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EXCLUSIVE DECAYS OF HEAVY MESONS INTO BARYONS

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A B S T R A C T

We analyze the decays of heavy meson resonances into baryon-antibaryon final states within the formalism of exclusive processes in QCD. The decay of $J^{PC} = 2^{++}$ states into proton-antiproton pairs is examined in detail. Predictions are presented for the branching ratios with the proton wave function normalized by data for $\psi \rightarrow p\bar{p}$. Predictions for the cross-section of charmed resonance production in $p\bar{p}$ collisions are given. We discuss the feasibility of studying $b\bar{b}$ spectroscopy in $p\bar{p}$ collisions as well as the reliability of our predictions.

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1. - INTRODUCTION

Experimental data on exclusive decay channels of heavy meson resonances (and particularly those of the charmed states) have reached a remarkable degree of accuracy. After the discovery¹⁾⁻⁵⁾ that several - or perhaps most - of these processes can be analyzed by the renormalization group within the context of perturbative QCD, it is a challenge to theorists interested in confronting QCD with experiment to make predictions for such decays.

Several exclusive decays of heavy meson states have already been considered in the literature. Duncan and Mueller⁵⁾, in one of the early papers on the application of the renormalization group to exclusive processes, discussed the decay of $J^{PC} = 0^{++}$ states into meson-antimeson pairs. The same analysis was applied by Chao⁶⁾ to a study of T decays into meson-antimeson pairs. A related paper is that of Chernyak and Zhitnitsky⁷⁾. More recently one of us⁸⁾ has computed exclusive decays of heavy meson states into glueballs.

Of more relevance to the present paper is the calculation of Brodsky and Lepage⁹⁾ of the decay $\phi \rightarrow p\bar{p}$ (isospin violating contributions to this decay were subsequently considered by Claudson, Glashow and Wise¹⁰⁾). The applicability of the formalism of exclusive processes to processes involving baryons was for some time in a bit of dispute^{2),4)} due to the apparent appearance of non-renormalization group controlled logarithms in higher orders⁴⁾. It is now widely accepted that such endpoint singularities are controlled by Sudakov effects.

In this paper we extend the study by Brodsky and Lepage of $\phi \rightarrow p\bar{p}$ decay by examining the exclusive baryon-antibaryon decays of other charmed states.

Interestingly enough, the most immediate test of our results may not be seen in the heavy meson decays, but rather in the inverse process: charmed resonance production in proton-antiproton collisions. Here the formalism of exclusive processes directly gives relations among the different cross-sections. Scaling relations to states involving quarks of heavier masses follow as a simple by-product.

The paper is organized as follows. In Section 2 we derive an expression for the exclusive decay amplitudes following the standard analysis of such processes from perturbative QCD. As will be explained shortly, we confine ourselves to processes that proceed via a two-gluon intermediate state. In order to avoid collinear divergences in the decay rates we exclude the two-gluon decay of the

$J = 1$ p-wave resonance (the χ_1 state), and concentrate on χ_2 decay. A standard factorization procedure is applied to the subprocess $c\bar{c} \rightarrow p\bar{p}$, and this subprocess is then finally projected onto a $J = 2$ p-wave state of polarization $E_{\mu\nu}$. This introduces the derivative of the heavy meson wave function, evaluated at the origin: $R'_p(0)$. As the same factor appears in the total decay rate, it cancels when we compute branching ratios, and our lack of knowledge about the heavy meson wave function is therefore of no relevance. A similar cancellation can be said to take place for the case of the proton-antiproton wave functions, although in this case a comparison with one more process is required. This is explained in detail in Section 2. In Section 3 we compute the 24 Feynman diagrams that contribute to the amplitude $\chi_2 \rightarrow p\bar{p}$, and present the predictions for the decay rate. With a knowledge of the total decay rate $\Gamma(\chi_2 \rightarrow \text{all})$, we can then compute the Breit-Wigner form of the cross-section $\sigma(p\bar{p} \rightarrow \chi_2 \rightarrow \text{all})$. As shown in this section, the extension of these results to states involving heavier quarks (the b quark, and, in principle, the t quark) is trivial. Finally, Section 4 contains a brief summary and a discussion of the reliability of our results. In Appendix A we collect information regarding spinor representations and identities useful for the calculation reported in Section 3.

2. - FORMALISM

The analysis of exclusive hard scattering processes within the framework of perturbative QCD was initiated and developed several years ago¹⁾⁻⁴⁾. The wide range of applicability of this perturbation theory analysis has particularly been stressed by Brodsky and Lepage²⁾. Their notation and formalism is convenient and intuitively easy to understand, and we will therefore make use of that formalism in this paper. A more formal, but basically completely equivalent, analysis can be found in Ref. 4).

A heavy meson decay is, strictly speaking, not a hard scattering process. However, in this paper we shall only be concerned with heavy mesons whose masses are much larger than any low energy hadronic mass scale, which is typically of the order of 0.5-1.0 GeV. If, in this case, the heavy meson is forced to decay via annihilation diagrams, either because of quantum number mismatch between initial and final states or because the decay takes place below threshold, then the process becomes short-distance dominated and all constituents participating

in the decay will have propagators pulled off-shell by an amount of the order of M^2 .

This short-distance dominance can be exploited in a systematic way by a factorization procedure: one separates the full decay amplitude into a short-distance decay amplitude T_H and soft distribution amplitudes $\phi(x_i)$ which are not calculable from perturbation theory. As all propagators involved are off-mass shell by an amount $\sim M^2$, the short-distance decay amplitude T_H can be computed directly from QCD perturbation theory. The distribution amplitudes $\phi(x_i)$, on the other hand, contain all the low energy information about the initial and final state hadrons. This low energy information can only be provided by a comparison with experimental data. However, the distribution amplitudes $\phi(x_i)$ are universal; they do not depend on the particular process one is considering. Information obtained from one known process can therefore be used directly to give predictions for a whole class of different exclusive processes involving the same type of hadrons.

