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A METHOD FOR ESTIMATING MUON PRODUCTION AND PENETRATION THROUGH A SHIELD

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S U M M A R Y

An approximate expression is derived that describes muon production by high-energy protons and the subsequent attenuation of the muons in a shield. It is shown that the muon flux at x metres ahead of an interaction by a proton of energy ϵ GeV and where pions have a path length of Δ metres in which to decay will be given by:

$$\phi = 8.5 \times 10^{-2} \frac{\epsilon \Delta}{x^2} e^{-(\alpha t / \epsilon)} \mu/m^2$$

where t is the shield thickness in metres and α is an effective muon energy loss rate which has a value of 22 GeV/m for iron and 7.8 GeV/m for concrete. It is further shown that the effective muon attenuation mean-free path is equivalent to 1/16 of the range of a muon with the energy of the interacting proton.

The width of the muon beam that passes through the shield is also considered and it is shown that the beam profile approximates to a gaussian distribution with a diameter at half maximum intensity of:

$$d = \frac{4.6x}{\sqrt{\epsilon \alpha t}} \text{ metres}$$

Calculated muon fluxes are shown to reasonably correspond with those obtained by more sophisticated computer methods for proton energies up to at least 30 GeV and over the entire range of shield thicknesses of interest for radiation safety. Results of measurements of muon levels behind beam dumps under various conditions are presented and are shown to be in reasonable agreement with predictions based on the above model.

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1. INTRODUCTION

High-energy muons have a much greater penetrating power than do protons, neutrons, and other hadrons and hence have to be given special consideration when determining the shield in the forward direction from targets or beam dumps in high-energy particle beams. Muon production, initiated by a high-energy proton interaction, is a complicated multistage process and flux calculations involve integrals that until now have had to be treated by computer methods. For radiation safety applications this approach is not always convenient and there is a need for an approximate analytic method of calculation to enable reasonable estimates to be made of muon fluxes through shields. Such a method would also show the relation between the parameters involved in muon production and attenuation which would facilitate the appraisal of any muon problem and enable optimized shields to be designed.

2. MUON PRODUCTION

An interaction of a high-energy proton with a nucleus may produce a number of energetic pions which go on to further interact with nuclei and contribute to the secondary particle cascade associated with the primary interaction. Pions are unstable particles and a very small fraction will decay into muons (plus a neutrino) before they have time to interact. The spectral and angular distribution of pions produced in proton interactions have been computed on the basis of an interaction model and published as a spectrum atlas (1). It has been noted that the pion spectra, in the forward direction from different energy proton interactions all overlap if production cross-section per GeV of incident proton energy is plotted against pion energy expressed in units of the incident proton energy. (The spectra are in fact of the momentum distribution but the substitution of energy for momentum will not significantly affect the arguments presented here.) Furthermore the spectra show little difference whether the proton strikes beryllium or copper. Computed pion spectra, scaled as indicated above, are shown in Fig. 1 for 50 and 19.2 GeV protons on copper and for 30 GeV protons on beryllium. Also shown is an analytical "best fit" to this universal spectrum which is reasonably

valid for pion energies from 40% of the proton energy up to a supposed cut-off at 93% of the proton energy. This analytical expression - which has the important property of being subsequently integrable - expresses pion production in the forward direction per unit energy interval and solid angle by:

$$\frac{d^2 N}{dE d\Omega} = 500 \epsilon(E/\epsilon)^2 e^{-16(E/\epsilon)} \quad (1)$$

where E is the pion energy and ϵ the primary proton energy.

A fraction of these pions will decay into muons while in flight. The probability of decay for a pion of energy E GeV (strictly should be momentum) travelling a distance of Δ metres is given by:

$$P = (\Delta/55E) \quad (2)$$

For beam stoppers the effective decay path is estimated to be 1.8 times the proton mean free path (2), which is taken to be 0.36 m for iron or 0.9 m for concrete.

The muons produced in the decay of a pion of energy $E\pi$ can have any energy between $0.43 E\pi$ and $E\pi$. Hence the average number of muons per GeV interval in this energy range will be $1/0.57 E\pi$ muons per GeV per pion decay. This muon spectrum has to be summed over the pion spectrum to obtain the muon energy spectrum resulting from the proton interaction. This spectrum will be approximately given by combining eqs. (1) and (2), which, after converting to particles per square metre at a distance of x metres ahead of the site of interaction becomes:

$$\phi(E)dE = \frac{1}{x^2} \int_E^{0.93\epsilon} \frac{1}{0.57E} \cdot \frac{\Delta}{55E} \cdot 500\epsilon(E/\epsilon)^2 e^{-16(E/\epsilon)} dE \quad (3)$$

$$= \frac{1.14\Delta}{x^2} [e^{-16(E/\epsilon)} - e^{-15}] \text{ muons/m}^2/\text{GeV} \quad (4)$$

Equation (4) will grossly overestimate the low-energy part of the muon spectrum as low-energy pions are overestimated by eq. (1). Also the upper limit of integration in eq. (3) should be $2.3 E$ rather than 0.93ϵ where $E < 0.4 \epsilon$ which again will cause an over-estimation in

the low-energy part of the spectrum. However, the low-energy muons will be stopped in the shield required to absorb hadrons coming from the proton interaction and will have a minimal influence on the muon shielding required under normal circumstances.

