$ term. The QCD prediction [19] according to the formula given in Table 7 also gives a good representation of the distribution.

c) Forward-Backward Multiplicity Correlations

The correlations between the forward and backward charged multiplicities has been studied. The average backward multiplicity $\langle n_B \rangle$ was calculated for a fixed forward multiplicity n_F in four W bins and these are presented in Fig. 11. It has been shown in pp experiment [20] that the average multiplicity in one hemisphere is a linear function of the number of particles in the opposite hemisphere, of the form

$$\langle n_B \rangle = a_B + b_B n_F .$$

The results of such linear fits are given in Table 8. The values of the slope b_B indicate little correlation between forward and backward multiplicities for all W bins. This indicates that within the framework of the Quark Parton Model, quark and diquark jets fragment relatively independently of each other.

Conclusions

Multiplicities of secondary charged hadrons in μ^+p interactions in the W range 4 - 20 GeV have been studied. The main conclusions are:

- The average charged multiplicity $\langle n \rangle$ rises linearly with $\ln W^2$ and no variation faster than this logarithmic term is needed to describe the energy dependence within the energy range of this experiment. The measured values are in good agreement with lower energy $\bar{\nu}p$ data. KNO scaling holds to within 20%. The dispersion D is a linear function of $\ln W^2$ and the ratio $\langle n \rangle / D$ is approximately independent of energy.
- The average forward charged multiplicity $\langle n_F \rangle$ exceeds that in the backward direction in the whole W range and shows a stronger rise with energy. The multiplicity distribution resulting from the fragmentation of the diquark are wider than those from the fragmentation of the quark. The average forward and backward multiplicities, $\langle n_F \rangle$ and $\langle n_B \rangle$, are well described by a linear function of $\ln W^2$. The forward multiplicity has a dependence on energy which is similar to the e^+e^- data in the energy range of this experiment.
- The correlations between the forward and backward charged multiplicities are quite weak, implying rather independent fragmentation of the quark and diquark system.

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Table 1

The multiplicity distributions in terms of probabilities
for various W ranges.

n	P(n)	P(n)	P(n)	P(n)
	4<W<6 GeV	6<W<8 GeV	8<W<10 GeV	10<W<12 GeV
1	0.087 ± 0.019	0.063 ± 0.014	0.036 ± 0.009	0.014 ± 0.005
3	0.319 ± 0.035	0.285 ± 0.026	0.217 ± 0.020	0.144 ± 0.019
5	0.366 ± 0.039	0.314 ± 0.029	0.371 ± 0.031	0.381 ± 0.035
7	0.208 ± 0.034	0.240 ± 0.035	0.210 ± 0.031	0.292 ± 0.038
9	0.021 ± 0.020	0.082 ± 0.022	0.144 ± 0.028	0.109 ± 0.029
11		0.015 ± 0.014	0.020 ± 0.007	0.050 ± 0.024
13			0.002 ± 0.001	0.010 ± 0.010
	12<W<14 GeV	14<W<16 GeV	16<W<18 GeV	18<W<20 GeV
1	0.014 ± 0.004	0.014 ± 0.003	0.010 ± 0.002	0.008 ± 0.001
3	0.140 ± 0.017	0.119 ± 0.013	0.094 ± 0.011	0.129 ± 0.012
5	0.263 ± 0.031	0.223 ± 0.024	0.272 ± 0.025	0.218 ± 0.023
7	0.325 ± 0.038	0.324 ± 0.035	0.309 ± 0.039	0.256 ± 0.034
9	0.182 ± 0.037	0.179 ± 0.047	0.135 ± 0.038	0.236 ± 0.039
11	0.059 ± 0.027	0.106 ± 0.041	0.140 ± 0.039	0.072 ± 0.036
13	0.018 ± 0.010	0.031 ± 0.028	0.033 ± 0.031	0.060 ± 0.046
15		0.002 ± 0.002	0.008 ± 0.008	0.015 ± 0.012
17			0.002 ± 0.002	0.011 ± 0.012

Table 2

Average charged multiplicities and values of χ^2 per degree of freedom for different W ranges.

W[GeV]	$\langle n \rangle$	χ^2/NDF
4 - 6	4.51 ± 0.09	7.6/4
6 - 8	5.10 ± 0.05	4.5/5
8 - 10	5.55 ± 0.04	6.0/6
10 - 12	6.06 ± 0.04	7.2/6
12 - 14	6.55 ± 0.06	12.0/6
14 - 16	6.95 ± 0.08	17.5/7
16 - 18	7.18 ± 0.05	8.0/8
18 - 20	7.38 ± 0.10	32.0/8

Table 3

Fitted values of the parameters in the expression
 $\langle n \rangle = a + b \ln W^2$ and comparison with νp , $\bar{\nu} p$ and e^+e^- results

Reaction	W[GeV]	a	b	χ^2/NDF	Ref.
μ^+p	4 - 20	0.31 ± 0.16	$1.21 \pm 0.04 \pm 0.12$ (stat) (syst)	9.8/6	This experiment
νp	3.3 - 11	-0.05 ± 0.11	1.43 ± 0.04	1.5	[2]
	2 - 15	$0.05 \pm .08$	1.42 ± 0.03	0.48	[1]
$\bar{\nu} p$	3.3 - 11	-0.56 ± 0.25	1.42 ± 0.08	0.15	[2]
	2.45 - 11.83	0.02 ± 0.20	1.28 ± 0.08		[3]
e^+e^-	3 - 18	0.04	1.39		[21]

Table 4

Dispersion D and the ratio $\langle n \rangle / D$ for different W ranges.

