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ELECTROPRODUCTION OF SUPERSYMMETRIC PARTICLES
AND GAUGE BOSONS

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ABSTRACT

The electroproduction of scalar leptons and gauginos at HERA and at super collider energies is studied. In particular, the channel $ep \rightarrow \tilde{e}(\rightarrow e+\tilde{\gamma})\tilde{\gamma}X$ is analyzed in full detail. The exact analytic expressions for the cross-section and the spectrum of the final electron are derived and evaluated for $\sqrt{s} \approx (0.15 \pm 3)$ TeV and different values of the scalar electron and photino masses. Since the exact formulae are relatively complicated, simplified expressions in the Weizsäcker-Williams approximation are also given and their accuracy is discussed by comparison with the exact formulae. The standard processes $ep \rightarrow Z(\rightarrow \nu\bar{\nu})eX$ and $ep \rightarrow W(\rightarrow e\bar{\nu})\nu X$ which also lead to one final electron and two invisible particles are considered in parallel. The corresponding cross-sections and distributions are discussed in comparison with the supersymmetric signal. The Z^0 cross-section is found to be quite smaller than was reported previously. To clarify this point the Z^0 production at the leptonic vertex was computed both exactly and in the Weizsäcker-Williams approximation with quite consistent results.

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1. - INTRODUCTION

Electroproduction at large enough energies is an optimal reaction for production and detection of possible new particles carrying the electron lepton number. In this paper we concentrate our attention on the search for scalar partners of electrons and neutrinos, as predicted by supersymmetric (SUSY) theories¹⁾. A scalar lepton ($\tilde{e}$ or $\tilde{\nu}$) is always expected to be produced in association with a second SUSY particle because of R-invariance which is normally respected in SUSY models. The second SUSY particle can be either a scalar quark ($\tilde{q}$) or a gaugino, i.e., a photino ($\tilde{\gamma}$), a gluino ($\tilde{g}$), a W-ino ($\tilde{w}$) or a Z-ino ($\tilde{z}$).

The associated production of a scalar lepton and a scalar quark, arising from the parton subprocesses

$$e + q \rightarrow \tilde{e} + \tilde{q} \quad (1)$$

$$e + q \rightarrow \tilde{\nu} + \tilde{q}' \quad (2)$$

was computed in Ref. 2). Numerical predictions at HERA energies ($\sqrt{s} \sim 300$ GeV) and beyond (including e-P supercolliders with $\sqrt{s} \approx 1.2$ TeV) have recently been derived^{3),4)}. These processes have detectable cross-sections, as shown in Fig. 1, provided that both the scalar lepton and the scalar quark are sufficiently light. Scalar electrons are more easily detectable than scalar neutrinos, through their dominant decay mode:

$$\tilde{e} \rightarrow e + \tilde{\gamma} \quad (3)$$

As for the scalar neutrinos, they are mostly expected⁵⁾ to decay into the invisible channel

$$\tilde{\nu} \rightarrow \nu + \tilde{\gamma} \quad (4)$$

On the other hand, if the scalar quarks are much heavier than some of the gauginos, then the associated production of a scalar lepton and a gaugino may become the most important process. At the parton level the most promising channel of this class appears to be:

$$e + q \rightarrow \tilde{e} + \tilde{\gamma} + q \quad (5)$$

$$\quad \quad \quad \downarrow$$

$$\quad \quad \quad e + \tilde{\gamma}$$

The relevant Feynman diagrams are shown in Fig. 2a,b. This channel is particularly favourable because the photino is expected to be the lightest gaugino. Moreover, in this case (as opposite to the case of gluinos), the production of both SUSY particles can take place at the leptonic vertex (Fig. 2a) via the exchange of a photon of small virtual mass which leads to a larger cross-section.

A numerical evaluation of process (5) at HERA energies has been discussed in Ref. 6). In this article we present a detailed analytic study of the production cross-section and of the momentum distributions for the final $\tilde{e}$ decay. We work on the assumption that the scalar quarks are quite heavy, so that the associated production channel $\tilde{e} + \tilde{q}$ is forbidden or strongly suppressed by phase space. In this case, as already stated, the $\tilde{e} + \tilde{\gamma} + q$ channel becomes the dominant production process. One can then neglect the production of the $\tilde{\gamma}$ (or $\tilde{g}$) at the quark vertex (Fig. 1b). We derive the exact analytic expression for the lowest order cross-section and the momentum distributions for $\tilde{e} + \tilde{\gamma}$ production at the leptonic vertex. Since the resulting formulae are rather complex, we furthermore discuss in detail the Weizsäcker-Williams (W-W) approximation for the same process which leads to a simple and reasonably accurate description of the cross-section and the interesting distributions. By comparison with the exact results, we can establish the quantitative accuracy of the W-W approximation.

The discussion of $\tilde{e} + \tilde{\gamma}$ production is then completed by considering the standard processes that lead to a similar observable signal. We thus consider the possibility of W or Z^0 production followed by suitable leptonic decays, according to the following chains of production and decay:

$$e + q \rightarrow e + Z^0 + q \quad (6)$$

$$\quad \quad \quad \downarrow$$

$$\quad \quad \quad \nu + \bar{\nu}$$

$$e + q \rightarrow \nu + W + q \quad (7)$$

$$\quad \quad \quad \downarrow$$

$$\quad \quad \quad e + \nu$$

As is the case for the SUSY process (5), also these standard channels can lead to an electron plus two invisible particles in the final state. The cross-section for production at the leptonic vertex of W and Z^0 (Figs. 3a and 4a

respectively) are estimated and compared with the SUSY signal. For values of the $\tilde{e}$ mass close to the W and Z^0 masses, the rates of the three chains (5), (6) and (7) are found to be of the same order. The signal from $\tilde{e}$ production can in general be disentangled from that arising from W or Z^0 production by studying the energy spectrum or the angular distribution of the final electron. For example, we derive detailed analytic and numerical predictions for the electron energy spectrum in the three cases. An additional discrimination criterion can be obtained on the basis of the other decay modes of W 's and Z^0 's. For example, W 's and Z^0 's decay with equal probability into electrons and muons, while $\tilde{e}$'s only decay into electrons.

The calculation of the W and Z^0 production cross-sections is interesting in itself. In particular we find that the Z^0 production cross-section has been much overestimated in the literature^{7),8)}. In order to clarify this issue, we have computed the cross-section for Z^0 production at the leptonic vertex exactly and in the W - W approximation with completely consistent results. The Z^0 can also be produced at the hadronic vertex (Fig. 4b), with comparable contribution to the total cross-section^{*)}. At sufficiently high energies, the two components peak in

different regions of phase space and the interference is small. The contributions from the leptonic and the hadronic vertices can then be distinguished on the basis of the electron transverse momentum distribution. We mainly concentrate our attention on the production at the leptonic vertex, because it is precisely this component that has been overestimated in the past. Also it is the one that can best simulate the SUSY process of $\tilde{e} + \tilde{\gamma}$ production.

The values of all the total cross-sections discussed here are affected by some uncertainty which is attached to the photon exchange diagram, i.e., the dominant contribution. In fact, the limit of integration over the virtual photon mass cannot be extended up to zero (the value appropriate for massless quarks and leptons) because the integral diverges. It is of course important to handle this divergence correctly, as we now discuss.

Consider the general diagram of photon exchange referring to a given production process at the leptonic vertex (Fig. 5). We neglect the electron and

*) Note that also W production can occur at the hadronic vertex via one-photon exchange. In this case the final lepton is an electron and not a neutrino: $ep \rightarrow eW^+X$. This process was studied in Refs. 7) and 9). In the last paper, the cross-section is certainly overestimated.