It is convenient, but of course not crucial, to use the language of light cone quantization and to work in a physical gauge such as the light cone gauge, $A^+ = 0$. Hadrons are then generally assigned momenta $(P^+, P^-, 0_\perp)$ as in the infinite momentum frame, and the constituent quarks (and/or gluons) are labelled by their relative longitudinal momenta $x_i \equiv k_i^+/P^+$ and their transverse momenta $k_{i\perp}^{(i)}$. Feynman rules, propagators and vertices in light cone quantization can be worked out in a standard way²⁾. In addition, one must include the effects of the binding of the quarks to form a hadron; this is taken care of by the bound state wave function $\phi(x_i, k_{i\perp}^{(i)})$. The graphical symbol for $\phi(x_i, k_{i\perp}^{(i)})$ is shown in Fig. 1a for a baryon in its three-quark valence state. Also shown in Fig. 1a is the bound state equation $\phi = SK\phi$ which the wave function $\phi(x_i, k_{i\perp}^{(i)})$ satisfies. Here S represents the three-particle propagator and K is a kernel built out of two-particle irreducible (2PI) graphs as illustrated in the Figure.

The wave function $\phi(x_i, k_{i\perp}^{(i)})$ is closely related to the hadronic distribution amplitude $\phi(x_i, Q^2)$, which plays a crucial rôle in this formalism. The lowest-order definition of $\phi(x_i, Q^2)$ for baryons is²⁾:

$$\phi(x_i, Q^2) \equiv \left\{ \ln(Q^2/\Lambda^2) \right\}^{-3\gamma_F/2\beta_0} \int \left[\frac{d^3 k_\perp}{16\pi^3} \right] \psi(x_i, k_\perp^{(i)}) \quad (2.1)$$

where the measure is

$$\left[\frac{d^2 k_{\perp}}{16\pi^3} \right] = \left(\prod_{i=1}^3 \frac{d^2 k_{\perp}^{(i)}}{16\pi^3} \right) 16\pi^3 \delta^2 \left(\sum_{j=1}^3 k_{\perp}^{(j)} \right) \quad (2.2)$$

The logarithmic factor in front of the integral is included in order to incorporate the effect of wave function renormalizations of the quarks. γ_F equals the (gauge dependent) anomalous dimension of the quark fields, and $\beta_0 = 11 - 2n_f/3$, with n_f being the effective number of flavours. As usual, Λ denotes the scale parameter of QCD. To the leading order in which we are working, it cannot be more precisely defined.

As $\phi(x_i, k_{\perp}^{(i)})$ can be viewed as the amplitude for finding n constituents inside the hadron, each carrying a fraction x_i of the hadron's longitudinal momentum and having transverse momentum $k_{\perp}^{(i)}$, it follows that the distribution amplitude $\phi(x_i, Q^2)$ itself represents the probability amplitude for finding the hadron split into n constituents, each carrying fractional longitudinal momenta x_i and collinear up to a scale given by Q^2 .

Although the wave function $\phi(x_i, k_{\perp}^{(i)})$, and hence $\phi(x_i, Q^2)$, cannot be computed from perturbation theory, a Q^2 evolution equation can be derived from QCD perturbation theory²⁾. We have illustrated the derivation of this evolution equation graphically in Fig. 1b. When large Q^2 flows through the wave function $\phi(x_i, k_{\perp}^{(i)})$, the perturbative expansion - inherent to the formalism of the bound state equation - is meaningful, since the associated running coupling constant,

$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda^2)} + \dots \quad (2.3)$$

will be small. Furthermore, the dominant contribution to the $k_{\perp}$ -integrations will come from the region $k_{\perp}^2 < Q^2$. To leading order, then, we can neglect $k_{\perp}^2$ with respect to Q^2 in all propagators, and the kernel, approximated by its value at $k_{\perp} = 0$, can be factored out of the integral. As can be seen, Fig. 1b now illustrates an evolution equation^{*)}:

$$Q^2 \frac{\partial}{\partial Q^2} \phi(x_i, Q^2) = \frac{\alpha_s(Q^2)}{4\pi} \int_0^1 [dy] V(x_i, y_i) \phi(y_i, Q^2) / y_1 y_2 y_3 \quad (2.4)$$

where $V(x_i, y_i)$ to lowest order is a symmetric one-gluon exchange kernel,

*) The measure equals $[dy] = dy_1 dy_2 dy_3 \delta(\sum_{i=1}^3 y_i - 1)$, where the delta function simply ensures (longitudinal) momentum conservation.

$$V(x_i, y_i) = 2x_1 x_2 x_3 \sum_{i \neq j} \Theta(y_i - x_i) \delta(y_k - x_k) \frac{y_i}{x_j} \left(\frac{\delta_{h_i h_j}}{x_i + x_j} + \frac{\Delta}{y_i - x_i} \right) \quad (2.5)$$

Here $\delta_{h_i h_j}$ is a Krönecker delta in quark helicities and Δ is defined by the way it operates on $\tilde{\phi} \equiv \phi/x_1 x_2 x_3$:

$$\Delta \tilde{\phi}(y_i, Q^2) \equiv \tilde{\phi}(y_i, Q^2) - \tilde{\phi}(x_i, Q^2) \quad (2.6)$$

The evolution equation (2.4) and the kernel (2.5) were first derived by Brodsky and Lepage²⁾. Introducing a set of polynomials $\tilde{\phi}_n(x_1)$, orthogonal on the unit cube with respect to the weight function $W(x_1) = x_1 x_2 x_3 \delta(\sum_{i=1}^3 x_i)$, they also found the solution to Eq. (2.4)²⁾:

$$\phi(x_i, Q^2) = x_1 x_2 x_3 \sum_{n=0}^{\infty} a_n \tilde{\phi}_n(x_i) [\ln(Q^2/\Lambda^2)]^{-\gamma_n/\beta_0} \quad (2.7)$$

The exponents γ_n , computed directly by diagonalizing the kernel (2.5), were subsequently¹¹⁾ shown to equal the anomalous dimensions of the lowest twist three-quark operators with the baryon quantum numbers, as expected.

Although the formal solution (2.7) for $\phi(x_1, Q^2)$ can be found, the coefficients a_n in the expansion (2.7) are unknown and not computable in perturbation theory. They represent (soft) matrix elements between the baryons and appropriate three-quark operators. Nevertheless, as the polynomials $\tilde{\phi}_n(x_1)$ form a complete orthogonal basis, once $\phi(x_1, Q^2)$ is known at some fixed $Q^2 = Q_0^2$, $\phi(x_1, Q^2)$ can be computed at any other Q^2 from the expansion (2.7).

Note also that for asymptotically large Q^2 the distribution amplitude for the proton takes the following form:

$$\phi(x_i, Q^2) \rightarrow C x_1 x_2 x_3 [\ln(Q^2/\Lambda^2)]^{-2/3\beta_0} \quad (2.8)$$

as $Q^2 \rightarrow \infty$. Here C is an unknown constant.