3. MUON ATTENUATION

Muons below 100 GeV are slowed down by ionisation losses at a reasonably constant rate, and hence range can be assumed to be proportional to energy. Iron is the most frequently used shielding material for muons and for iron of density 7.2 g/cm^3 the range of a muon of energy E GeV is approximated by:

$$R = 0.72E \text{ m of iron .} \quad (5)$$

Hence for a shield of thickness t metres of iron all muons with initial energy greater than E will get through where:

$$E = 1.38t \text{ GeV .} \quad (6)$$

The muon flux through the shield per proton interaction is therefore

$$\phi_{\mu} = \int_{1.38t}^{0.93\epsilon} \phi(E) dE \text{ muons/m}^2 \quad (7)$$

Carrying out this integration with the muon spectrum given in Eq. (4) the flux per interacting proton becomes:

$$\phi_{\mu} = \frac{7.1 \times 10^{-2} \Delta}{x^2} \epsilon (e^{-22t/\epsilon} - C) \text{ muons/m}^2 \quad (8)$$

where C is a term that puts $\phi_{\mu} = 0$ when t equals the range of the highest energy muon in the spectrum. Shields that reduce muon fluxes to safe limits are normally much less than that required to range out the maximum energy muon and hence C can be ignored for practical applications. The flux given by Eq. (8) is that due to muons from positive pion decay only, and should be increased by about 20% to account for other mechanisms of muon production.

The muon flux at a distance x metres ahead of a proton interaction of ϵ GeV and with a pion decay path of Δ metres through a shield of t metres of iron therefore becomes:

$$\phi_{\mu} = \frac{8.5 \times 10^{-2} \epsilon \Delta}{x^2} e^{-22t/\epsilon} \quad \text{muons/m}^2 \quad (9)$$

The effective attenuation mean free path, (λ) , of muons from a proton interaction of energy ϵ GeV is therefore:

$$\lambda = \epsilon/22 \text{ metres of iron} . \quad (10)$$

This mean free path is proportional to the muon range per unit energy and if R is the range of a muon of energy equal to that of the interacting proton, then using Eq. (5) the effective muon attenuation mean free path becomes:

$$\lambda = R/16 . \quad (11)$$

This relation should hold for any shielding material. For practical applications it is better to express mean free path in terms of the proton energy and the shielding material used and can be expressed as:

$$\lambda = \epsilon/\alpha \text{ metres} \quad (12)$$

where α is an effective muon energy loss rate that has the estimated values as given in Table 1 for common shielding materials.

Table 1
 Muon shielding effectiveness of various materials

Material	Density g/cm ³	α GeV/m
Iron	7.2	22
Lead	11.3	29
Concrete	2.35	7.8
Heavy concrete	3.5	11.7
Earth	2.0	6.8
Water	1.0	3.9

4. THE DIMENSIONS OF THE MUON BEAM

To a first approximation the diameter of a muon beam in a shield will depend on the angular distribution of the pions emitted in the proton interaction. Inspection of pion spectra as a function of angle (1) suggests that the form of the spectra stay reasonably constant with increasing angle but with a reduced maximum energy. This limiting pion energy at an angle θ is approximately proportional to the effective proton energy ϵ_θ , needed to produce the same spectrum but in the forward direction. For the purposes of this analysis the limiting pion energy has been taken as that energy where the pion intensity has dropped to 3% of the maximum intensity in the spectrum. In Fig. 2 the ratio of primary to effective proton energy at an angle θ , ϵ/ϵ_θ , has been plotted against the square of primary proton energy times angle of emission $(\epsilon\theta)^2$ over a range of angles and of primary energies from 19.2 to 50 GeV. As can be seen this leads to an empirical straight-line relation from which it can be shown that

$$\epsilon_\theta = \frac{\epsilon}{1 + 0.13 (\epsilon\theta)^2} \quad (13)$$

up to at least $\epsilon\theta = 6$ GeV radians.