quark masses in the following and denote by t the opposite of the virtual photon mass squared

$$t = -q^2 \quad (8)$$

We note that for all inelastic channels, the double pole at $t = 0$ from the photon propagator squared is reduced to a single pole by the corresponding vanishing of the numerator at $t = 0$ due to gauge invariance. In fact, the hadronic tensor obtained by squaring the quark current is proportional to

$$H_{\mu\nu} \approx p_\mu k_\nu + p_\nu k_\mu - g_{\mu\nu} \frac{t}{2} \quad (9)$$

where k and p are the initial and final quark four-momenta respectively (see Fig.5). The point $t = 0$ corresponds to the forward direction. In the inelastic case, the photon four momentum $q = k - p$ is different from zero in the forward direction and k , p and q are all proportional to each other. Thus at $t = 0$, $H_{\mu\nu}$ is proportional to $q_\mu q_\nu$. Consequently, by gauge invariance, the product of $H_{\mu\nu}$ with the leptonic tensor vanishes in the forward direction (contrary to what is true in the elastic case). Apart from possible logarithms (which are present in the Z^0 case, for example), the differential cross-section near $t \approx 0$ is at most as singular as:

$$\frac{d\sigma}{dt} \approx \frac{A(s)}{t} \quad (10)$$

This result is the basis for the W-W approximation which we consider in Section 3. Analytic or numeric mistreatments of this delicate behaviour near $t \approx 0$ may lead to gross overestimates of the cross-sections. This reduced singularity near $t \approx 0$, also alleviates the need of a precise identification of the lower limit (denoted by t_{cut}) of the integration over t . The dependence on t_{cut} is in fact logarithmic as a consequence of Eq. (10). Fixing t_{cut} by giving, for example, a finite mass to the initial quark, taken as free, does not make much sense because the parton model eventually breaks down at small t . Thus a value of t_{cut} of order 1 GeV^2 appears more reasonable. In fact, on the one hand the parton model is still sufficiently accurate at such a scale. On the other hand, 1 GeV^2 is close enough to all possible hadronic scales (target mass, final jet inelasticity and the like) that eventually act as regulators. Thus all numerical calculations are given in the following for

$$t_{\text{cut}} \approx 1 \text{ GeV}^2 \quad (11)$$

This article is organized as follows. In Section 2 we collect the exact analytic expressions for electroproduction of $\tilde{e} + \tilde{\gamma}$. In Section 3 we discuss the three processes of e , Z^0 and W production in the W - W approximation. In Section 4 we discuss the numerical results. Section 5 is devoted to our conclusions. Finally the results of the exact calculation of Z^0 production at the leptonic vertex are described in the Appendix.

2. - ANALYTIC RESULTS FOR $\tilde{e} + \tilde{\gamma}$ PRODUCTION

We now describe the analytic results for the process

$$e^- + p \rightarrow \tilde{e}^- + \tilde{\gamma} + X \quad (12)$$

Two scalars¹⁾ $\tilde{f}_L, \tilde{f}_R$ are associated to each fermion f (either a quark or a lepton), one for each helicity state of f . The index L or R refers to the left or right-handed helicity of the fermion partner, and $\tilde{f}_L$ and $\tilde{f}_R$ have definite couplings with the specified chirality. In general, the mass eigenstates are superpositions of $\tilde{f}_L$ and $\tilde{f}_R$. Since the mass splittings between the two mass eigenstates are expected to be much smaller than the average masses, we shall neglect them in the following. In the limit of degenerate masses, we shall give the separate cross-section for $\tilde{e}_L$ and $\tilde{e}_R$ production, but all the numerical results will be shown for the sum of the $\tilde{e}_L$ and $\tilde{e}_R$ cross-sections.

The relevant diagrams for process (12) are shown in Fig. 2. The corresponding couplings¹⁾ are given by:

$$\begin{aligned} \mathcal{L} = & -e Q_f \bar{f} \gamma_\mu f A^\mu - e Q_f (\tilde{f}_L^\dagger i \overleftrightarrow{\partial}_\mu \tilde{f}_L + \tilde{f}_R^\dagger i \overleftrightarrow{\partial}_\mu \tilde{f}_R) A^\mu - \\ & - i \sqrt{2} e Q_f \tilde{\gamma} (p_L \tilde{f}_L^\dagger + p_R \tilde{f}_R^\dagger) f + \\ & + \frac{2e}{\sin 2\theta_W} \bar{f} \gamma_\mu \left[\left(I_f^3 - Q_f \sin^2 \theta_W \right) p_L - Q_f \sin^2 \theta_W p_R \right] f Z^\mu + \\ & + \frac{2e}{\sin 2\theta_W} \left[\left(I_f^3 - Q_f \sin^2 \theta_W \right) \tilde{f}_L^\dagger i \overleftrightarrow{\partial}_\mu \tilde{f}_L - Q_f \sin^2 \theta_W \tilde{f}_R^\dagger i \overleftrightarrow{\partial}_\mu \tilde{f}_R \right] Z^\mu \end{aligned} \quad (13)$$

Here Q_f and I_f^3 are the charge and the 3rd weak isospin component of the f supermultiplet, θ_W is the electroweak angle, $p_{L,R}$ are the chirality projectors

$P_{L,R} = (1 \mp \gamma_5)/2$, $\tilde{\gamma}$ is the photino spinor field and A_μ , Z_μ^0 are the photon and Z^0 vector fields respectively.

The resulting matrix elements for $\tilde{e}_L^-$ and $\tilde{e}_R^-$ production from e^- -quark collisions, denoted by M_L and M_R respectively, can be written down in the form (an overall phase factor being omitted)

$$M_{L,R} = \sqrt{2} e^3 \bar{u}(p) \gamma_\mu \left[\frac{Q_q}{(p-k)^2} + a_{L,R} \frac{(I_q^3 \frac{1-\gamma_5}{2} - Q_q \sin^2 \theta_w)}{(p-k)^2 - M_Z^2} \right] u(k).$$

$$\cdot \bar{u}(p_y) P_{L,R} \left[\frac{\hat{p}_y + \hat{p}_s}{(p_y + p_s)^2} \gamma^\mu + \frac{(p_s + \ell - p_y)^\mu}{(\ell - p_y)^2 - \tilde{M}^2} \right] u(\ell) \quad (14)$$

where the labelling of momenta is shown in Fig. 2, Q_q and I_q^3 refer to the target quark, M_Z , $\tilde{M}$ are the Z^0 and $\tilde{e}$ masses respectively, (the $\tilde{\gamma}$ mass will be denoted by $m_{\tilde{\gamma}}$) and

$$a_L = \frac{2 \cos 2\theta_w}{\sin^2 2\theta_w} \quad (15)$$

$$a_R = - \frac{1}{\cos^2 \theta_w} \quad (16)$$

The electron and quark masses are neglected everywhere in this paper.

It is a matter of simple but long algebra to derive the $\tilde{e}$ invariant distribution starting from the matrix elements in Eq. (14). The scalar electron differential production rate can then be folded with the decay distribution for $\tilde{e} \rightarrow e\tilde{\gamma}$ in order to obtain the properties of the observed electron.

The resulting exact $\tilde{e}^-$ distribution, which follows from Eq. (14) after convolution with the quark densities in the target, summed over $\tilde{e}_L^-$ and $\tilde{e}_R^-$, is given by:

$$\begin{aligned}
 E'_s \frac{d\sigma}{d^3p'_s} = & \frac{\alpha^3}{\pi^2 s'} \int_{x_{\min}}^1 \frac{dx}{x} \int_{t_{\min}}^{t_{\max}} \frac{dt}{P_s} \sum_f \left\{ \left[2 \frac{Q_{q_f}^2}{t^2} + (a_L + a_R) \frac{Q_{q_f} I_{q_f}^3 2Q_{q_f} \sin^2 \theta_w}{t(t+M_Z^2)} + \right. \right. \\
 & \left. \left. + \frac{(a_L^2 + a_R^2) (I_{q_f}^3)^2 - 2Q_{q_f} I_{q_f}^3 \sin^2 \theta_w + 2Q_{q_f}^2 \sin^4 \theta_w}{2(t+M_Z^2)^2} \right] (q_f(x, P^2) + \bar{q}_f(x, P^2)) f(s, t, E_s, p_{sL}, p_{sT}) + \right. \\
 & \left. + \left[\frac{(a_L - a_R) I_{q_f}^3 Q_{q_f}}{t(t+M_Z^2)} + \frac{(a_L^2 - a_R^2) I_{q_f}^3 (I_{q_f}^3 - 2Q_{q_f} \sin^2 \theta_w)}{2(t+M_Z^2)^2} \right] \right\}. \quad (17)
 \end{aligned}$$

$$\left(q_f(x, P^2) - \bar{q}_f(x, P^2) \right) g(s, t, E_s, p_{sL}, p_{sT}) \Big\}$$