Having thus briefly reviewed how one can describe the proton-antiproton final state in this formalism, we turn our attention to the actual decay $\chi_2 \rightarrow p \bar{p}$ of the heavy meson. Since we consider only decays into systems containing u and d quarks, we may view the final state quarks as massless at high momentum transfer. In this limit quarks and antiquarks can annihilate through gluons only when they have opposite helicities, which in turn means that the produced $p \bar{p}$ pair has

$|\lambda - \bar{\lambda}| = +1$, λ being the helicity of the proton. Helicity rules out the 3P_0 state (suppressed by $\sim m^2/Q^2$). Similarly, since $S = |\lambda - \bar{\lambda}| = +1$, the CP of the $p\bar{p}$ system is $(-1)^{L+S}(-1)^{L+1} = +1$, whereas CP of the 1P_1 state is -1 . Thus $p\bar{p}$ decays of this state are ruled out also. In our approximation, the final p and $\bar{p}$ are treated as massless.

We omit a discussion of similar decays from the 3P_1 state, even though, as far as quantum numbers are concerned, such decays are allowed. However, in this case the total decay rate is complicated by collinear divergences^{12),13)} and calculations involving this state will not be reliable.

As the quantum numbers of the χ_2 state are $J^{PC} = 2^{++}$, the χ_2 will decay strongly to lowest order in $\alpha_s(Q^2)$ via a two-gluon intermediate state, as shown in Fig. 2.

The decay $\chi_2 \rightarrow p\bar{p}$ can be described by a factorization technique very similar to the one already discussed for the distribution amplitudes $\phi(x_1, Q^2)$. The exclusive decay amplitude for the χ_2 state is shown in Fig. 3a, T_H representing the short-distance decay amplitude where $Q^2 \approx M^2$ (M being the heavy quark mass) is forced to flow through all constituents. For the final state the factorization proceeds as before: in leading order the dominant contribution to the $k_\perp$ -integrations comes from the region $k_\perp^2 < Q^2 \approx M^2$. We can then ignore $k_\perp^2$ compared with Q^2 in all propagators and factor the $k_\perp$ -integrations out on the wave functions $\phi(x_i, k_\perp^{(i)})$ of the p and $\bar{p}$. On account of Eq. (2.1) this precisely builds up the distribution amplitudes $\phi(x_i, Q^2)$ for the baryons. We are then left with an amplitude

$$A = \int_0^1 [dx] [dy] \phi^*(y_i, Q^2) T_H(x_i, y_i) \phi(x_i, Q^2) \quad (2.9)$$

where T_H has an expansion to lowest order as shown in Fig. 3b. It represents the subprocess $c\bar{c} \rightarrow gg \rightarrow p\bar{p}$.

The decaying heavy meson state is treated as follows. First we note that as the masses involved in the quark-antiquark system are very large, it will be a good approximation to treat the heavy meson wave function in the non-relativistic limit^{14),15)}. In general we then have a non-relativistic wave function $\Psi_{LM}(\vec{k})$ with the angular momentum quantum numbers as indicated. To construct the full bound state wave function for the heavy quark-antiquark system we must couple the two spinors u and $\bar{v}$ (in either the singlet $S = 0$, $S_z = 0$ or triplet $S = 1$,

$S_Z = -1, 0, +1$ states) to produce the desired state of total angular momentum J and total angular momentum projection J_Z .

We use the centre-of-mass system for the heavy meson. The quark and anti-quark are given momenta $Q_1 = \frac{1}{2}P+k$ and $Q_2 = \frac{1}{2}P-k$, respectively. Note that $P^2 = 4M^2$ is the invariant mass of the heavy meson. Introducing the Clebsch-Gordon coefficient $\langle LM_L SS_Z | JJ_Z \rangle$, which connects the states (LM_L) and (SS_Z) to the total angular momentum state (JJ_Z) , we have the amplitude

$$T = (2M)^{-3/2} \int \frac{d^3k}{(2\pi)^3} \Psi_{LM_L}(\vec{k}) \langle LM_L SS_Z | JJ_Z \rangle \text{Tr} [A(\vec{k})(M+A_1) \Pi_{SS_Z}(M-A_2)] \quad (2.10)$$

where $A(\vec{k})$ is as defined in Eq. (2.9). Also, Π_{SS_Z} is a polarization projection operator,

$$\Pi_{00} = \gamma_5, \quad \Pi_{1S_Z} = \epsilon(S_Z)$$

for singlet and triplet states, respectively. $\epsilon_\mu(S_Z)$ is the standard spin-1 polarization vector.

For the $J = 2$ state we are interested in, the associated polarization tensor $E_{\mu\nu}(J_Z)$ has the following properties:

$$\begin{aligned} E_{\mu\nu} &= E_{\nu\mu} \\ E^\mu{}_\mu &= 0 \\ P^\mu E_{\mu\nu} &= 0 \\ \sum_{\text{pol}} E_{\mu\nu} E_{\sigma\rho} &= \frac{1}{2} [P_{\mu\sigma} P_{\nu\rho} + P_{\nu\sigma} P_{\mu\rho}] - \frac{1}{3} P_{\mu\nu} P_{\sigma\rho} \end{aligned} \quad (2.11)$$

where

$$P_{\mu\nu} = -g_{\mu\nu} + \frac{P_\mu P_\nu}{4M^2} \quad (2.12)$$

Note also that two integrals of the non-relativistic wave function $\Psi_{LM_L}(\vec{k})$ can be expressed in terms of the standard radial wave function and its derivative, at the origin:

$$\int \frac{d^3k}{(2\pi)^3} \Psi_{00}(\vec{k}) = \frac{1}{\sqrt{4\pi}} R_s(0) \quad (2.13)$$

$$\int \frac{d^3k}{(2\pi)^3} k^\alpha \Psi_{1M_1}(\vec{k}) = -i\sqrt{\frac{3}{4\pi}} R'_p(0) \epsilon^\alpha(M_1) \quad (2.14)$$

We perform a Taylor expansion of the integrand in Eq. (2.10) around $\vec{k} = 0$. This amounts to a (v/c) -expansion, valid in the non-relativistic limit:

$$\begin{aligned} A(\vec{k})(M+\not{Q}_1) \pi_{SS_2}(M-\not{Q}_2) &\simeq A(0)(M+\frac{1}{2}\not{P}) \pi_{SS_2}(M-\frac{1}{2}\not{P}) \\ &+ k^\alpha [A_\alpha(0)(M+\frac{1}{2}\not{P}) \pi_{SS_2}(M-\frac{1}{2}\not{P}) + A(0)(-\not{\chi}_\alpha) \pi_{SS_2}(M-\frac{1}{2}\not{P}) \\ &+ A(0)(M+\frac{1}{2}\not{P}) \pi_{SS_2}(-\not{\chi}_\alpha)] \end{aligned} \quad (2.15)$$

where we have introduced

$$A_\alpha(0) \equiv \left. \frac{\partial}{\partial k^\alpha} A(\vec{k}) \right|_{k=0} \quad (2.16)$$