W[GeV]	D	$\langle n \rangle / D$
4 - 6	1.90 ± 0.18	2.38 ± 0.22
6 - 8	2.20 ± 0.11	2.40 ± 0.12
8 - 10	2.26 ± 0.10	2.43 ± 0.11
10 - 12	2.28 ± 0.16	2.67 ± 0.17
12 - 14	2.41 ± 0.18	2.71 ± 0.19
14 - 16	2.65 ± 0.24	2.62 ± 0.25
16 - 18	2.84 ± 0.13	2.53 ± 0.13
18 - 20	3.04 ± 0.30	2.43 ± 0.30

Table 5

Forward and backward mean charged multiplicities and
dispersions as a function of W.

W[GeV]	$\langle n_F \rangle$	D_F	$\langle n_B \rangle$	D_B
4 - 6	2.26 ± 0.13	1.38 ± 0.15	2.25 ± 0.13	1.29 ± 0.13
6 - 8	2.79 ± 0.08	1.53 ± 0.08	2.31 ± 0.08	1.42 ± 0.08
8 - 10	2.98 ± 0.07	1.64 ± 0.09	2.57 ± 0.07	1.57 ± 0.08
10 - 12	3.26 ± 0.07	1.60 ± 0.09	2.80 ± 0.07	1.78 ± 0.11
12 - 14	3.62 ± 0.09	1.72 ± 0.12	2.93 ± 0.09	1.80 ± 0.12
14 - 16	3.87 ± 0.12	1.73 ± 0.18	3.08 ± 0.12	1.93 ± 0.19
16 - 18	3.99 ± 0.07	1.83 ± 0.09	3.19 ± 0.07	1.93 ± 0.09
18 - 20	4.04 ± 0.13	1.86 ± 0.20	3.34 ± 0.13	2.09 ± 0.20

Table 6

Parameters of the linear fit to the forward and backward multiplicities, $\langle n \rangle = a + b \ln W^2$.

	a	b	χ^2/NDF
$\langle n_F \rangle$	-0.14 ± 0.21	0.73 ± 0.05	6.0/6
$\langle n_B \rangle$	0.60 ± 0.20	0.45 ± 0.04	3.0/6

Table 7

Parameters of the fit of the QCD formula to the twice forward multiplicity.

Parameterisation	parameters	χ^2/NDF
$2\langle n_F \rangle = a + b \exp[c(\ln W^2/Q_o^2)^{\frac{1}{2}}]$	$a = 2.85 \pm 0.23$ $b = 0.091 \pm 0.005$	6.1/6
$Q_o^2 = 1 \text{ GeV}^2, N_f=4 \text{ flavours}$	$c = [72/(33-2N_f)]^{\frac{1}{2}}$	

Table 8

Fit of the relation $\langle n_B \rangle = a_B + b_B n_F$ for different W ranges.

W[GeV]	a_B	b_B	χ^2/NDF
4 - 8	1.89 ± 0.08	0.08 ± 0.03	4.6/4
8 - 12	2.68 ± 0.08	-0.02 ± 0.02	17.3/6
12 - 16	2.97 ± 0.09	0.02 ± 0.02	23.0/6
16 - 20	3.20 ± 0.09	0.05 ± 0.02	10.7/7

Figure Captions

1. Multiplicity distributions for:
 - a) $4 < W < 6$ GeV
 - b) $8 < W < 10$ GeV
 - c) $12 < W < 14$ GeV
 - d) $16 < W < 18$ GeVThe curves are the two Poisson distributions, described in the text, computed for the measured average charged multiplicity.
2. Average charged hadron multiplicity as a function of W^2 . The shaded area corresponds to the estimated systematic errors. The νp , $\bar{\nu} p$ points are taken from Ref. [2].
3. Comparison of the $\mu^+ p$ average charged multiplicity with $e^+ e^-$ [4,5,10], pp , and $p\bar{p}$ multiplicities [9,14,15] uncorrected for K_s^0 and Λ , $\bar{\Lambda}$ decays.
4. The dispersion D of the $\mu^+ p$ charged multiplicity distributions as a function of W and its comparison with $e^+ e^-$ [10], (K_s^0 , Λ , $\bar{\Lambda}$ being subtracted).
5. The ratio $\langle n \rangle / D$ as a function of W for $\mu^+ p$ compared to $e^+ e^-$, pp and $p\bar{p}$ [10,14], K_s^0 , Λ , $\bar{\Lambda}$ contributions being subtracted for $e^+ e^-$.
6. The KNO scaling function $P(n) \langle n \rangle$ as a function of the ratio $n / \langle n \rangle$ for different W regions. The full curve for $p\bar{p}$ annihilation comes from ref. [15]. The theoretical dashed curve comes from ref. [16] and the dash-dotted curve from ref. [17].
7. The average forward ($x_F > 0$) and backward ($x_F < 0$) multiplicities as a function of W^2 . The solid lines are fits of the form $a + b \ln W^2$ (table 6).
8. Comparison of the charged $\mu^+ p$ forward and backward multiplicities with νp , $\bar{\nu} p$ data [2] as a function of W^2 .
9. Dispersions $D_{F,B}$ as a function of the average forward (backward) charged multiplicities. The linear fit is from ref. [18].
10. Twice the average forward $\mu^+ p$ multiplicity and its comparison with $e^+ e^-$ data after subtraction of the K_s^0 , Λ , $\bar{\Lambda}$ contributions to the charged multiplicities [4,5,10].
11. The averaged charged backward multiplicity $\langle n_B \rangle$ as a function of the forward multiplicity n_F for four W bins.

FIG. 1

FIG.2

FIG. 3

FIG. 4

FIG. 5

FIG. 6

FIG. 7

FIG. 8

FIG. 9

FIG. 10

FIG. 11