The separate distributions for $\tilde{e}_L$ and $\tilde{e}_R$ can also be read from Eq. (17) by halving the photon exchange term, proportional to $Q_{q_f}^2$, and by only keeping the a_L or the a_R terms, respectively. $q_f(x, P^2)(\bar{q}_f(x, P^2))$ are the quark (antiquark) densities in the proton, with f a flavour index and P a suitable mass scale to be discussed later. In the electron-quark rest frame, E_s is the $\tilde{e}$ energy, p_{sL} and p_{sT} are the longitudinal and transverse components of the $\tilde{e}$ momentum with respect to the axis specified by the initial electron and p_s is the absolute value of the $\tilde{e}$ momentum:

$$p_s = \sqrt{p_{sL}^2 + p_{sT}^2} = \sqrt{E_s^2 - \tilde{M}^2} \quad (18)$$

The relations of E_s , p_{sL} , p_{sT} , in terms of the corresponding quantities in the electron-proton rest frame, denoted by primed symbols, are given by:

$$E_s = \frac{1}{2\sqrt{x}} [(1+x) E'_s - (1-x) p'_{sL}] \quad (19)$$

$$p_{sL} = \frac{1}{2\sqrt{x}} [(1+x) p'_{sL} - (1-x) E'_s] \quad (20)$$

$$p_{ST} = p_{ST}' \quad (21)$$

The allowed range of E'_s is specified in Eqs. (43) and (44). The kernels f and g are given by:

$$\begin{aligned} f(s, t, E_s, p_{sL}, p_{sT}) = & \left[t(-\tilde{M}^2 + m_Y^2 - (p_s \cdot k) - \frac{t^2}{2}) \right] \mathcal{J}_1 + \\ & + (\tilde{M}^2 - m_Y^2) \left[t(\tilde{M}^2 - 2(p_s \cdot k)) - 4(p_s \cdot k)^2 \right] \mathcal{J}_2 + \frac{1}{2} \left[t(\tilde{M}^2 - m_Y^2 - s + 4(p_s \cdot k) + \right. \\ & + 2(p_s \cdot l)) + t^2 \left. \right] \mathcal{J}_3 + (\tilde{M}^2 - m_Y^2) \left[-s^2 + st - \frac{t^2}{2} \right] \mathcal{J}_4 + \left[4s(\tilde{M}^2 - m_Y^2)(p_s \cdot k) + \right. \\ & + t \left((\tilde{M}^2 - m_Y^2)(-\tilde{M}^2 + m_Y^2 + s - 2(p_s \cdot k)) + 2(p_s \cdot k)(s - 2(p_s \cdot k)) \right) \left. \right] + \quad (22) \\ & + \frac{t^2}{2} \left(-\tilde{M}^2 + 3m_Y^2 + s - 6(p_s \cdot k) - \frac{t^2}{2} \right) \mathcal{J}_5 \end{aligned}$$

$$\begin{aligned} g(s, t, E_s, p_{sL}, p_{sT}) = & t \left\{ -(\tilde{M}^2 + (p_s \cdot k) + \frac{t}{2}) \mathcal{J}_1 + \frac{1}{2} \left[t + \tilde{M}^2 - m_Y^2 - \right. \right. \\ & - s + 4(p_s \cdot k) + 2(p_s \cdot l) \left. \right] \mathcal{J}_3 + (\tilde{M}^2 - m_Y^2) \left(\frac{t}{2} - s \right) \mathcal{J}_4 + \\ & + \left[\tilde{M}^2 (2s - 2(p_s \cdot k)) + 2m_Y^2 (p_s \cdot k) + \frac{t}{2} (s - 3\tilde{M}^2 + m_Y^2 - 2(p_s \cdot k)) - \right. \\ & \left. \left. - \frac{t^2}{2} \right] \mathcal{J}_5 \right\} \quad (23) \end{aligned}$$

Here $\sqrt{s}$ is the e-q centre-of-mass subenergy

$$s = x, S \quad (24)$$

p_{sk} and p_{sl} are given by

$$2(p_s \cdot k) = \sqrt{s} (E_s + p_{sL}) \quad (25)$$

$$2(p_s \cdot l) = \sqrt{s} (E_s - p_{sL}) \quad (26)$$

The quantities $\mathcal{J}_i$ are defined by:

$$\mathcal{J}_1 = \frac{\pi p_s}{2s(\sqrt{s} - E_s - p_{sL}) \sqrt{(\delta - p_1)(\delta - p_2)}} \quad (27)$$

$$\mathcal{J}_2 = \frac{\mathcal{J}_1}{2\sqrt{s}} \left(\frac{1}{\delta - p_1} + \frac{1}{\delta - p_2} \right) \quad (28)$$

$$\mathcal{J}_3 = \frac{\pi p_s}{2s(\sqrt{s} - E_s - p_{sL}) \sqrt{\left(\frac{\sqrt{s}}{2} - p_1\right)\left(\frac{\sqrt{s}}{2} - p_2\right)}} \quad (29)$$

$$\mathcal{J}_4 = \frac{\mathcal{J}_3}{4\sqrt{s}} \left[\frac{1}{\frac{\sqrt{s}}{2} - p_1} + \frac{1}{\frac{\sqrt{s}}{2} - p_2} \right] \quad (30)$$

$$\mathcal{J}_5 = \frac{\mathcal{J}_1 - \mathcal{J}_3}{(\sqrt{s} - 2\delta)\sqrt{s}} \quad (31)$$

where

$$\delta = (s + \tilde{\Pi}^2 - m_f^2 - \sqrt{s} E_s + \sqrt{s} p_{sL} + t) / 2\sqrt{s} \quad (32)$$

$$p_{1,2} = \frac{B \mp \Delta}{A} \quad (33)$$

$$B = (s + \tilde{M}^2 - m_{\tilde{\gamma}}^2 - 2\sqrt{s} E_s)(\sqrt{s} - E_s - p_{sL}) / 2 p_s^2 + t \left[p_s - \frac{p_{sL}(\sqrt{s} - E_s)}{p_s} \right] / p_s \sqrt{s} \quad (34)$$

$$\Delta = \frac{p_{sT}}{p_s} \left[\frac{t}{\sqrt{s} p_s^2} (s + \tilde{M}^2 - m_{\tilde{\gamma}}^2 - 2\sqrt{s} E_s)(\sqrt{s} - E_s - p_{sL}) - \frac{t^2}{s} \left[\left(\frac{\sqrt{s} - E_s}{p_s} \right)^2 - 1 \right] \right]^{1/2} \quad (35)$$

$$A = (\sqrt{s} - E_s - p_{sL})^2 / p_s^2 \quad (36)$$

The integration limits in Eq. (17) are fixed as follows. The lower limit of the t integration, t_{cut} , is specified in Eq. (11). The upper limit is given by:

$$t_{\text{max}} = \frac{\sqrt{s} (s + \tilde{M}^2 - m_{\tilde{\gamma}}^2 - 2\sqrt{s} E_s) / (\sqrt{s} - E_s - p_{sL})}{s + \tilde{M}^2 - 2\sqrt{s} E_s} \quad (37)$$

The value of x_{min} in Eq. (17) is determined by the condition

$$t_{\text{max}} \geq t_{\text{cut}} \quad (38)$$

For example, in the case $m_{\tilde{\gamma}} \approx 0$ the result is

$$x_{\text{min}} = \text{MAX} \left\{ \frac{\sqrt{s} (E_s' - p_{sL}') - \tilde{M}^2}{\sqrt{s} (\sqrt{s} - E_s' - p_{sL}')} ; \frac{t_{\text{cut}}}{\sqrt{s} (\sqrt{s} - E_s' - p_{sL}')} \right\} \quad (39)$$

The distribution of the final electron arising from the decay $\tilde{e} \rightarrow e \tilde{\gamma}$ is obtained by:

$$E_e \frac{d\sigma}{d^3k_e} = \frac{\tilde{M}^2}{\pi E_e (\tilde{M}^2 - m_\gamma^2)} \int_{(E'_s)_{\min}}^{(E'_s)_{\max}} dE'_s \int_{c_{\min}}^{c_{\max}} dc \frac{E'_s \frac{d\sigma}{d^3p'_s}}{[(c_{\max} - c)(c - c_{\min})]^{1/2}} \quad (40)$$

where $E'_s(d\sigma/d^3p'_s)$ is given in Eq. (17). In the (initial) electron-proton rest frame, E_e and $\vec{k}_e$ are the final electron energy and momentum, and

$$c_{\max, \min} = \cos \theta_e \cos \chi \pm \sin \theta_e \sin \chi \quad (41)$$

where θ_e is the angle between the initial and final electron and

$$\cos \chi = \frac{2E'_s E_e - \tilde{M}^2 + m_\gamma^2}{2E_e p'_s} \quad (42)$$

The integration limits on the $\tilde{e}$ energy are given by:

$$(E'_s)_{\max} = \frac{s + \tilde{M}^2 - m_\gamma^2}{2\sqrt{s}} \quad (43)$$

$$(E'_s)_{\min} = \frac{4\tilde{M}^2 E_e^2 + (\tilde{M}^2 - m_\gamma^2)^2}{4(\tilde{M}^2 - m_\gamma^2) E_e} \quad (44)$$

Finally, we note that the allowed range of E_e is specified by:

$$E_e^{\max, \min} = \frac{(\tilde{M}^2 - m_\gamma^2)}{4\sqrt{s'} \tilde{M}^2} \left[s' + \tilde{M}^2 + m_\gamma^2 \pm \sqrt{(s' + \tilde{M}^2 - m_\gamma^2)^2 - 4s' \tilde{M}^2} \right] \quad (45)$$