Specializing to the case at hand, $L = S = 1$ and $J = 2$, and noting that the first term in the expansion (2.15) only contributes to S-wave decays, we can perform the k -integration in Eq. (2.10):

$$\begin{aligned} T &= (2M)^{-3/2} (-i\sqrt{\frac{3}{4\pi}} R'_p(0) \epsilon^\alpha(M_L)) \langle 1M_L 1S_2 | 2J_2 \rangle \times \\ &T_F [A_\alpha(0)(M+\frac{1}{2}\not{P}) \not{\epsilon}(S_2)(M-\frac{1}{2}\not{P}) \\ &+ A(0) \{ (M+\frac{1}{2}\not{P}) \not{\epsilon}(S_2)(-\not{\chi}_\alpha) + (-\not{\chi}_\alpha) \not{\epsilon}(S_2)(M-\frac{1}{2}\not{P}) \}] \end{aligned} \quad (2.17)$$

Now

$$(M+\frac{1}{2}\not{P}) \not{\epsilon}(S_2)(M-\frac{1}{2}\not{P}) = M \not{\epsilon}(S_2)(2M-\not{P}) \quad (2.18)$$

since $\epsilon_\mu(S_2)$ is purely spacelike.

Furthermore,

$$(M+\frac{1}{2}\not{P}) \not{\epsilon}(S_2) \not{\chi}_\alpha + \not{\chi}_\alpha \not{\epsilon}(S_2)(M-\frac{1}{2}\not{P}) = 2M \epsilon_\alpha(S_2) + \frac{1}{2}\not{P} [\not{\chi}_\beta, \not{\chi}_\alpha] \epsilon^\beta(S_2) \quad (2.19)$$

Then, using

$$\epsilon_\mu(S_2)\epsilon_\nu(M_L)\langle 1M_L 1S_2 | 2J_2 \rangle = \epsilon_{\mu\nu}(J_2) \quad (2.20)$$

as well as Eq. (2.11), we see that the last two terms in Eq. (2.17) vanish identically. As can be seen, the vanishing of these two terms is simply a consequence of the fact that the Clebsch-Gordon coefficient connecting two spin-1 systems to one spin-2 system is symmetric and traceless.

Using Eq. (2.18) we then finally have the full decay amplitude

$$T = -i M^{-1/2} \sqrt{\frac{3}{8\pi}} R'_p(0) \text{Tr} [A_\alpha(0) \delta_\beta (M - \frac{1}{2}\not{P})] \epsilon^{\alpha\beta} \quad (2.21)$$

with $A_\alpha(0)$ as defined in Eqs. (2.9) and (2.16).

3. - RESULTS AND COMPARISON WITH EXPERIMENT

3.1. - The Exclusive Amplitude

The set of lowest order Feynman diagrams contributing to the exclusive $\chi_2 \rightarrow p\bar{p}$ decay amplitude is shown in Fig. 4. There are 24 diagrams altogether. We define a dimensionless reduced short-distance decay amplitude $T_H(x,y)$ by

$$T = -i M^{-1/2} \sqrt{\frac{3}{8\pi}} R'_p(0) (g^{1\beta} - i g^{2\beta}) g^{3\alpha} \epsilon_{\alpha\beta} \int [dx][dy] \phi^*(y) T_H(x,y) \phi(x) \quad (3.1)$$

where T is as given in Eq. (2.21). Here $g^{\alpha\beta}$ is the metric tensor. The decay amplitude $T_H(x,y)$ defined in this manner is completely gauge invariant. This is obvious to lowest order. In higher orders a careful treatment of the heavy meson wave function, as well as the baryonic distribution amplitudes, is needed.

Working in the convenient Feynman gauge, with massless u and d quarks, we use techniques summarized in Appendix A to derive the following amplitudes for the diagrams A-F:

$$T_H(A) = \frac{1}{2} C_T g^6 \frac{x_1 + y_1}{x_1 y_1 x_3 y_3 (1-x_1)(1-y_1) [x_1(1-y_1) + y_1(1-x_1)]^2} \quad (3.2)$$

$$T_H(B) = T_H(A) \quad (3.3)$$

$$T_H(C) = \frac{1}{2} C_T g^6 \frac{x_1 + y_1}{x_1 y_1 x_2 y_2 (1-x_1)(1-y_1) [x_1(1-y_1) + y_1(1-x_1)]^2} \quad (3.4)$$

$$T_H(D) = T_H(C) \quad (3.5)$$

$$T_H(E) = 0 \quad (3.6)$$

$$T_H(F) = 0 \quad (3.7)$$

Here $C_T = 1/8\sqrt{n_c}((n_c+1)/n_c)^2$, n_c being the number of colours, is the common colour factor for all diagrams.

To get the full decay amplitude we then add the diagrams with the "external" gluons I and II interchanged, as well as all diagrams with the quark lines permuted. It turns out that, diagram by diagram, $T_H(I,II) = T_H(II,I)$, so that the full lowest order $T_H(x,y)$ is simply

$$T_H(x,y) = 2C_T g^6 \left\{ \frac{1}{x_3 y_3} + \frac{1}{x_2 y_2} \right\} \frac{x_1 + y_1}{x_1 y_1 (1-x_1)(1-y_1) [x_1(1-y_1) + y_1(1-x_1)]^2} \quad (3.8)$$

Note that $T_H(x,y) = T_H(y,x)$ is symmetric, as it should be.

3.2. The $\chi_2 \rightarrow p\bar{p}$ decay rate

In order to compute the amplitude T from Eq. (3.1), we require the functional form and absolute normalization of the proton distribution amplitude $\phi(x, Q^2)$. As explained in the previous section, this can be accomplished by a comparison with one other process involving protons/antiprotons for which the corresponding calculation is free from ambiguities, and for which reliable data exist. The most obvious choice may be the electromagnetic form factor of the proton²⁾. However, as was discussed in detail in the first paper of Ref. 16), there are several reasons for not choosing this process ($\gamma^* p \rightarrow p$) as the "reference process". Instead, we shall follow Ref. 16), and use the exclusive charmonium decay $\psi \rightarrow p\bar{p}$

to provide us with information⁹⁾ about the distribution amplitude $\phi(x, Q^2)$ at fixed $Q^2 \approx 9.6 \text{ GeV}^2$ ^{*}). This furthermore has the advantage that the logarithmic evolution of $\phi(x, Q^2)$ with Q^2 [as dictated by the evolution equation (2.4)] can be ignored as far as other charmonium resonances are concerned, since $Q_\psi^2 = Q_\chi^2$.