If θ is replaced by r/x , where r is the beam radius at a distance x down beam from the proton interaction, and ϵ_0 substituted for ϵ in the muon transport equation, [Eq. (9)], then the resulting muon flux will be that at a radius r in the beam at a distance x straight-ahead from the point of interaction. Further, if ϕ_0 is the muon flux on the beam axis as given by Eq. 9, then the flux at a radius r metres in the muon beam will reduce to:

$$\phi_r = \frac{\phi_0}{1 + 0.13 \left(\frac{\epsilon x}{x}\right)^2} e^{-0.13 \epsilon x t \frac{r^2}{x^2}} \quad (14)$$

The denominator in the linear term has little effect on the beam profile compared to the exponential term. Neglecting it will slightly increase the predicted beam size which is in the right direction to compensate for small angle muon scattering. The exponential term indicates a gaussian beam profile where the beam diameter at half maximum intensity, d , will be given by:

$$d = \frac{4.6x}{\sqrt{\epsilon x t}} \text{ metres} \quad (15)$$

The above expression is very approximate but will give a useful indication of the dimensions of a muon beam and how it is likely to vary through different shields. In particular the constant, derived from the empirical fit in Fig. 2, will vary slightly depending on how ϵ_0 is defined. This introduces an uncertainty in the value of d of the order of $\pm 30\%$.

The constraint that $\epsilon\theta$ should be less than 6 GeV radians in Eq. 13 leads to the requirement that the shield has to be at least two muon mean-free paths thick (as defined by Eq. 12). This requirement will always be met in conditions where the muon transport equation is valid.

The diameter of the muon beam, given by Eq. 15, applies to a shielding material with a muon energy loss rate α . It is understood that α should be replaced by the sum of α for all components in a composite shield.

5. COMPARISON WITH OTHER DATA

The derived expression for muon fluxes can be compared directly with data obtained from detailed computer calculations. Such a comparison is made in Fig. 3 which shows muon flux as a function of thickness of an iron beam dump for proton beams of energies from 10 to 50 GeV. The curves are muon fluxes calculated using Eq. 9 and the dashed lines are determined from a computer program MUSTOP (3). As can be seen, agreement is very good up to 30 GeV and over at least 6 decades of muon attenuation. At 50 GeV there appears to be a difference between the two calculations of about a factor of 5. The fact that the two curves are parallel suggest this discrepancy is in the source term, the energy dependence of which comes directly from the pion production dependence on proton energy. Hence, the difference in result between the two methods of calculation is more likely to be due to differences in the pion production models used than in the subsequent analysis, and no reason for the discrepancy is evident on the basis of the arguments presented.

Comparison of the derived expressions with experimental data will give a more realistic assessment of their predictive power. The results of a series of measurements of muon fluxes behind a beam dump with and without targets in a beam of 24 GeV protons (4) are summarized in Table 2. Also given in this table is the measured flux behind a larger dump using 26 GeV protons. In this case the beam profile was also determined giving a beam width (at half maximum) of 1.0 m to be compared with 1.1 m calculated using Eq. 15. A comparison between the measured and calculated fluxes in Table 2 gives an indication of the precision with which Eq. 9 predicts muon beam intensities under realistic accelerator conditions.

Table 2
Measured and calculated muon fluxes
behind beam dumps

Proton Energy GeV	Target	Beam dump		Muons/m ² per 10 ⁶ incident protons	
		iron m	+ concrete m	Measured	Calcul.
26	None	7.2	5.6	3	1.7
24	None	4.8	4.8	9	19
24	2 cm tungsten	4.8	4.8	19	37
24	3.5 cm tungsten	4.8	4.8	23	48

6. CONCLUSIONS

Although muon production and attenuation is a complicated process, it is shown to be possible to express levels in a conventional way with a source term and an exponential attenuation term that is applicable over the range of shield thicknesses normally used for radiation safety. In particular, it is of interest to note that the effective mean-free path of muons produced by proton beams is 1/16 of the range of a muon with the same energy as the incident proton. Although the estimation of the diameter of a muon beam is relatively crude, it serves to show how narrow muon beams result from the angular distribution of pions produced in proton interactions and gives a good indication of the lateral dimensions to be expected.

Predictions of muon levels in shields appear to reasonably coincide with those obtained from detailed computer programs and comparison with measurements show that the derived formulae are adequate to enable near optimum shields to be designed for muons from proton beams of energy up to at least 30 GeV.

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Fig. 1 Normalized pion spectra from proton interactions: (a) 50 GeV on copper, (b) 19.2 GeV on copper, (c) 30 GeV on beryllium and (d) empirical relation used to describe pion spectra.

Fig. 2 The ratio of effective to primary proton energy at an angle θ versus the square of the product of primary energy and angle. The line demonstrates the validity of the proposed relation between effective and primary energy.

Fig. 3 Comparison between calculated muon fluxes in an iron dump (solid line) and those obtained with computer program MUSTOP.