This completes the discussion of the differential scalar electron distribution and of the spectrum of the observed electron. We now consider the total cross-section for $\tilde{e}$ production. For this purpose, we found it convenient to rearrange the integrations in the following way:

$$\sigma = \int_{t_{\min}}^{s' - (\tilde{M} + m_\gamma)^2} dt \int_{\frac{1}{s'} t + (\tilde{M} + m_\gamma)^2}^1 dx \int_{(\tilde{M} + m_\gamma)^2}^{s' - t} dQ^2 \frac{d\sigma}{dQ^2 dt} \quad (46)$$

Q is the invariant mass of the $\tilde{e} - \tilde{\gamma}$ system. $d\sigma/dQ^2 dt$ can be written down in the following form, analogous to Eq. (17) for the differential distribution:

$$\begin{aligned} \frac{d\sigma}{dQ^2 dt} = & \frac{\alpha^3}{4\pi s'^2} \frac{\Delta(Q^2, \tilde{M}^2, m_\gamma^2)}{Q^2} \sum_f \left\{ \left[2 \frac{Q_f^2}{t^2} + (a_L + a_R) \frac{Q_{q_f} [I_{q_f}^3 - 2Q_{q_f} \sin^2 \theta_W]}{t(t + M_Z^2)} + \right. \right. \\ & \left. \left. + \frac{a_L^2 + a_R^2}{2} \frac{(I_{q_f}^3 - 2Q_{q_f} I_{q_f}^3 \sin^2 \theta_W + 2Q_{q_f}^2 \sin^4 \theta_W)}{(t + M_Z^2)^2} \right] (q_f(x, p^2) + \bar{q}_f(x, p^2)) \right\} h(s, t, Q^2) \end{aligned} \quad (47)$$

$$+ \left[\frac{(a_L - a_R) I_{q_f}^3 \mathcal{Q} q_f}{t(t + M_Z^2)} + \frac{a_L^2 - a_R^2}{2} \frac{I_{q_f}^3 (I_{q_f}^3 - 2\mathcal{Q} q_f \sin^2 \theta_w)}{(t + M_Z^2)^2} \right]. \quad (47)$$

cont.

$$\cdot \left(q_f(x, p^2) - \bar{q}_f(x, p^2) \right) K(s, t, Q^2) \Big\}$$

Many of the notations have already been explained in connection with Eq. (17).

In addition we have

$$\Delta(x, y, z) = (x^2 + y^2 + z^2 - 2xy - 2xz - 2yz)^{1/2} \quad (48)$$

$$\begin{aligned} h(s, t, Q^2) = & \frac{-t}{Q^2} (2J_1 + 4J_4 + J_8) + \left[\frac{\tilde{M}^2 - m_Y^2}{Q^2} (2s^2 - 2st - \right. \\ & \left. - (\tilde{M}^2 - m_Y^2)t) + \frac{t^2}{2Q^2} (\tilde{M}^2 + m_Y^2) - t(\tilde{M}^2 - m_Y^2) - \frac{1}{2}st \right] J_2 + \\ & + \frac{1}{Q^2} \left[(2t - 4s)(\tilde{M}^2 - m_Y^2) + 2st - t^2 + tQ^2 \right] J_3 + \\ & + (\tilde{M}^2 - m_Y^2)(st + \tilde{M}^2 t - s^2) J_5 + 2(\tilde{M}^2 - m_Y^2)(2s - t) J_6 - \\ & - 4(\tilde{M}^2 - m_Y^2) J_7 + \left[\frac{\tilde{M}^2 - m_Y^2}{Q^4} (st - s^2 - \frac{t^2}{2}) + \frac{t}{2Q^2} (s + \tilde{M}^2 - m_Y^2) + \right. \\ & \left. + \frac{t}{2} \right] J_9 \end{aligned} \quad (49)$$

$$\begin{aligned}
 k(s, t, Q^2) = & t \left\{ -\frac{1}{Q^2} (2J_1 + J_8) + \left[\frac{\tilde{M}^2 + m^2}{Q^2} \left(s - \frac{t}{2} \right) - \tilde{M}^2 - \frac{s}{2} \right] J_2 + \right. \\
 & + \frac{1}{Q^2} \left(t + 2(\tilde{M}^2 - m^2) + Q^2 \right) J_3 + \left[\frac{\tilde{M}^2 - m^2}{Q^4} \left(\frac{t}{2} - s \right) + \frac{1}{2Q^2} \left(s + \tilde{M}^2 - m^2 \right) + \right. \\
 & \left. \left. + \frac{1}{2} \right] J_9 \right\}
 \end{aligned} \tag{50}$$

The explicit forms of the integrals J_i ($i = 1, \dots, 9$) are listed in Table 1. Note that at $t = 0$ both h and k vanish linearly with t . While this property is directly manifest from Eq. (50) for k , in the case of h it takes some little algebra to verify it from Table 1.

3. - THE WEIZSÄCKER-WILLIAMS APPROXIMATION

For many practical purposes, it may be useful to derive simple approximate forms of the previous formulae as they are rather complicated. A convenient framework in this respect is provided by the well-known Weizsäcker-Williams (W-W) approximation, which is found to be rather accurate in the present case. In this section we shall describe the W-W approximation for the $\tilde{e} + \tilde{\gamma}$ production process of interest. By comparison with the exact calculation of the previous section, we shall be able to specify the accuracy limits of the W-W approximation. We shall also derive the W-W formulae for the standard competitor processes of W or Z^0 production at the leptonic vertex so that a comparison of the respective cross-sections and distributions will be possible in Section 4.

We have already seen that for an inelastic process of the form depicted in Fig. 5, the double pole at $t = 0$ in the rate arising from the photon propagator is reduced to a simple pole by a corresponding zero in the numerator. The remaining singularity leads to a logarithmic term in the cross-section. This logarithm becomes large when the available c.o.m. energy is increased at fixed t_{cut} . When the logarithmic term from photon exchange is large enough, the cross-section is well approximated by only taking correctly into account this term.

This is precisely what is done in the W-W approximation where the cross-section is described by a convolution of the probability for emitting a photon from the lower vertex with the corresponding real photon cross-section at the upper vertex.

For a process of deep inelastic electroproduction on protons of the form of interest to us, i.e.: $e + p \rightarrow A(\tilde{M}) + B(\tilde{m}) + X$ where $\tilde{M}$ and $\tilde{m}$ are the masses of the produced particles, the W-W approximation gives:

$$\frac{d\sigma(E)}{d\Omega_{AB}} = \int_{x_{min}}^1 dx \sum_f q_f(x, P^2) \int_{\eta_{min}}^{\eta_{max}} d\eta P_{\gamma q_f}(\eta) \frac{d\sigma^\gamma(x\eta E)}{d\Omega_{AB}} \quad (51)$$

where $d\Omega_{AB}$ refers to some measurement on the particles A and/or B, q_f are the quark densities in the proton, $P_{\gamma q_f}$ is related to the probability of finding a photon in the quark q_f :

$$P_{\gamma q_f}(\eta) = \frac{\alpha}{2\pi} Q_{q_f}^2 \frac{1+(1-\eta)^2}{\eta} \ln \frac{t_{max}}{t_{cut}} \quad (52)$$

and $d\sigma^\gamma(x\eta E)$ is the cross-section for production of particles A and B by a real photon of energy $x\eta E$ (where E is the energy of the proton) in a collision with the incoming electron. For the particular case of the energy spectrum of particle A in the electron-proton centre-of-mass, which is important in the following, one obtains:

$$s^{3/2} \frac{d\sigma}{d\tilde{E}} = \int_{x_{min}}^1 dx \sum_f q_f(x, P^2) \int_{\eta_{min}}^{\eta_{max}} d\eta P_{\gamma q_f}(\eta) \frac{A(\hat{s}, \hat{t}, \hat{u})}{x\eta(1-x\eta)} \quad (53)$$

where $A(\hat{s}, \hat{t}, \hat{u})$ is proportional to the invariant matrix element squared, summed and averaged over the polarizations, and

$$\begin{aligned}
 \hat{s} &= x\gamma s' \\
 \hat{t} &= \tilde{M}^2 - \sqrt{s'} (\tilde{E} - \tilde{p}_L) \\
 \hat{u} &= \tilde{M}^2 + \tilde{m}^2 - \hat{s} - \hat{t}
 \end{aligned} \tag{54}$$

with

$$\tilde{p}_L = \frac{-x\gamma s' + (1+x\gamma)\sqrt{s'} \tilde{E} - \tilde{M}^2 + \tilde{m}^2}{(1-x\gamma)\sqrt{s'}} \tag{55}$$