According to Ref. 9), the branching ratio for $\psi \rightarrow p\bar{p}$ can be expressed as^{**)}

$$\frac{\Gamma(\psi \rightarrow 3g \rightarrow p\bar{p})}{\Gamma(\psi \rightarrow 3g)} = 3.2 \cdot 10^6 \alpha_s(Q^2)^3 \frac{|\vec{p}_{c.m.}|}{\sqrt{Q^2}} \frac{\langle T \rangle^2}{(Q^2)^4} \quad (3.9)$$

where, near threshold, $|\vec{p}_{c.m.}|/\sqrt{Q^2} \approx 0.4$ and $Q^2 \approx 9.6 \text{ GeV}^2$. Here, the short-distance decay amplitude is to lowest order

$$\langle T \rangle = \int_0^1 [dx] [dy] \frac{\phi^*(y, Q^2)}{y_1 y_2 y_3} \frac{x_1 y_3 + x_3 y_1}{[x_1(1-y_1) + y_1(1-x_1)][x_3(1-y_3) + y_3(1-x_3)]} \frac{\phi(x, Q^2)}{x_1 x_2 x_3} \quad (3.10)$$

Therefore, in order to obtain the absolute normalization of $\phi(x, Q^2)$ we evaluate Eq. (3.9) for a particular choice of $\phi(x, Q^2)$ and compare it to the experimental average of $0.22 \pm 0.02\%$. Note that in doing so we must be careful in distinguishing $\psi \rightarrow 3g$ from $\psi \rightarrow \text{all}$. In fact, at $Q^2 \approx 10 \text{ GeV}^2$, $\Gamma(\psi \rightarrow 3g) \approx 0.7\Gamma(\psi \rightarrow \text{all})$.

Knowing that as $Q^2 \rightarrow \infty$, $\phi(x, Q^2) \sim x_1 x_2 x_3$ and taking into account that a simple equipartition of longitudinal momentum yields $\phi(x, Q^2) \sim \delta(x_1 - 1/3)\delta(x_2 - 1/3)\delta(x_3 - 1/3)$ we choose as a convenient parametrization of $\phi(x, Q^2)$

$$\phi(x, Q^2) = C(x_1 x_2 x_3)^{\frac{1}{2}} \quad (3.11)$$

This parametrization has several appealing features. It is totally symmetric in the x_i 's, and it interpolates smoothly between the two extremes mentioned above: at $\eta = 1$ we obtain the asymptotic distribution amplitude, and as $\eta \rightarrow \infty$ we recover the δ -function parametrization.

*) Note that had we instead used the proton's electromagnetic form factor to normalize our amplitudes, we would have been able to give simultaneous predictions for $\Gamma(\psi \rightarrow p\bar{p})$ and $\Gamma(\chi_2 \rightarrow p\bar{p})$.

**) We ignore isospin violating contributions through the process $\psi \rightarrow \gamma^* \rightarrow p\bar{p}$ (10).

For reliable predictions one would hope for relative insensitivity to η , and fortunately this appears to be the case. In Fig. 5 we show the ratio $R = \Gamma(\chi_2 \rightarrow p\bar{p})/\Gamma(\phi \rightarrow p\bar{p})$ as a function of η ^{*}). Despite almost ten orders-of-magnitude of difference in the absolute values of each Γ between $\eta = 1$ and $\eta = 4$, the ratio turns out to be very close to constant. The region $\eta \lesssim 0.8$ is a region where potential endpoint singularities could be important, thus invalidating the standard factorization procedure. Such endpoint singularities have already been observed in form factor calculations ²⁾, and it is quite possible that they will appear in other processes as well.

Averaging over the $(2J+1) = 5$ initial states, and summing over the two-final states, we obtain a decay rate

$$\Gamma(\chi_2 \rightarrow p\bar{p}) = \frac{2}{5} \frac{|\mathcal{T}|^2}{32\pi M} \quad (3.12)$$

Finally, using the known result ¹²⁾ for $\Gamma(\chi_2 \rightarrow 2g)$,

$$\Gamma(\chi_2 \rightarrow 2g) = \frac{1}{5} \frac{8\alpha_s(Q^2)^2}{M^4} |R'_p(0)|^2 \quad (3.13)$$

and $\alpha_s(Q^2) = 0.2$, we derive

$$\begin{aligned} \frac{\Gamma(\chi_2 \rightarrow p\bar{p})}{\Gamma(\chi_2 \rightarrow 2g)} &= 1.2 \cdot 10^{-4}, \quad \eta = 1 \\ \frac{\Gamma(\chi_2 \rightarrow p\bar{p})}{\Gamma(\chi_2 \rightarrow 2g)} &= 0.8 \cdot 10^{-4}, \quad \eta = 4 \end{aligned} \quad (3.14)$$

Theoretical uncertainty arises not only from lack of knowledge about $\phi(x_1 x_2 x_3)$ (here parametrized by η), but also from the value used for $\alpha_s(Q^2)$. There are in fact two uncertainties: the error in the experimentally determined value for Λ_{QCD} , and the choice of Q^2 used in evaluating $\alpha_s(Q^2)$. For fixed $Q^2 = 10 \text{ GeV}^2$, we can estimate the uncertainty in $\alpha_s(Q^2)$ by choosing different values of Λ_{QCD} : (a) $\Lambda = 0.1 \text{ GeV}$, (b) $\Lambda = 0.2 \text{ GeV}$ and (c) $\Lambda = 0.3 \text{ GeV}$. At $\eta = 1$ the branching ratio (3.14) becomes

^{*}) We have taken out all irrelevant helicity factors, etc., and plot only the bare integrals.

$$\begin{aligned} (a) \quad \text{Br}(\chi_2 \rightarrow p\bar{p}) &= 1.28 \cdot 10^{-4} \\ (b) \quad \text{Br}(\chi_2 \rightarrow p\bar{p}) &= 1.61 \cdot 10^{-4} \\ (c) \quad \text{Br}(\chi_2 \rightarrow p\bar{p}) &= 1.88 \cdot 10^{-4} \end{aligned}$$

(3.15)

Summing up the uncertainties involved in this lowest order QCD perturbation calculation, we conclude that perturbative QCD predicts a branching ratio for $\chi_2 = p\bar{p}$ in the range of 1 to 2×10^{-4} .