The integration limits are specified as follows:

$$\begin{aligned}
 x_{\min} &= z_{\min} \\
 \eta_{\min} &= \frac{z_{\min}}{x} \\
 \eta_{\max} &= \min \left\{ 1, \frac{z_{\max}}{x} \right\}
 \end{aligned} \tag{56}$$

where

$$\begin{aligned}
 z_{\max, \min} &= \frac{(\sqrt{s} - \tilde{E})(\sqrt{s} \tilde{E} - \tilde{M}^2 + \tilde{m}^2) - \sqrt{s} (\tilde{E}^2 - \tilde{M}^2)}{\sqrt{s} (s - 2\sqrt{s} \tilde{E} + \tilde{M}^2)} \cdot \\
 &\cdot \left[1 \pm \left(1 + \frac{(s - 2\sqrt{s} \tilde{E} + \tilde{M}^2)(s (\tilde{E}^2 - \tilde{M}^2) - (\sqrt{s} \tilde{E} - \tilde{M}^2 + \tilde{m}^2)^2)}{((\sqrt{s} - \tilde{E})(\sqrt{s} \tilde{E} - \tilde{M}^2 + \tilde{m}^2) - \sqrt{s} (\tilde{E}^2 - \tilde{M}^2))^2} \right)^{1/2} \right]
 \end{aligned} \tag{57}$$

The available range for the energy $\tilde{E}$ is given by

$$\begin{aligned}\tilde{E}_{\min} &= \tilde{M} \\ \tilde{E}_{\max} &= \frac{\sqrt{1 + \tilde{M}^2 - \tilde{m}^2}}{2\sqrt{s}}\end{aligned}\quad (58)$$

Finally the total cross-section is clearly given by:

$$\sigma_{TOT} = \int_{\tilde{E}_{\min}}^{\tilde{E}_{\max}} \frac{d\sigma}{d\tilde{E}} d\tilde{E} \quad (59)$$

However, if one is only interested in the total cross-section then the following alternative expression is more convenient:

$$\sigma_{TOT} = \frac{1}{s} \int_{(\tilde{M}+\tilde{m})^2}^1 dx \sum_f q_f(x, P^2) \int_{(\tilde{M}+\tilde{m})^2}^1 d\eta P_{\gamma q_f}(\eta) \int_{\hat{t}_-}^{\hat{t}_+} d\hat{t} \frac{A(\hat{s}, \hat{t}, \hat{u})}{2\hat{s}^2} \quad (60)$$

with

$$\hat{t}_{\pm} = \frac{1}{2} \left[\tilde{M}^2 + \tilde{m}^2 - \hat{s} \pm \Delta \right] \quad (61)$$

where $\Delta \equiv \Delta(\hat{s}, \tilde{M}^2, \tilde{m}^2)$ is defined in Eq. (48).

Note that $d\sigma/d\tilde{E}$ and σ_{TOT} depend on t_{cut} which appears in Eq. (52) for $P_{\gamma q_f}$. Also note that, as is the case for all leading logarithmic approximations, there is some ambiguity on the choice of t_{max} which also appears in Eq. (52) for $P_{\gamma q_f}$. We have taken

$$t_{max} = x s - (\tilde{M} + \tilde{m})^2 \quad (62)$$

This value corresponds to the maximum value of t at fixed x (and variable η). However, we stress that any other quantity of the same order of magnitude is equally acceptable for t_{max} within the limits of the W-W approximation.

We now discuss the explicit expressions of $A(\hat{s}, \hat{t}, \hat{u})$ for the processes under study. For the $\tilde{e} + \tilde{\gamma}$ case, $\tilde{M}$ is the $\tilde{e}$ mass and $\tilde{m} \equiv m_{\tilde{\gamma}}$. Starting from the diagrams in Fig. 2a, neglecting the Z^0 exchange and considering the real photon subprocess $\gamma + e \rightarrow \tilde{e} + \tilde{\gamma}$, one obtains

$$A_{\tilde{e}+\tilde{\gamma}}(\hat{s}, \hat{t}, \hat{u}) = 4\pi N \alpha^2 \left[\frac{2\tilde{M}^2 - \tilde{m}^2 - \hat{t}}{2\hat{s}} - \frac{(\tilde{M}^2 - \tilde{m}^2)^2}{\hat{s}(\tilde{M}^2 - \hat{u})} + \frac{(\tilde{M}^2 - \tilde{m}^2)\hat{u}}{(\tilde{M}^2 - \hat{u})^2} \right] \quad (63)$$

where $N = 1$ or 2 if only one or both of the separate contributions of $\tilde{e}_L$ and $\tilde{e}_R$ are included.

In the W-W approximation for the $W + \nu$ case, one neglects the diagrams in Fig. 3b where a W is exchanged and similarly those diagrams in Fig. 3a where a Z^0 is exchanged. One then considers the real photon subprocess $\gamma + e \rightarrow \nu + W$. Note that, although the diagrams of Fig. 3a are not in general gauge invariant by themselves, the W-W approximation is consistent because the gauge breaking terms vanish with the photon virtual mass. One obtains:

$$A_{\nu+W}(\hat{s}, \hat{t}, \hat{u}) = \frac{\pi \alpha^2}{\sin^2 \theta_W} \left[\frac{2\tilde{M}^2 - \hat{t}}{\hat{s}} + \frac{2(\hat{t} - \tilde{M}^2 - \frac{\tilde{M}^4}{\hat{s}})}{\hat{s} + \hat{t}} + \frac{2\tilde{M}^4 - 2\hat{s}\hat{t}}{(\hat{s} + \hat{t})^2} \right] \quad (64)$$

In this case $\tilde{M} \equiv M_W$ and $\tilde{m} \equiv m_\nu = 0$.

The $e + Z^0$ case presents one additional subtlety. The diagrams in Fig. 4a describe Z^0 production at the leptonic vertex. Neglecting the Z^0 exchange and considering the photon subprocess $\gamma + e \rightarrow Z^0 + e$ one obtains:

$$A_{e+Z^0}(\hat{s}, \hat{t}, \hat{u}) = \frac{\pi \alpha^2}{\sin^2(2\theta_W)} [(1-4\sin^2\theta_W)^2 + 1] \left[\frac{-\hat{u}}{\hat{s}} - \frac{\hat{s} + \frac{2\tilde{m}^2 \hat{t}}{\hat{s}}}{\hat{u} - |q^2| \frac{\tilde{m}^2}{\hat{s}}} \right] \quad (65)$$

where $\tilde{M} \equiv m_e \approx 0$, $\tilde{m} \equiv M_Z$ and $|q^2|$ is a (small) photon mass. The novel feature is that in the second diagram of Fig. 4a the electron propagator produces a singularity at $u = 0$ if the electron mass is neglected and the photon is real. This singularity is inside the integration domain of Eq. (60) and would produce a logarithmic divergence of the integral. Since the hadronic inelasticity is taken to be larger than $t_{\text{cut}} \sim 0(1 \text{ GeV}^2)$, it follows that $-q^2 \gtrsim t_{\text{cut}}$. As $t_{\text{cut}} \gg m_e$, m_e being the electron mass, the dominant regulating effect is the photon mass and not the electron mass. This is why the photon mass is kept in the singular denominator in Eq. (65). $-|q^2|(\tilde{m}^2/\hat{s})$ is the minimum value of this denominator so that $\hat{u}$ still starts from zero in Eq. (65). Thus, in the $e + Z^0$ case, the following replacement must be made in Eqs. (53) and (60)

$$P_{\gamma q_j}(\eta) A(\hat{s}, \hat{t}, \hat{u}) \rightarrow \frac{\alpha}{2\pi} Q_j^2 \frac{1+(1-\eta)^2}{\eta} \int_{t_{\text{cut}}}^{t_{\text{max}}} \frac{d|q^2|}{|q^2|} A(\hat{s}, \hat{t}, \hat{u}) \quad (66)$$

This form is well behaved at $\hat{u} = 0$. The net effect of the near singularity from the electron propagator is that the cross-sections obtained after integration over $\hat{u}$ behave as $\ln^2 t_{\text{cut}}$ as opposite to the usual dependence as $\ln t_{\text{cut}}$.

Note that in this case, the calculation of the visible electron spectrum is directly obtained from Eq. (53). In the $\tilde{e} + \tilde{\gamma}$ and $W + \nu$ cases one must instead make the convolution with the $\tilde{e}$ and W decays. This is done in general by using Eq. (40). In particular, for the visible electron energy spectrum one finds after integration over the angles:

$$\frac{d\sigma}{dE_e} = \frac{\tilde{M}^2}{\tilde{M}^2 - \tilde{m}^2} \int_{\tilde{E}_{\min}}^{\tilde{E}_{\max}} \frac{d\tilde{E}}{\tilde{p}} \frac{d\sigma}{d\tilde{E}} \quad (67)$$

where $d\sigma/d\tilde{E}$ is given by Eq. (53) and $\tilde{E}$, $\tilde{M}$, $\tilde{p} = \sqrt{\tilde{E}^2 - \tilde{M}^2}$ refer either to the $\tilde{e}$ or to the W . The phase space boundaries can be read off from Eqs. (43)-(45) with the replacements $E'_s \rightarrow \tilde{E}$ and $m_\gamma \rightarrow \tilde{m}$.

4. - RESULTS

We now describe in detail the quantitative results of physical interest that are derived from our analysis. For all the numerical calculations discussed in the present section, the following specifications are valid. For $\tilde{e} + \tilde{\gamma}$ production we always consider the sum of the two cross-sections for $\tilde{e}_L$ and $\tilde{e}_R$ production, with $\tilde{e}_L$ and $\tilde{e}_R$ assumed to be degenerate in mass. The quark and antiquark densities are taken from the recent work of Duke-Owens (set 1)¹⁰⁾. The scale p^2 of parton densities has been chosen in nearly equivalent but different ways in the exact calculations and in the W-W approximations. In the exact formulae we have taken $p^2 = -q^2$, where q is the four momentum of the exchanged photon (or Z^0) which probes the proton structure. This is in fact the definition of the scale in the ordinary leptonproduction experiments where the parton densities are actually measured. In the W-W approximation the cross-section is evaluated for a real photon (the Z^0 exchange is neglected). The only remnant of the virtual photon mass is contained in the logarithmic factor $\ln(t_{\max}/t_{\text{cut}})$ in Eq. (52) which arises from the integration over the dq^2/q^2 distribution of equivalent photon emission from the quark. In the W-W approximate formulae we then identify p^2 with a suitably defined average value of $-q^2$. For the $\tilde{e} + \tilde{\gamma}$ and $W + \nu$ cases we directly take the average over the dq^2/q^2 distribution

$$p^2 = \frac{t_{\max} - t_{\text{cut}}}{\ln t_{\max}/t_{\text{cut}}} \quad (68)$$

In the $Z^0 + e$ case we have seen that an additional $\ln C/|q^2|$ factor is obtained at small q^2 from the leading term of the total cross-section of the $\gamma + l \rightarrow Z^0 + e$ subprocess. The value of C is given by:

$$C = \frac{(\hat{s} - M_Z^2) \hat{s}}{|q^2| M_Z^2} \quad (69)$$

Correspondingly we have set in this case:

$$P^2 = \frac{\int_{t_{\text{cut}}}^{t_{\text{max}}} dq^2 \ln C/q^2}{\int_{t_{\text{cut}}}^{t_{\text{max}}} \frac{dq^2}{q^2} \ln C/q^2} \approx \frac{2 t_{\text{max}} \ln(x S'/M_Z^2)}{\ln^2(t_{\text{max}}/t_{\text{cut}})} \quad (70)$$

where the last expression is valid for $xS \gg M_Z^2$ and $t \gg t_{\text{cut}}$.

Finally, if the contrary is not explicitly stated, we have fixed t_{cut} according to Eq. (11), namely $t_{\text{cut}} \approx 1 \text{ GeV}^2$.

In Fig. 6 the values of the total cross-sections for the process of $\tilde{e} + \tilde{\gamma}$ associated production are plotted for $m_{\tilde{\gamma}} = 0$ and different $\tilde{e}$ masses in the energy range $\sqrt{s} \approx (0.15 \div 3) \text{ TeV}$. Also shown are the total cross-sections for $e + Z^0$ and $\nu + W$ production, multiplied by the relevant branching ratios. The Z^0 curve corresponds to $M_Z \approx 93.8 \text{ GeV}$, $\sin^2 \theta_W \approx 0.217$, $\beta(Z^0 \rightarrow \nu \bar{\nu}) \approx 0.18$ and the exact formula in Eq. (11) which describes Z^0 production at the leptonic vertex. The W curve refers to $M_W \approx 84 \text{ GeV}$, $B(W \rightarrow e \nu) \approx 0.083$ and is obtained from the W - W formulae in Eqs. (60)-(64). We see that at HERA energies, $\sqrt{s} \approx 300 \text{ GeV}$, and realistic integrated luminosities one can aim at observing $\tilde{e}$'s produced via this mechanism if $\tilde{M} \lesssim 100 \text{ GeV}$. In this range of masses for the scalar electrons and for relatively light photinos, the signals from Z^0 and W channels have comparable cross-sections and can be separated away. In particular, we stress that this statement is only true because we found values of the Z^0 production cross-section

much below the level quoted in Refs. 7) and 8)*) . It is also to be noted that at higher energies, in the supercollider domain, one does not gain much in sensitivity to higher $\tilde{e}$ masses, because of the Z^0 and W backgrounds which are very large in comparison with the signal for $\tilde{M} \gtrsim (150 \div 200)$ GeV. It is important in this respect to observe the steeper increase of the $\nu + W$ cross-section in comparison with the other ones in Fig. 6. This behaviour is due to the diagram with the W - W - γ vertex in Fig. 3 where a spin one particle is exchanged in the t channel of the subprocess $e + \gamma \rightarrow \nu + W$.

The dependence of the $\tilde{e} + \tilde{\gamma}$ production cross-section on the photino mass is displayed in Fig. 7 for $\sqrt{s} \approx 300$ GeV and different values of the $\tilde{e}$ mass.

An important question is the stability of the results versus reasonable changes of the value of t_{cut} . As an example, we report in Table 2 the cross-section for $\tilde{e} + \tilde{\gamma}$ production at $\sqrt{s} \approx 300$ GeV, with $\tilde{M} = 50$ GeV and $m_{\tilde{\gamma}} = 0$ as a function of t_{cut} . It is clear from Table 2 that the ambiguity due to t_{cut} is not important. As discussed in the introduction, the actual regulating mechanisms depend on external masses, bound state parameters and so on. In the range of energy considered, a value of t_{cut} of the order of magnitude of 1 GeV^2 is such that the parton model is still reliable and the typical size of hadronic scales is approached. From Table 2 we see that the remaining uncertainty is well inside the overall theoretical error due to other unknowns (parton densities, scaling violation scale, QCD parameters and so on).

The accuracy of the W - W approximation for the total cross-sections can be estimated by a comparison with the exact calculations, which is shown in Tables 3 and 4. The approximation is in general quite satisfactory. It is particularly good when one is far from the kinematical limits and the energy is not too high. For very large values of $\sqrt{s}$ the importance of the neglected Z^0 exchange contribution and of the different treatment of scaling violations is increased and the W - W approximation is less precise.

*)For Z^0 production at the hadronic vertex we find from the W - W approximation values of the total cross-section which are in reasonable agreement with the results of Ref. 8). We obtain, for example, at $\sqrt{s} = 300$ GeV, 500 GeV, 1 TeV, $\sigma(ep \rightarrow eZ^0 X)_{\text{hadronic}} \approx 7.2 \cdot 10^{-38} \text{ cm}^2$, $1.9 \cdot 10^{-37} \text{ cm}^2$, $4.7 \cdot 10^{-37} \text{ cm}^2$, respectively. Note that in this case the value of t_{cut} entering the formula for the equivalent number of photons is of order m_e^2 , where m_e is the electron mass. Correspondingly, most of the final electrons are at very small angles and are lost in the beam pipe.

The channels $\tilde{e} + \tilde{\gamma}$, $e + Z^0$ and $\nu + W$ all lead to an electron at large angles plus missing energy. The separation of the three processes must be made on the basis of the energy spectrum and the transverse momentum distribution of the observed electron, and also by taking advantage of the alternative decay modes of W 's and Z^0 's. As an example, we present here a comparative study of the electron energy spectrum in the three cases. The normalized spectrum of the final electron from $\tilde{e} \rightarrow e + \tilde{\gamma}$ decay, obtained by using the exact formulae in Eqs. (17) and (40), is shown in Figs. 8 and 9 for some typical values of the reaction energy and the $\tilde{e}$ mass. A sample of the corresponding values of the absolute differential cross-sections is reported in Tables 5 and 6. We have taken a unit branching ratio for the mode $\tilde{e} \rightarrow e + \tilde{\gamma}$. The energy distributions are peaked at $E_e \approx \tilde{M}/2$ (for $m_{\tilde{\gamma}} \sim 0$), where E_e is the electron energy in the e - p centre-of-mass frame.