3.3. Cross-Sections and Angular Distributions

The results for the hard scattering matrix elements can be used to compute the magnitude of the total cross-section of heavy quark resonance production. In this subsection we shall briefly discuss how this total cross-section is obtained, and how perturbative QCD can be used to give predictions for the angular distribution of various exclusive differential cross-sections. Finally, at the end of this section we shall comment on the question of heavy onium resonance production involving quarks heavier than the c quark.

For the total cross-section of $p\bar{p} \rightarrow \chi_2$ we use the Breit-Wigner distribution formula

$$\sigma = \frac{4\pi(2J+1)}{4M_\chi^2} \frac{\Gamma_{p\bar{p}} \Gamma_f}{(\sqrt{s} - M_\chi)^2 + \frac{1}{4}\Gamma^2} \quad (3.16)$$

where $\Gamma_f = \Gamma(\chi_2 \rightarrow \text{observed final state})$, $\Gamma = \Gamma(\chi_2 \rightarrow \text{all})$ and we have averaged over the initial $p, \bar{p}$ spins. The total width of the χ_2 is not very well known, but independent measurements from unpublished SLAC ball data as well as more recent ISR data¹⁷⁾ place the total width between 2 MeV and 3 MeV. With $\Gamma_{p\bar{p}}$ between $0.8 \cdot 10^{-4}\Gamma$ and $1.8 \cdot 10^{-4}\Gamma$ all parameters in the cross-section (3.16) are fixed, and the predicted cross-section can be compared with experiment. The data from the ISR experiment on $p\bar{p} \rightarrow \text{charmonium resonances}$ appear to be in reasonable agreement with predictions¹⁷⁾.

The same helicity selection rules which helped us determine (ideally) which heavy onium states could be produced in $p\bar{p}$ collisions can also be used to provide

useful information about angular distributions in the decay. As an example, consider a χ_2 ($J=2$) state prepared in a pure state of angular momentum projection M . The angular distribution of, for example, the decay $\chi_2 \rightarrow p\bar{p}$ will, due to the helicity conserving interactions, be governed solely by the rotation matrix elements $d_{M,\pm 1}^2$ which read:

$$\begin{aligned}
 M=2 \quad & \begin{cases} d_{2,1}^2 = -\frac{1}{2}(1+\cos\theta)\sin\theta \\ d_{2,-1}^2 = -\frac{1}{2}(1-\cos\theta)\sin\theta \end{cases} \\
 M=1 \quad & \begin{cases} d_{1,1}^2 = \frac{1}{2}(1+\cos\theta)(2\cos\theta-1) \\ d_{1,-1}^2 = \frac{1}{2}(1-\cos\theta)(2\cos\theta+1) \end{cases} \\
 M=0 \quad & \begin{cases} d_{0,1}^2 = \sqrt{\frac{3}{2}}\sin\theta\cos\theta \\ d_{0,-1}^2 = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta \end{cases}
 \end{aligned} \tag{3.17}$$

with θ denoting the angle with respect to the axis along which M is quantized.

For example, if the χ_2 state is prepared with $M = 1$, its subsequent decay $\chi_2 \rightarrow p\bar{p}$ will have an angular distribution given by

$$\begin{aligned}
 W(\theta) & \propto |d_{1,1}^2|^2 + |d_{1,-1}^2|^2 \\
 & = \frac{1}{2}(1+\cos^2\theta) - 2\cos^2\theta\sin^2\theta
 \end{aligned} \tag{3.18}$$

All other distributions can be obtained in a similar way: the only requirement is a knowledge of the initial density matrix of the χ_2 and the desired final state.

We close this section with an estimate of the production cross-section for $p\bar{p} \rightarrow b\bar{b}$ resonances. Ignoring the logarithmic Q^2 evolution of the baryonic distribution amplitudes $\phi(x_1 x_2 x_3)$ as well as the logarithmic Q^2 dependence of the coupling constant $\alpha_s(Q^2)$ we see from Eqs. (3.1), (3.12) and (3.13) that the branching ratio for $[\text{heavy quark resonance}]_{J=2} \rightarrow p\bar{p}$ scales as M^{-8} . Using $M_b \approx 5$ GeV and $M_c \approx 1.5$ GeV, we predict a branching ratio for the 3P_2 $b\bar{b}$ state into an exclusive $p\bar{p}$ final state of roughly $(M_c/M_b)^8 \times 1.5 \cdot 10^{-4} \approx 10^{-8}$. This

exceedingly small branching ratio will be difficult to verify experimentally. The total resonance production cross-section for the 3P_2 $b\bar{b}$ state will be suppressed by a similar factor, as can be seen from Eq. (3.16).

4. - DISCUSSION

As we have demonstrated, perturbative QCD can be applied to a wide set of exclusive processes involving heavy quarkonium states. With a few assumptions, absolute predictions can be obtained for branching ratios for exclusive baryon-antibaryon decays, as well as for the magnitude of annihilation cross-sections. In this paper, for reasons of comparison with experimental data, we have focused mainly on charmonium states and their coupling to proton-antiproton states. This involves applying perturbative QCD in the Q^2 range of $Q^2 \approx 10 \text{ GeV}^2$, and it is perhaps not a priori obvious that this is justified, especially when the baryons have masses near 1 GeV. However, other implicit assumptions and approximations were used in the calculation, and an assessment of the reliability of this calculation is called for. Let us consider two of the approximations.

First, the heavy meson was treated in the non-relativistic limit in which v^2/c^2 corrections are neglected. Although the success of quarkonium potential models/spectroscopy may justify this approximation even in the case of charmed mesons, one might worry that a significant bias has been introduced. Furthermore, the heavy meson was pictured as consisting of simply a bare quark-antiquark pair, with the binding energy ignored. Is this a valid approximation? One way to gain insight into these questions is to compare¹⁸⁾ theoretical predictions for the ratio of total hadronic decay widths of, say, $\Gamma(\chi_0 \rightarrow 2g)$ and $\Gamma(\chi_2 \rightarrow 2g)$. For this ratio, again, factors involving $|R'(0)|$ cancel out, and one has $\Gamma(\chi_0 \rightarrow 2g)/\Gamma(\chi_2 \rightarrow 2g) = (15/4)(1+12\alpha_s(Q^2)/\pi+\dots)$ including the $O(\alpha_s(Q^2))$ corrections¹³⁾. For $\alpha_s(Q^2) = 0.2$ this ratio yields 6.6, which is to be compared with the experimental average¹⁹⁾ of $7.1^{+3.6}_{-1.8}$. This agreement between theoretical expectations and data tends to make one confident in the non-relativistic approximation involved.