In Fig. 10 the exact electron spectrum from $\tilde{e}$ decay for given values of $\sqrt{s}$ and $\tilde{M}$ is compared with the corresponding W - W approximation. A similar comparison is also shown in Fig. 11 for different values of $\sqrt{s}$ and $\tilde{M}$. As is usually the case, the W - W approximation works better for totally inclusive quantities than for differential distributions. In the present case, it turns out to be reasonably reliable even at large values of E_e where the rate is small.

In Figs. 11 and 12 the spectrum of the electron from $\tilde{e}$ production is compared with the electron spectra from Z^0 or W production. All the spectra in these figures are computed in the W - W approximation, apart from one exact curve for the $\tilde{e} + \tilde{\gamma}$ reaction which is shown for comparison in Fig. 11. One sees, for example, that for $\sqrt{s} \approx 300$ GeV and $\tilde{M} \lesssim 100$ GeV, the Z^0 background is confined at rather small values of the electron energy.

5. - CONCLUSION

We have studied the electroproduction of scalar electrons in association with photinos in the energy range $\sqrt{s} \approx (0.15 \div 3)$ TeV. The cross-sections are rather small. At HERA energies one can observe $\tilde{e}$'s produced by this mechanism if $\tilde{M} < 100$ GeV. The observation of $\tilde{e}$'s of higher mass not only would require non-realistic values of the integrated luminosity but is also made problematic by the background from W and Z^0 production. In this respect, the situation is not much changed by going at higher energies because the background grows at the same rate or faster (in the W case) than the signal. Actually, the results of the present

calculation are more favourable than those one would have obtained from the existing literature. We have in fact demonstrated by an exact calculation that the cross-section for Z^0 production at the leptonic vertex is smaller than previously reported.

If the $\tilde{e}$'s are relatively light, i.e., $\tilde{M} \lesssim 50$ GeV, they will probably be discovered in e^+e^- reactions before HERA operation starts or at LEP I. If $50 \text{ GeV} \lesssim \tilde{M} \lesssim 100 \text{ GeV}$ and the scalar quarks have comparable masses than the scalar electrons, then the associated production of $\tilde{e} + \tilde{q}$ is the best channel for detecting scalar partners of fermions at HERA, as seen from Fig. 1. Finally, if scalar quarks are much heavier than scalar electrons and $\tilde{M} \sim 100 \text{ GeV}$, then the associated production of $\tilde{e} + \tilde{\gamma}$ becomes the most suitable channel for observing scalar electrons at HERA.

APPENDIX

We report the exact results for Z^0 production at the leptonic vertex, as computed from the diagrams in Fig. 4a. The total cross-section for $t > t_{\text{cut}}$ [see Eq. (11)] is given by:

$$\sigma_{Z^0} = \int_{t_{\text{cut}}}^{s-M_Z^2} dt \int_{\frac{t+M_Z^2}{s}}^1 dx \int_{M_Z^2}^{xs-t} dQ^2 \frac{d\sigma_{Z^0}}{dQ^2 dt}(x, t, Q^2, M_Z^2) \quad (\text{A.1})$$

where

$$\begin{aligned} \frac{d\sigma_{Z^0}}{dQ^2 dt} = & \frac{\alpha^3}{4\pi s} \frac{Q^2 - M_Z^2}{Q^2} \sum_f \left\{ \left[\frac{Q_f^2}{t^2} (v_e^2 + a_e^2) + \frac{(v_{q_f}^2 + a_{q_f}^2)(v_e^4 + 6v_e^2 a_e^2 + a_e^4)}{(t+M_Z^2)^2} - \right. \right. \\ & \left. \left. - \frac{2Q_f v_{q_f} v_e (v_e^2 + 3a_e^2)}{t(t+M_Z^2)} \right] (q_f(x, P^2) + \bar{q}_f(x, P^2)) h_Z(s, t, Q^2) + \right. \\ & \left. + 2 \left[\frac{-Q_f q_{q_f} a_e (a_e^2 + 3v_e^2)}{t(t+M_Z^2)} + \frac{4v_e a_e v_{q_f} a_{q_f} (v_e^2 + a_e^2)}{(t+M_Z^2)^2} \right] \right. \\ & \left. (q_f(x, P^2) - \bar{q}_f(x, P^2)) K_Z(s, t, Q^2) \right\} \quad (\text{A.2}) \end{aligned}$$

Most of the notations have already been specified in connection with Eqs. (17) and (47). Here we have in addition the following definitions. The vector and axial couplings of the Z^0 to the fermions are given by:

$$\sigma_e = \frac{-(1 - 4\sin^2\theta_w)}{2\sin(2\theta_w)} \quad ; \quad a_e = \frac{-1}{2\sin(2\theta_w)} \quad (A.3)$$

$$v_{q_f} = \frac{I_{q_f}^3 - 2Q_f\sin^2\theta_w}{\sin(2\theta_w)} \quad ; \quad a_{q_f} = \frac{I_{q_f}^3}{\sin(2\theta_w)} \quad (A.4)$$

The kernels h_{Z^0} and k_{Z^0} are given by the following expressions:

$$h_{Z^0}(s, t, Q^2) = \frac{2t}{Q^2} (2J_1 + 4J_4 + J_8) + \left[\frac{1}{Q^2} (4s(t + M_Z^2)(s - t) + 2t^2 \cdot (t - M_Z^2) + 2M_Z^4 t) + t(-2M_Z^2 + Q^2 - 2s + 2t) \right] J_2 + \frac{4}{Q^2} (M_Z^2 + t)(t - 2s) J_3 +$$

(A.5)

$$+ M_Z^2 (2st - 2s^2 - t^2) J_5 + M_Z^2 (8s - 4t) J_6 - 8M_Z^2 J_7 +$$

$$+ \frac{1}{Q^2} \left(\frac{M_Z^2}{Q^2} (2st - 2s^2 - t^2) + t(M_Z^2 - 2s) \right) J_9$$

$$k_{Z^0}(s, t, Q^2) = t \left\{ \frac{2}{Q^2} (2J_1 + J_8) + \left(\frac{2}{Q^2} (M_Z^2 - t)(t - 2s) + 2M_Z^2 - Q^2 + 2s - 2t \right) J_2 + \right. \\ \left. + \frac{4}{Q^2} (M_Z^2 - t) J_3 + M_Z^2 (t - 2s) J_5 + 4M_Z^2 J_6 + \frac{1}{Q^2} \left(\frac{M_Z^2}{Q^2} (t - 2s) + M_Z^2 - 2s \right) J_9 \right\}$$

(A.6)

where the integrals J_i ($i = 1, \dots, 9$) can be obtained from Table 1 by the replacements $\tilde{M} \rightarrow 0$, $m_\gamma \rightarrow M_Z$.

TABLE CAPTIONS

Table 1 : Explicit form of the integrals J_i , which appear in Eqs. (49) and (50). The following relations hold

$$\begin{aligned} 2(\ell Q) &= Q^2 + t ; \quad 2(kQ) = s - t ; \\ c_x &= 1 - \frac{2sQ^2}{(Q^2 + t)(s - t)} ; \quad s_x^2 = 1 - c_x^2 ; \\ \beta &= 1 + \frac{(\tilde{M}^2 - m_\gamma^2)Q^2}{(\ell Q)(Q^2 - \tilde{M}^2 + m_\gamma^2)} ; \quad \Delta = \Delta(Q^2, \tilde{M}^2, m_\gamma^2) \end{aligned}$$

[defined in Eq. (48)].

Table 2 : Dependence of the total cross-section for $ep \rightarrow e\tilde{\gamma}X$ from t_{cut} .

Table 3 : Comparison of the exact results with the corresponding W-W approximations for the total cross-sections (in nb) of $ep \rightarrow e\tilde{\gamma}X$ and $ep \rightarrow eZ^0X$ ($t_{\text{cut}} \approx 1 \text{ GeV}^2$) [times $B(Z^0 \rightarrow \nu\bar{\nu}) \approx 0.18$].

Table 4 : Comparison of the exact results with the corresponding W-W approximations (in brackets) for the total cross-section of $ep \rightarrow e\tilde{\gamma}X$ ($t_{\text{cut}} \approx 1 \text{ GeV}^2$).

Table 1

$$J_1 = 2\pi(Q^2 - \tilde{M}^2 + m_\gamma^2) \frac{(k \cdot Q)}{Q^2}$$

$$J_2 = \frac{2\pi Q^2}{\Delta(\ell \cdot Q)} \ln \frac{Q^2(\tilde{M}^2 - m_\gamma^2) + (\ell \cdot Q)(Q^2 - \tilde{M}^2 + m_\gamma^2 + \Delta)}{Q^2(\tilde{M}^2 - m_\gamma^2) + (\ell \cdot Q)(Q^2 - \tilde{M}^2 + m_\gamma^2 - \Delta)}$$

$$J_5 = 4\pi(Q^2)^2 \left\{ \left[Q^2(\tilde{M}^2 - m_\gamma^2) + (\ell \cdot Q)(Q^2 - \tilde{M}^2 + m_\gamma^2) \right]^2 - (\ell \cdot Q)^2 \Delta^2 \right\}^{-1}$$

$$J_8 = \frac{2\pi(\ell \cdot Q)}{Q^2} (Q^2 - \tilde{M}^2 + m_\gamma^2)$$

$$J_9 = 4\pi$$

$$J_3 = J_2 \frac{(k \cdot Q)}{2Q^2} (Q^2 - \tilde{M}^2 + m_\gamma^2) (1 - c_x \beta) + 2\pi \frac{(k \cdot Q)}{(\ell \cdot Q)} c_x$$

$$J_4 = J_2 \left(\frac{(k \cdot Q)}{2Q^2} \right)^2 \left[(Q^2 - \tilde{M}^2 + m_\gamma^2)^2 (1 - 2c_x \beta + \beta^2 (c_x^2 - \frac{s_x^2}{2})) + \frac{\Delta^2 s_x^2}{2} \right] + \\ + \frac{\pi(k \cdot Q)^2 (Q^2 - \tilde{M}^2 + m_\gamma^2)}{Q^2(\ell \cdot Q)} \left[2c_x - \beta (c_x^2 - \frac{1}{2} s_x^2) \right]$$

$$J_6 = J_5 \frac{(k \cdot Q)}{2Q^2} (Q^2 - \tilde{M}^2 + m_\gamma^2) (1 - c_x \beta) + J_2 \frac{(k \cdot Q)}{2(\ell \cdot Q)} c_x$$

$$J_7 = J_5 \left(\frac{(k \cdot Q)}{2Q^2} \right)^2 \left[(Q^2 - \tilde{M}^2 + m_\gamma^2)^2 (1 - 2c_x \beta + (c_x^2 - \frac{1}{2} s_x^2) \beta^2) + \frac{\Delta^2 s_x^2}{2} \right] + \\ + J_2 \frac{(k \cdot Q)^2}{2Q^2(\ell \cdot Q)} (Q^2 - \tilde{M}^2 + m_\gamma^2) \left(c_x - (c_x^2 - \frac{1}{2} s_x^2) \beta \right) + \pi \left(\frac{(k \cdot Q)}{(\ell \cdot Q)} \right)^2 (c_x^2 - \frac{1}{2} s_x^2)$$

Table 2

t_{cut} (GeV)	$\sigma(t_{\text{cut}})$ (pb)
0.5	$1.6 \cdot 10^{-2}$
1	$2.35 \cdot 10^{-2}$
5	$1.8 \cdot 10^{-2}$
10	$1.6 \cdot 10^{-2}$
25	$1.3 \cdot 10^{-2}$

Table 3

$\tilde{M} \approx 100 \text{ GeV} \quad \tilde{m}_{\tilde{\gamma}} = 0$					
$\sqrt{s}$ (TeV)	$\tilde{e}$ (Exact)	$\tilde{e}$ (WW)	Z^0 (Exact)	Z^0 (WW)	
0.15	$7.1 \cdot 10^{-9}$	$5.0 \cdot 10^{-9}$	$2.7 \cdot 10^{-7}$	$1.8 \cdot 10^{-7}$	nb
0.30	$1.1 \cdot 10^{-6}$	$9.0 \cdot 10^{-7}$	$7.1 \cdot 10^{-6}$	$6.1 \cdot 10^{-6}$	nb
0.50	$4.2 \cdot 10^{-6}$	$4.3 \cdot 10^{-6}$	$2.15 \cdot 10^{-5}$	$1.9 \cdot 10^{-5}$	nb
1.0	$1.5 \cdot 10^{-5}$	$1.6 \cdot 10^{-5}$	$5.6 \cdot 10^{-5}$	$5.3 \cdot 10^{-5}$	nb
2.0	$3.5 \cdot 10^{-5}$	$3.9 \cdot 10^{-5}$	$1.1 \cdot 10^{-4}$	$1.1 \cdot 10^{-4}$	nb
3.0	$5.1 \cdot 10^{-5}$	$5.8 \cdot 10^{-5}$	$1.4 \cdot 10^{-4}$	$1.5 \cdot 10^{-4}$	nb

Table 4

$\sqrt{s} = 300 \text{ GeV}$					
	$\tilde{m}_{\tilde{\gamma}} = 0$	40	80	120	GeV
$\tilde{M} = 50 \text{ GeV}$	$2.35(2.35) \cdot 10^{-5}$	$9.2(8.3) \cdot 10^{-6}$	$2.05(1.65) \cdot 10^{-6}$	$3.2(2.3) \cdot 10^{-7}$	nb
$= 100 \text{ GeV}$	$1.1 (0.9) \cdot 10^{-6}$	$5.0(3.8) \cdot 10^{-7}$	$9.8 (6.75) \cdot 10^{-8}$	-	nb

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FIGURE CAPTIONS

- Fig. 1 : Total cross-sections³⁾ for the process $ep \rightarrow \tilde{e}qX$ of associated production of a scalar electron and a scalar quark for $m_{\tilde{e}} \approx m_{\tilde{q}}$. The masses of the exchanged gaugino were taken as $m_{\tilde{\gamma}} = 0$, $m_{\tilde{Z}} = 90$ GeV.
- Fig. 2 : Diagrams for the associated production of $\tilde{e} + \tilde{\gamma}$ (or $\tilde{g}$).
- Fig. 3 : Diagrams for W production via $e + q \rightarrow \nu + W + q$ at (a) the leptonic vertex or (b) the hadronic vertex.
- Fig. 4 : Diagrams for Z^0 production via $e + q \rightarrow e + Z^0 + q$ at (a) the leptonic vertex or (b) the hadronic vertex.
- Fig. 5 : General diagram for a production process taking place at the leptonic vertex through photon exchange.
- Fig. 6 : Total cross-sections for $e^-p \rightarrow \tilde{e}\tilde{\gamma}X$ with $t_{\text{cut}} \approx 1 \text{ GeV}^2$, $m_{\tilde{\gamma}} = 0$ and different values of the $\tilde{e}$ mass $\tilde{M}$. Also shown are the total cross-sections for $ep \rightarrow eZ^0X$ [multiplied by $B(Z^0 \rightarrow \nu\nu) \approx 0.18$] and $ep + \nu WX$ [times $B(W \rightarrow e\nu) \approx 0.083$]. The $\tilde{e}\tilde{\gamma}$ and Z^0e cross-sections are evaluated from the exact expressions in Eqs. (46) and (A.1). The Z^0e curve only includes Z^0 production at the leptonic vertex. The $W\nu$ cross-section is obtained in the W-W approximation.
- Fig. 7 : Dependence on the photino mass $m_{\tilde{\gamma}}$ of the total cross-section for $ep \rightarrow \tilde{e}\tilde{\gamma}X$ at $\sqrt{s} = 300$ GeV and different values of the $\tilde{e}$ mass $\tilde{M}$.
- Fig. 8 : Normalized electron spectrum for $ep \rightarrow \tilde{e}(\rightarrow e\tilde{\gamma})\tilde{\gamma}X$ at $\sqrt{s} \approx 300$ GeV, for $m_{\tilde{\gamma}} = 0$ and scalar electron mass values $\tilde{M} = 50$ GeV, 100 GeV. E_e is the electron energy in the e-p centre-of-mass frame.
- Fig. 9 : Normalized electron spectrum for $ep \rightarrow \tilde{e}(\rightarrow e\tilde{\gamma})\tilde{\gamma}X$ at $\sqrt{s} = 1$ TeV, for $m_{\tilde{\gamma}} = 0$ and scalar electron mass values $\tilde{M} = 100$ GeV, 300 GeV. E_e is the electron energy in the e-p centre-of-mass frame.
- Fig. 10 : The exact energy spectrum of the visible electron from the process $ep \rightarrow \tilde{e}(\rightarrow e\tilde{\gamma})\tilde{\gamma}X$ is compared with the W-W approximation for $\sqrt{s} = 1$ TeV and $\tilde{M} = 100$ GeV, $m_{\tilde{\gamma}} = 0$.
- Fig. 11 : Comparison of the visible electron spectrum obtained from the different channels as indicated, for $\sqrt{s} = 300$ GeV. A comparison of the exact spectrum and its W-W approximation is also shown in one case.
- Fig. 12 : Comparison of the visible electron spectrum obtained from the different channels as indicated for $\sqrt{s} = 2$ TeV.

Fig. 1

(a)

(b)

Fig. 2

(a)

(b)

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9

Fig. 10

Fig. 11

Fig. 12