Concerning the initial/final state baryon-antibaryon system, it is also worthwhile to reconsider the approximations we introduced. In the factorization procedure described in Section 2, we separated various terms that scale as m^2/Q^2 , m being a "typical hadronic scale". (Thus m^2 here generically represents

all light quark masses, the average $\langle k_{\perp}^2 \rangle$ of Fermi motion within the baryon, and terms arising from higher Fock states in the baryon wave function.) All of these terms can in principle be taken into account in a systematic way. It is not difficult to carry the factorization procedure through even in the case of (light) massive quarks, but the inclusion of only these terms is not meaningful. All terms of order m^2/Q^2 must be included, including higher twist contributions (which effectively correspond to higher Fock states in the baryon wave function). Including such terms is a laborious and difficult task, and we have not attempted to do so. Are such terms expected to be important? For the charmonium states, unfortunately, the answer seems to be yes. (It is very difficult to see how such terms could be of relevance for the corresponding $b\bar{b}$ decays, however.) We need only examine the associated helicity conservation laws - in the massless limit - to convince ourselves that there are difficulties. As discussed in Section 2, these helicity selection rules state that the coupling to $p\bar{p}$ states can only proceed via states of helicity $\lambda - \bar{\lambda} = \pm 1$. In particular, the η_c and the χ_0 state should not be able to decay into a single $p\bar{p}$ pair. This contradicts experimental results¹⁷⁾, which show that the η_c decays into a $p\bar{p}$ final state at a rate comparable in magnitude to that of $\Gamma(\chi_2 \rightarrow p\bar{p})$. This issue has recently been discussed by Martin, Olsson and Stirling²⁰⁾.

For the charge-conjugate odd states, such as the ψ , the situation is further complicated by a possible large electromagnetic contribution to $\psi \rightarrow p\bar{p}$ from the proton's electromagnetic form factor, a point which has been examined by Claudson, Glashow and Wise¹⁰⁾. Fortunately, such electromagnetic contributions cannot play any rôle for the J^{P+} states we have considered in this paper, due to the extra suppression factor of $\alpha_{EM}^2 \approx 10^{-4}$ for such states.

The main problem with our analysis for the charmonium states hence appears to be the neglect of $1/Q^2$ corrections. This is an interesting and perhaps somewhat surprising result, since at the charmonium states one is working at a Q^2 of roughly 10 GeV^2 . The extra m^2/Q^2 terms are not expected to play an important rôle for the heavy meson states around the T , for which our predictions therefore should be more reliable. Here the main problem will be experimental: measuring branching ratios of exclusive final states at the level of 10^{-8} . Among the reasons to believe that our predictions will be reliable for the $b\bar{b}$ resonances is the observed flatness of the curve in Fig. 5. This insensitivity to the parameter η makes us confident that the numbers we can quote are model-independent. Higher order corrections of order $\alpha_s(Q^2)^4$, or more, should not distort this picture.

While this study was nearing completion we became aware of two earlier related studies^{21),22)}. Our analytic result for $\Gamma(\chi_2 \rightarrow p\bar{p})$, evaluated in the limit $\eta \rightarrow 1$ in which comparison is possible, differs slightly from that of Ref. 21). Our numerical values are, however, in rough agreement with this reference, in the limit mentioned.

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APPENDIX

In this Appendix we will collect a set of explicit spinor representations, as well as some useful identities involving them. Our conventions follow those of Bjorken and Drell²³⁾, apart from the standard exception of spinor normalization. Our spinors are normalized such that

$$\bar{u}_{\uparrow\downarrow}(p)u_{\uparrow\downarrow}(p) = 2m, \quad \bar{v}_{\uparrow\downarrow}(p)v_{\uparrow\downarrow}(p) = -2m \quad (\text{A.1})$$

For massless fermions, where $\bar{u}(p)$ and $u(p)$ are orthogonal, the normalization is fixed by

$$u_{\uparrow\downarrow}^\dagger(p)u_{\uparrow\downarrow}(p) = 2p^0, \quad v_{\uparrow\downarrow}^\dagger(p)v_{\uparrow\downarrow}(p) = -2p^0 \quad (\text{A.2})$$

The explicit spinor representations we will construct now are related to, but differ from, the spinor representations of Appendix A in the last reference of 2)^{*}).

We first introduce the two helicity projection operators,

$$\Lambda_+ \equiv \frac{1}{2}(1 + \gamma^5), \quad \Lambda_- \equiv \frac{1}{2}(1 - \gamma^5)$$

Now, for eigenvectors

$$\Lambda_+ \psi^+ = \psi^+$$

choose

$$\chi(1) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad \xi(1) = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

and for eigenvectors

$$\Lambda_- \psi^- = \psi^-$$

^{*}) We thank Peter Lepage for discussions about this.

choose

$$\xi(\uparrow) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \quad \chi(\downarrow) = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}$$

Note that these "basis spinors" are spin eigenstates, too:

$$\begin{aligned} \sigma^3 \chi(\uparrow) &= \chi(\uparrow) & \sigma^3 \xi(\uparrow) &= \xi(\uparrow) \\ \sigma^3 \chi(\downarrow) &= -\chi(\downarrow) & \sigma^3 \xi(\downarrow) &= -\xi(\downarrow) \end{aligned} \quad (\text{A.3})$$

Furthermore

$$\begin{aligned} \Lambda_+ \xi(\uparrow) &= \Lambda_- \xi(\downarrow) = 0 \\ \Lambda_+ \chi(\downarrow) &= \Lambda_- \chi(\uparrow) = 0 \end{aligned} \quad (\text{A.4})$$

and we have the following orthogonality properties:

$$\begin{aligned} \xi^\dagger(\uparrow) \chi(\uparrow) &= \xi^\dagger(\uparrow) \chi(\downarrow) = 0 \\ \xi^\dagger(\downarrow) \chi(\uparrow) &= \xi^\dagger(\downarrow) \chi(\downarrow) = 0 \end{aligned} \quad (\text{A.5})$$

Also, as

$$\begin{aligned} \gamma^0 \chi(\uparrow) &= \xi(\uparrow) \\ \gamma^0 \chi(\downarrow) &= \xi(\downarrow) \end{aligned} \quad (\text{A.6})$$

we have, for example, $\bar{\xi}(\uparrow) \chi(\uparrow) = 1$.

In this basis we can now introduce a set of normalized spinors:

$$u_\uparrow(p) = (p^+)^{-1/2} (\not{p} + m) \xi(\uparrow) \quad (\text{A.7})$$

$$\bar{u}_\uparrow(p) = (p^+)^{-1/2} \chi^\dagger(\uparrow) (\not{p} + m) \quad (\text{A.8})$$

$$u_\downarrow(p) = (p^+)^{-1/2} (\not{p} + m) \xi(\downarrow) \quad (\text{A.9})$$

$$\bar{u}_\downarrow(p) = (p^+)^{-1/2} \chi^\dagger(\downarrow) (\not{p} + m) \quad (\text{A.10})$$

For antispinors use

$$V_{\uparrow}(p) = (p^+)^{-1/2} (-\not{p} + m) \zeta(\downarrow) \quad (\text{A.11})$$

$$\bar{V}_{\uparrow}(p) = (p^+)^{-1/2} \chi^{\dagger}(\downarrow) (-\not{p} + m) \quad (\text{A.12})$$

$$V_{\downarrow}(p) = (p^+)^{-1/2} (-\not{p} + m) \zeta(\uparrow) \quad (\text{A.13})$$

$$\bar{V}_{\downarrow}(p) = (p^+)^{-1/2} \chi^{\dagger}(\uparrow) (-\not{p} + m) \quad (\text{A.14})$$

Note that these spinors trivially satisfy the Dirac equation. Also note that all spinors become eigenstates of $\frac{1}{2}(1 \pm \gamma^5)$ in the chiral limit $m \rightarrow 0$.

A special problem arises when $p^+ = p^0 + p^3$ vanishes, in which case the above representation becomes meaningless. In that case one can use a " p^- basis" ($p^- = p^0 - p^3$):

$$u_{\uparrow}(p) = (p^-)^{-1/2} (\not{p} + m) \chi(\uparrow) \quad (\text{A.15})$$

$$\bar{u}_{\uparrow}(p) = (p^-)^{-1/2} \bar{\chi}^{\dagger}(\uparrow) (\not{p} + m) \quad (\text{A.16})$$

$$v_{\uparrow}(p) = (p^-)^{-1/2} (-\not{p} + m) \chi(\downarrow) \quad (\text{A.17})$$

$$\bar{v}_{\uparrow}(p) = (p^-)^{-1/2} \bar{\chi}^{\dagger}(\downarrow) (-\not{p} + m) \quad (\text{A.18})$$

- and similarly for "down" spinors.

When computing matrix elements the following identities are useful:

$$\begin{aligned}\gamma^1 \chi(\uparrow) &= \chi(\downarrow), & \gamma^1 \chi(\downarrow) &= -\chi(\uparrow) \\ \gamma^2 \chi(\uparrow) &= i \chi(\downarrow), & \gamma^2 \chi(\downarrow) &= i \chi(\uparrow) \\ \gamma^0 \gamma^3 \chi(\uparrow) &= \chi(\uparrow), & \gamma^0 \gamma^3 \chi(\downarrow) &= \chi(\downarrow)\end{aligned}\tag{A.19}$$

Similar identities can be worked out for the ξ -spinors.

If in-going and out-going fermion lines are joined into traces, the following outer products are useful:

$$\chi(\uparrow) \chi^\dagger(\uparrow) = \frac{1}{4} (1 + \gamma^5) (1 + \gamma^0 \gamma^3)\tag{A.20}$$

$$\chi(\downarrow) \chi^\dagger(\downarrow) = \frac{1}{4} (1 - \gamma^5) (1 + \gamma^0 \gamma^3)\tag{A.21}$$

$$\chi(\uparrow) \xi^\dagger(\uparrow) = \frac{1}{4} (1 + \gamma^5) (\gamma^0 - \gamma^3)\tag{A.22}$$

$$\chi(\downarrow) \xi^\dagger(\downarrow) = \frac{1}{4} (1 - \gamma^5) (\gamma^0 - \gamma^3)\tag{A.23}$$

$$\xi(\uparrow) \chi^\dagger(\uparrow) = \frac{1}{4} (1 - \gamma^5) (\gamma^0 + \gamma^3)\tag{A.24}$$

$$\xi(\downarrow) \chi^\dagger(\downarrow) = \frac{1}{4} (1 + \gamma^5) (\gamma^0 + \gamma^3)\tag{A.25}$$

$$\chi(\uparrow) \xi^\dagger(\downarrow) = \frac{1}{4} (1 + \gamma^5) \gamma^0 (i\gamma^2 - \gamma^5 \gamma^1)\tag{A.26}$$

$$\chi(\downarrow) \xi^\dagger(\uparrow) = \frac{1}{4} (1 - \gamma^5) \gamma^0 (i\gamma^2 - \gamma^5 \gamma^1)\tag{A.27}$$

Although this list is not exhaustive, the Dirac traces can usually be arranged so that no other outer products appear.

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FIGURE CAPTIONS

- Fig. 1 The bound state equation for the wave function $\phi(x, k_{\perp})$ (a). The kernel K is built out of two-particle irreducible (2PI) graphs and has a perturbative expansion as shown. When high Q^2 flows through the wave function (b), the bound state equation factorizes and one obtains an evolution equation as described in the text.
- Fig. 2 When the χ_2 state decays strongly, it does so through a two-gluon intermediate state to lowest order in $\alpha_s(Q^2)$.
- Fig. 3 The exclusive decay $\chi_2 \rightarrow p\bar{p}$ (a). The short-distance amplitude T_H can be expanded in a perturbation series (b).
- Fig. 4 All diagrams contributing to T_H in lowest order. The amplitude for each individual diagram in the Feynman gauge is listed in Section 3. The spin orientation labels in diagram A apply to all six diagrams A-F.
- Fig. 5 The ratio $R = T(\chi_2 \rightarrow p\bar{p})/T(\psi \rightarrow p\bar{p})$ normalized to unity at the asymptotic distribution amplitude ($\eta=1$). The hatched area shows the region which possibly is forbidden due to singular endpoint behaviour.

- Figure 1 -

- Figure 2 -

- Figure 3 -

- Figure 4 -

- Figure 5 -